Numerical simulations

in Astronomy based on the

Particle Hydrodynamics

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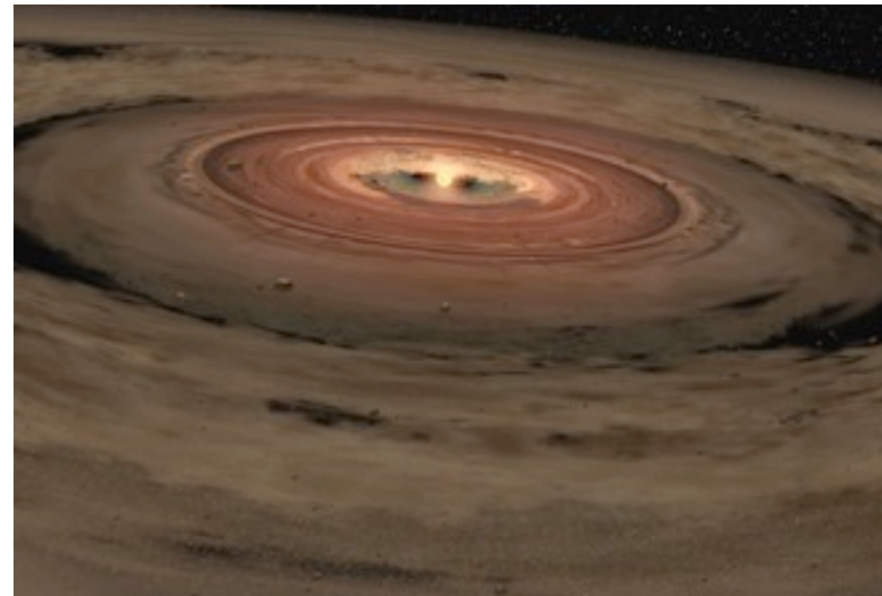
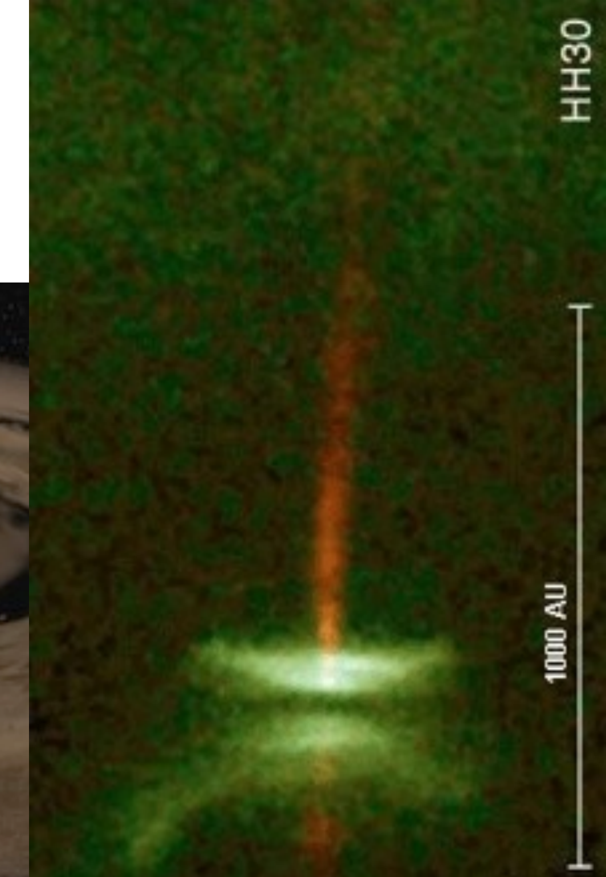
What we solve...

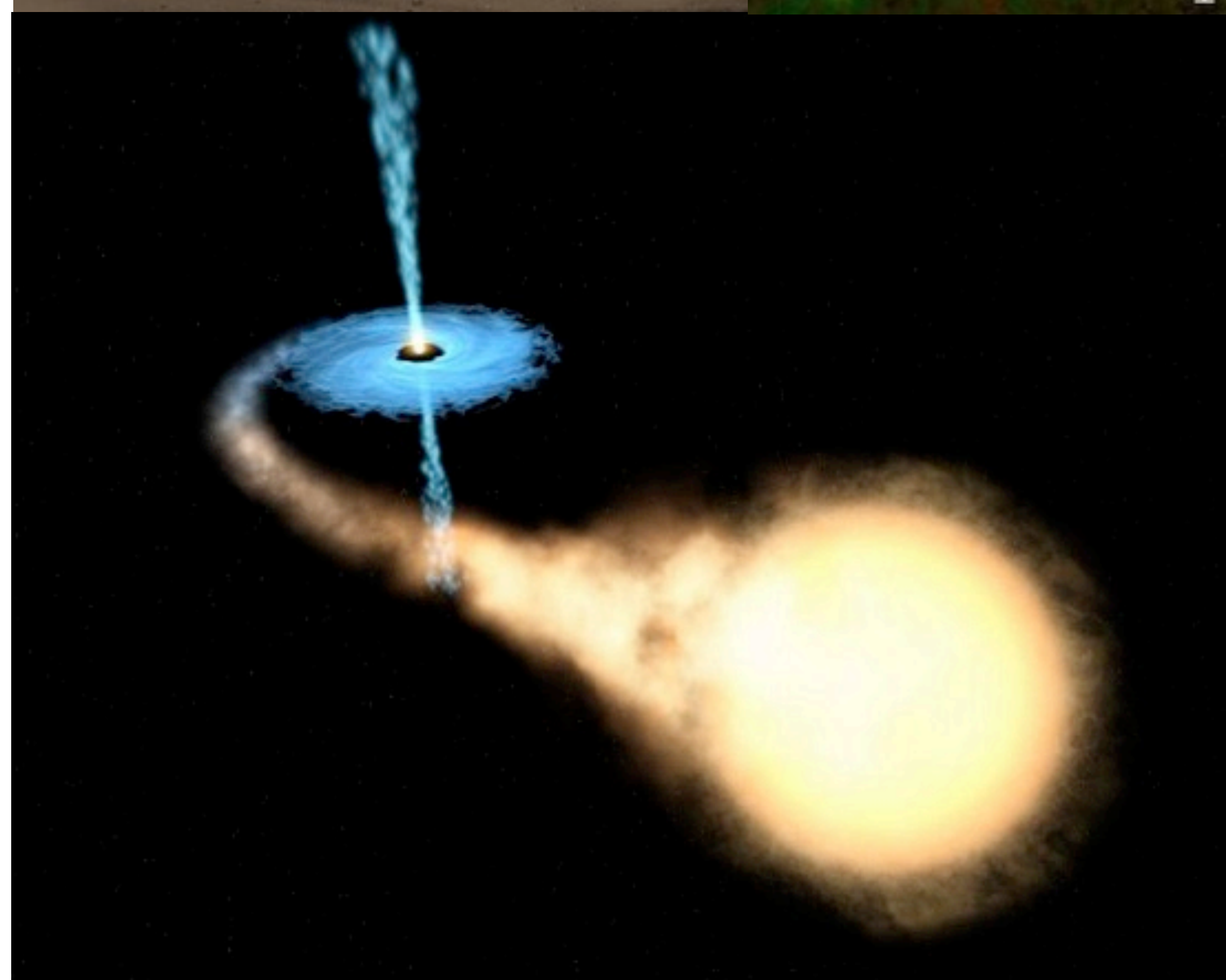




$$\frac{Dq}{Dt} = -V\boldsymbol{\nabla}S,$$

$$q = \begin{cases} V \\ \boldsymbol{v} \\ e \end{cases}, S = \begin{cases} \boldsymbol{v} \\ P \\ P\boldsymbol{v} \end{cases}$$

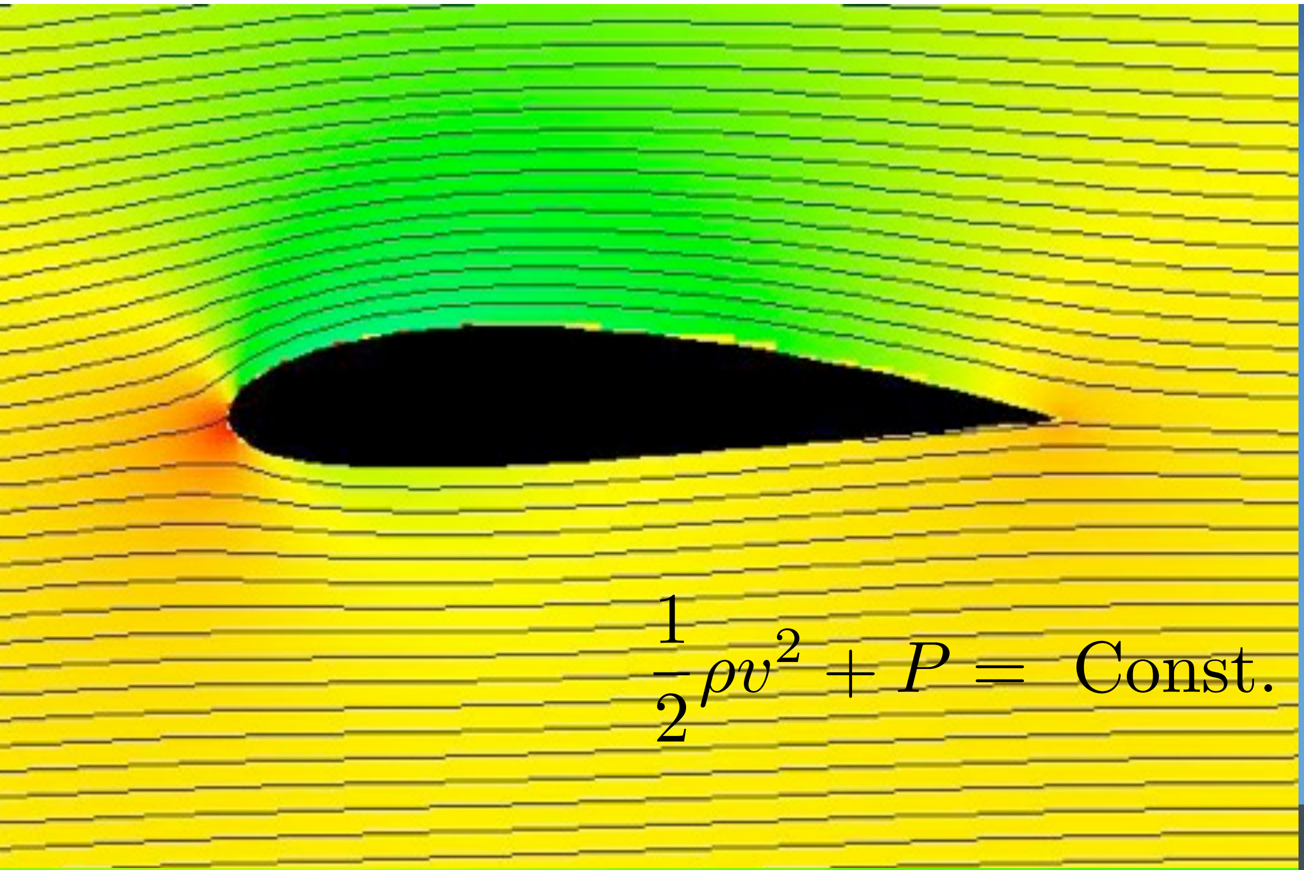
$\nabla^2\Phi = 4\pi G\rho$ and EOS.



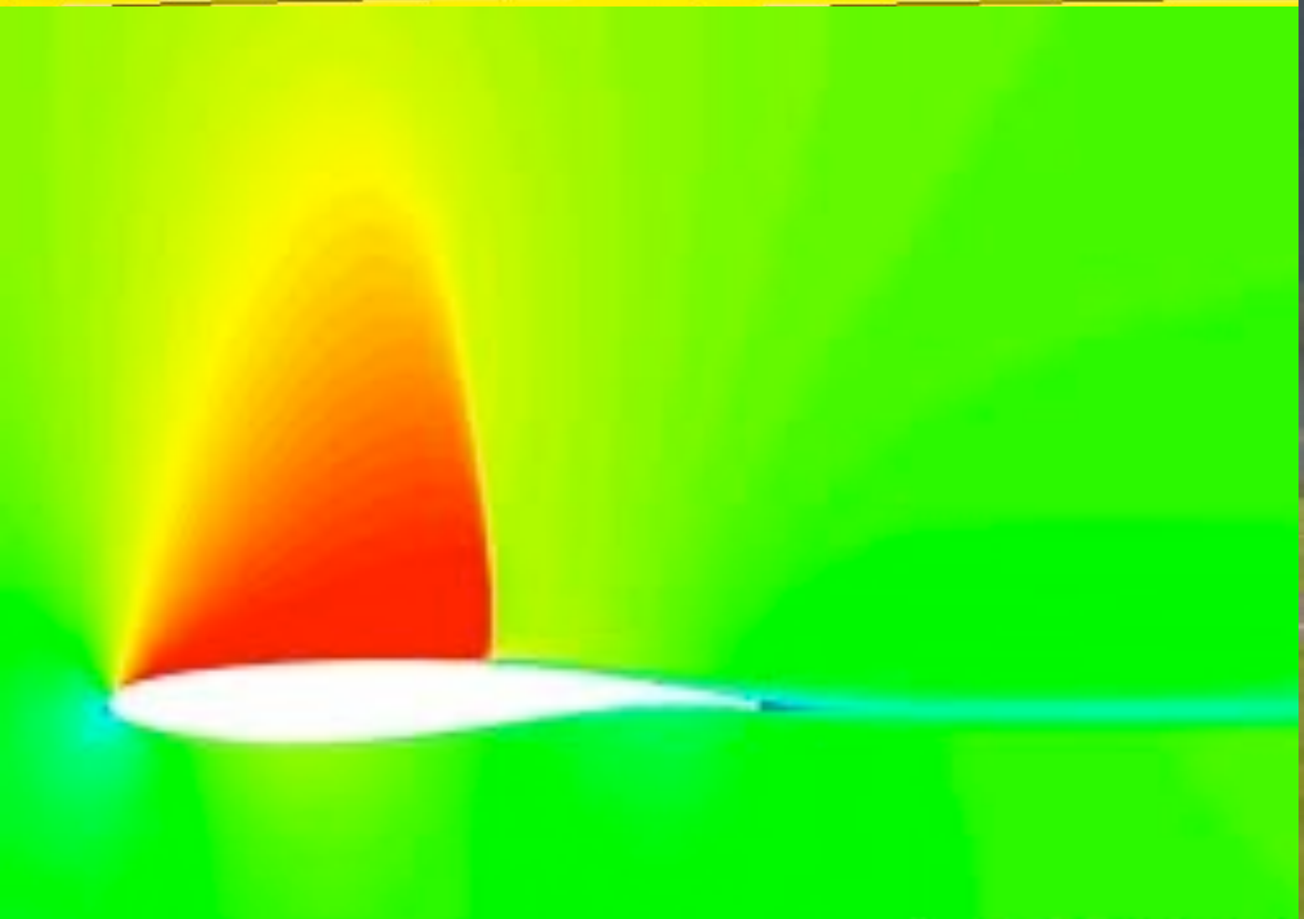
No general solution !

Transonic point

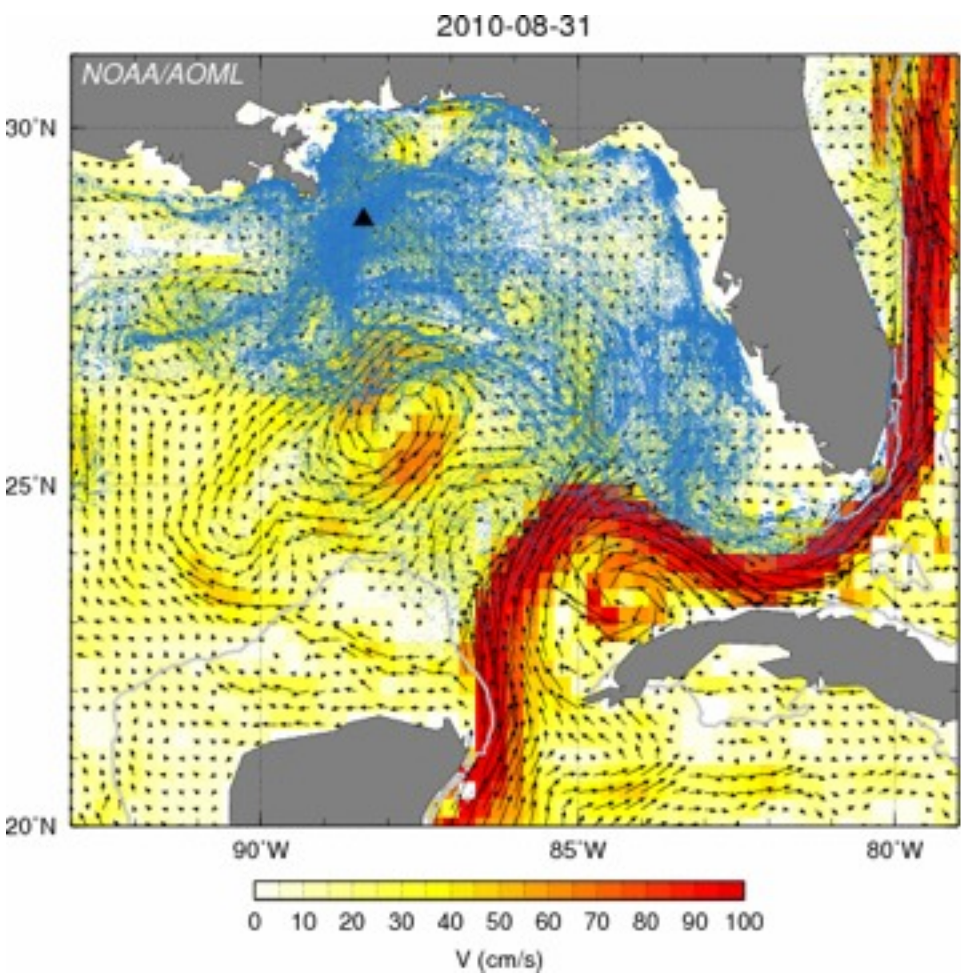
Bell X-1 exceeded
800 mph, M1.06 (1947)

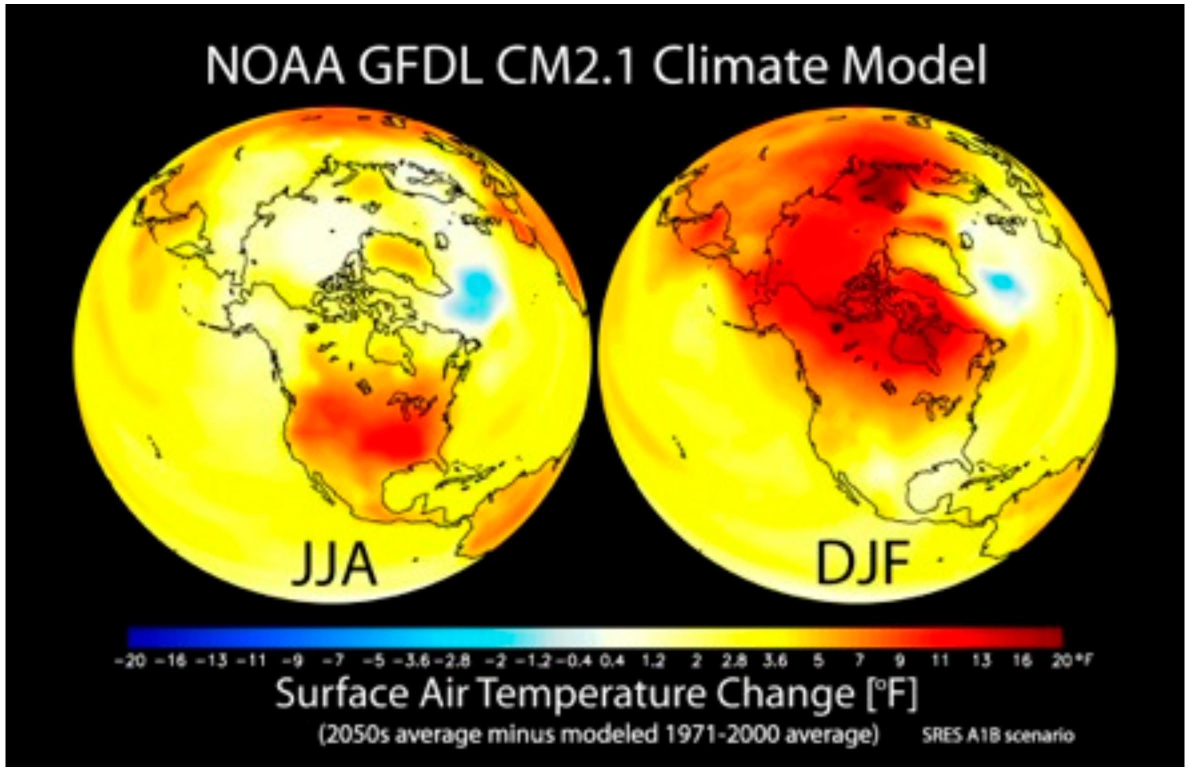


$$\frac{1}{2}\rho v^2 + P = \text{Const.}$$



All sciences use numerical simulations





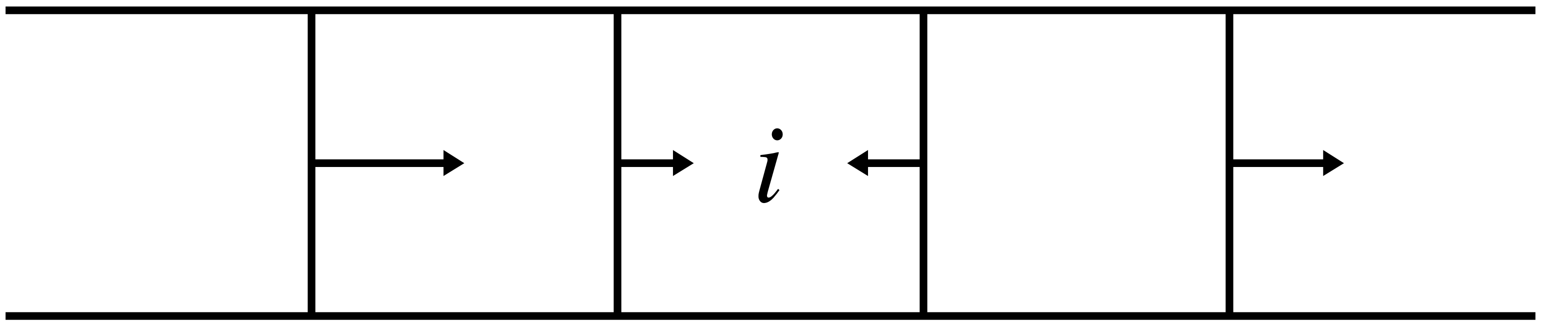
Astronomy is special in the strong shocks.

Why are the results different ?

Can I believe my simulation ?

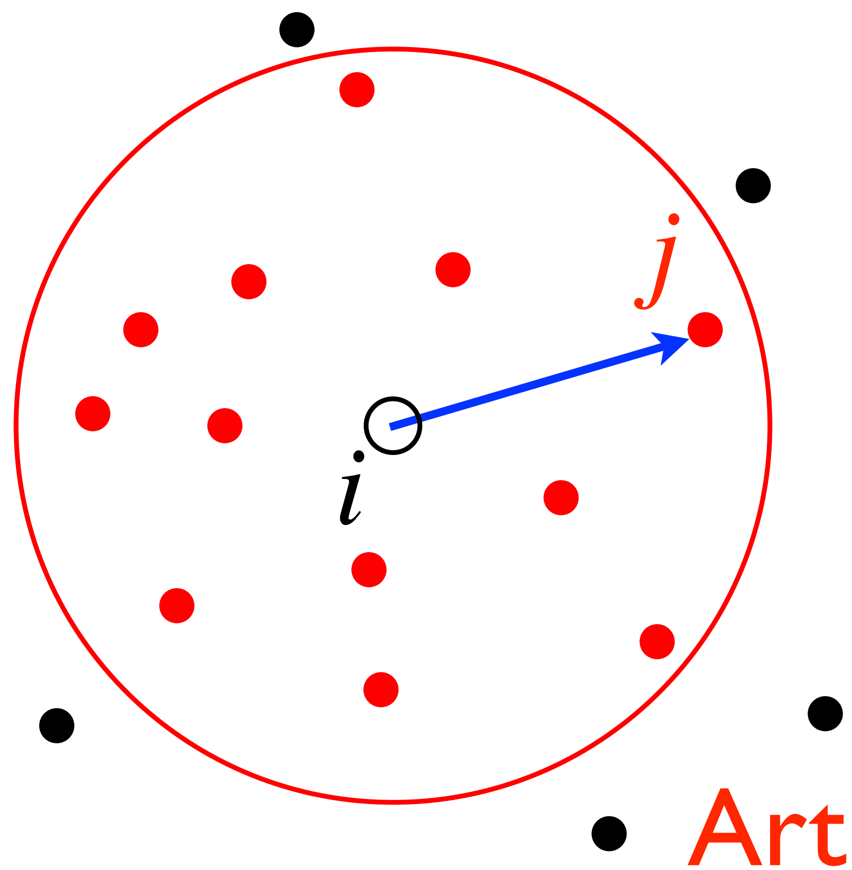
Finite Difference Method (FDM)

Grid-base code



$$\frac{\partial q_i}{\partial t} + \nabla F_i = 0$$

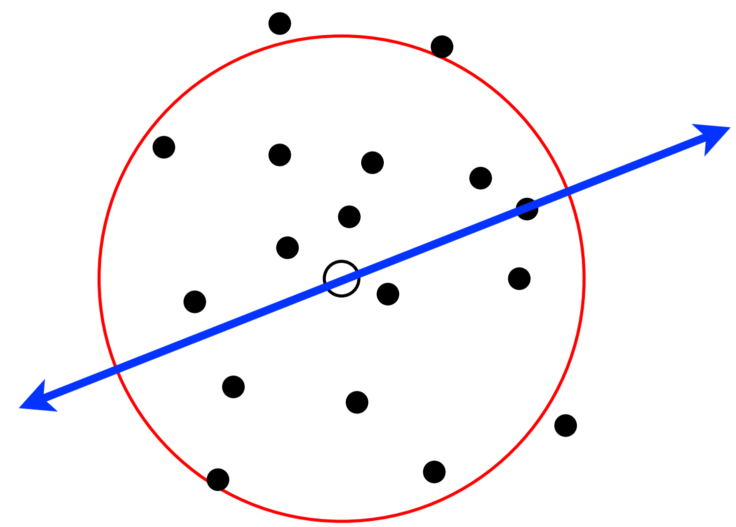
Smoothed Particle Hydrodynamics (SPH)



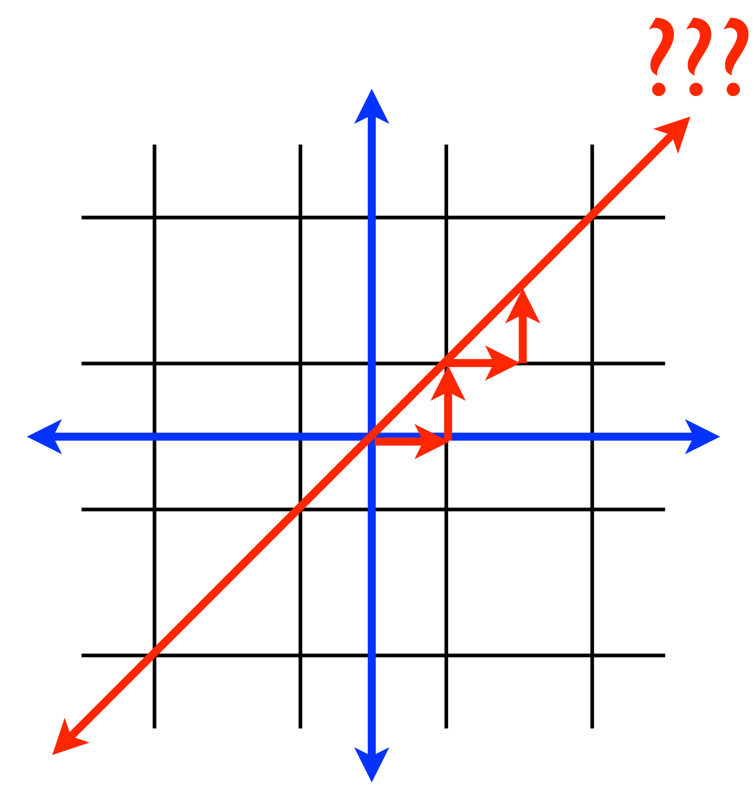
Artificial viscosity

$$\dot{\boldsymbol{v}}_i = -\sum_j m_j \left(\frac{P_i}{\rho_i} + \frac{P_j}{\rho_j} + \Pi_{ij} \right) \boldsymbol{\nabla}_i W(\boldsymbol{r}_i - \boldsymbol{r}_j, h_{ij})$$

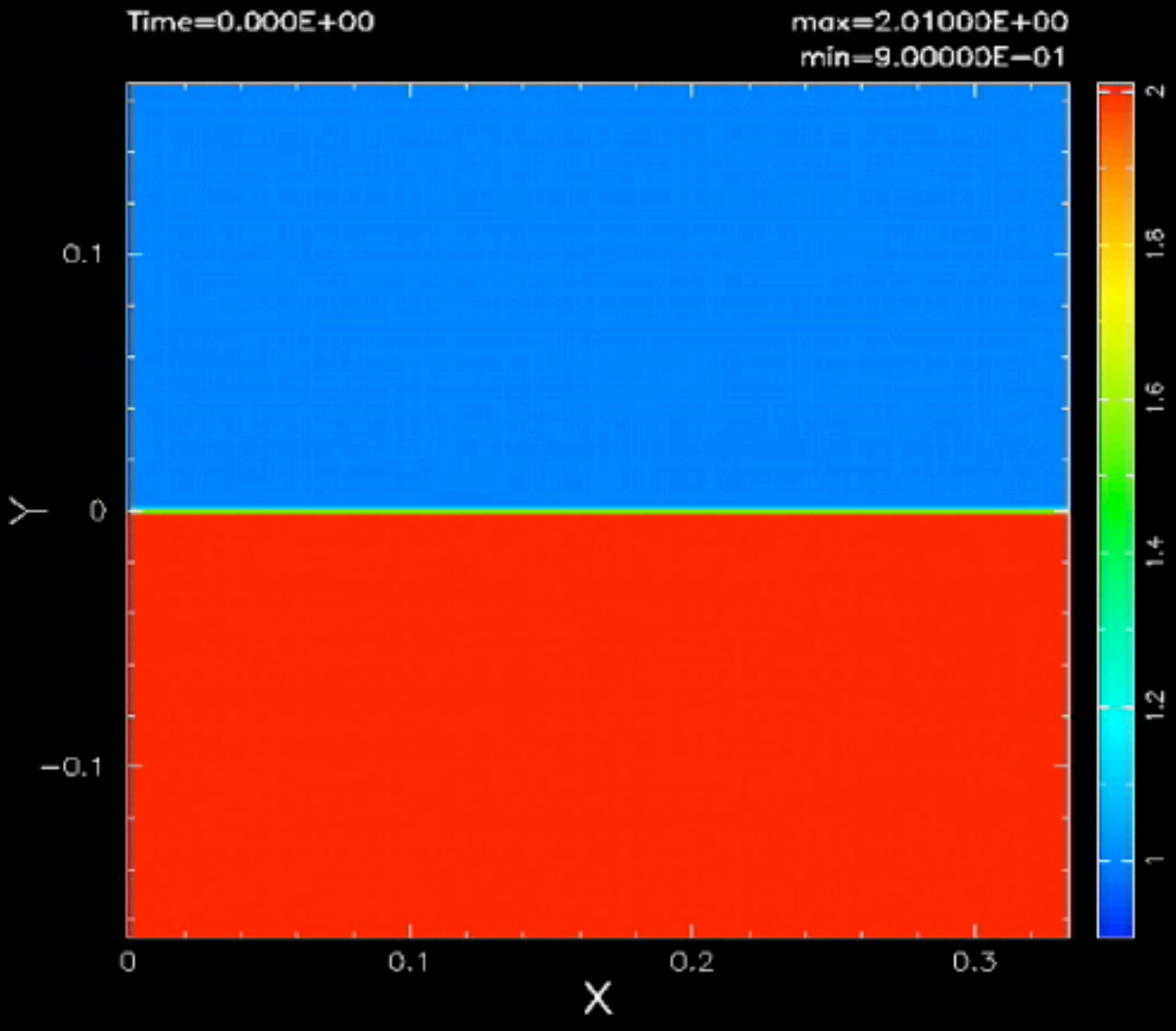
Why particle code ?
1. Multi-dimension

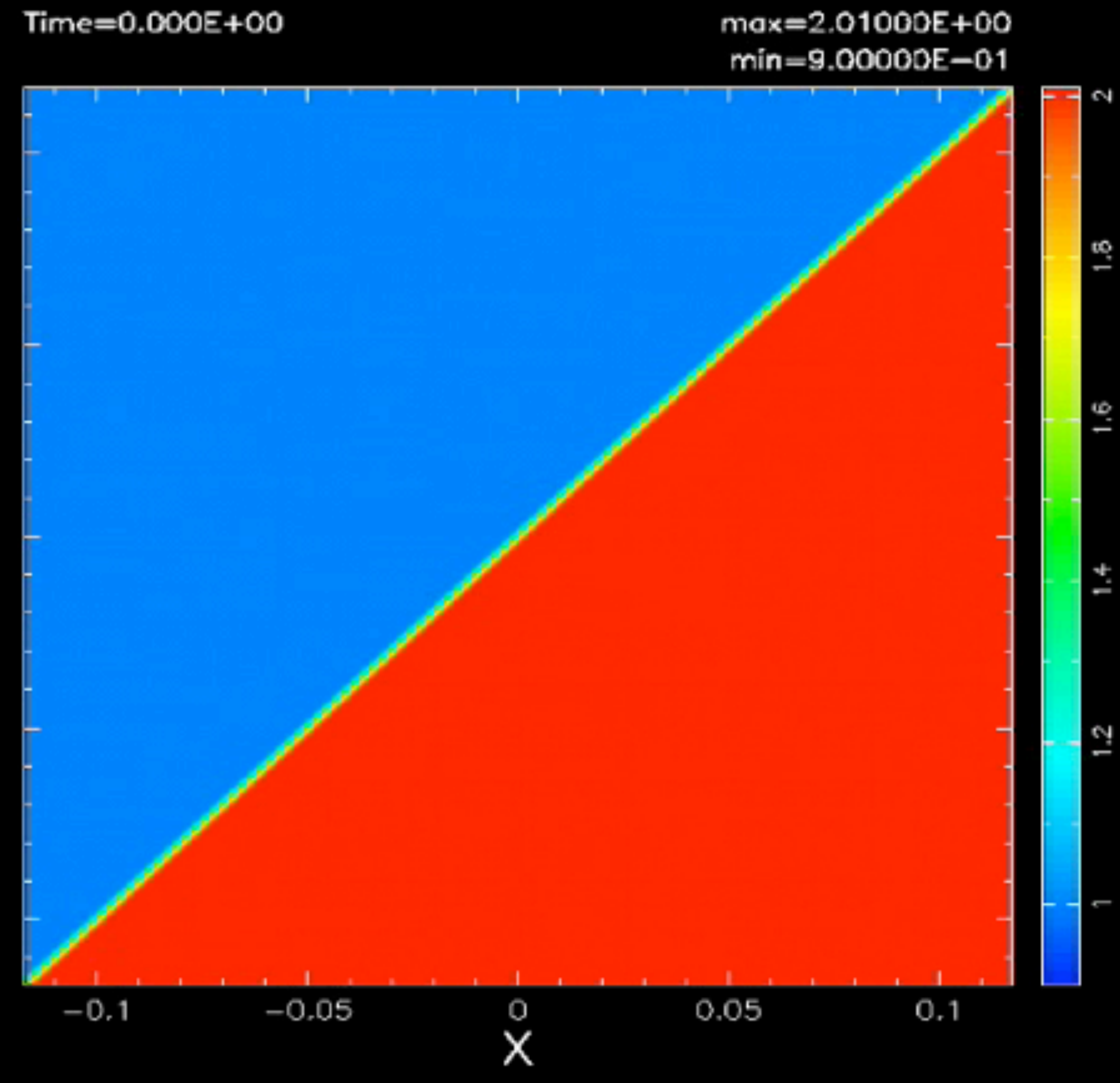


- 1-Dim. even in 3-Dim.
- Lagrangian .vs. Eurian



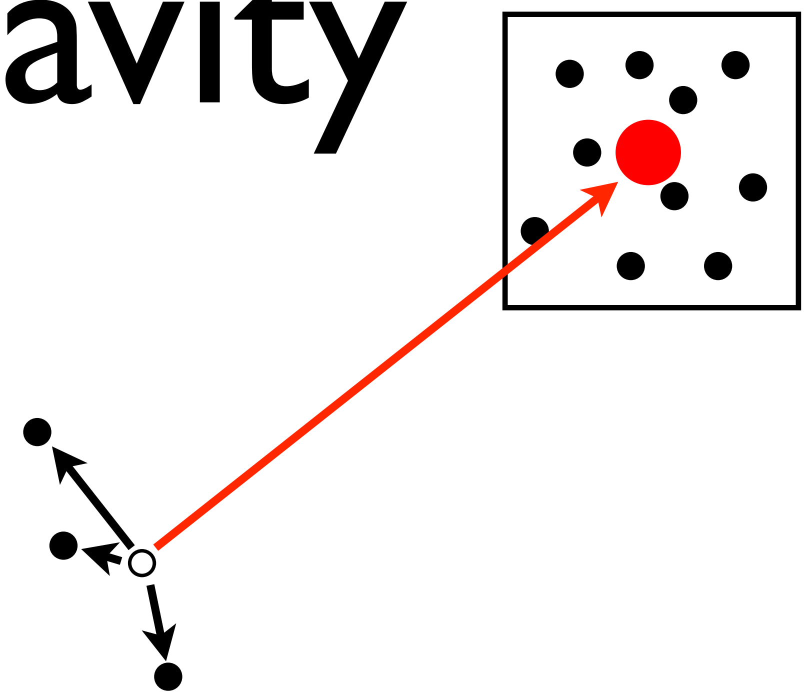
Kelvin-Helmholtz Inst.



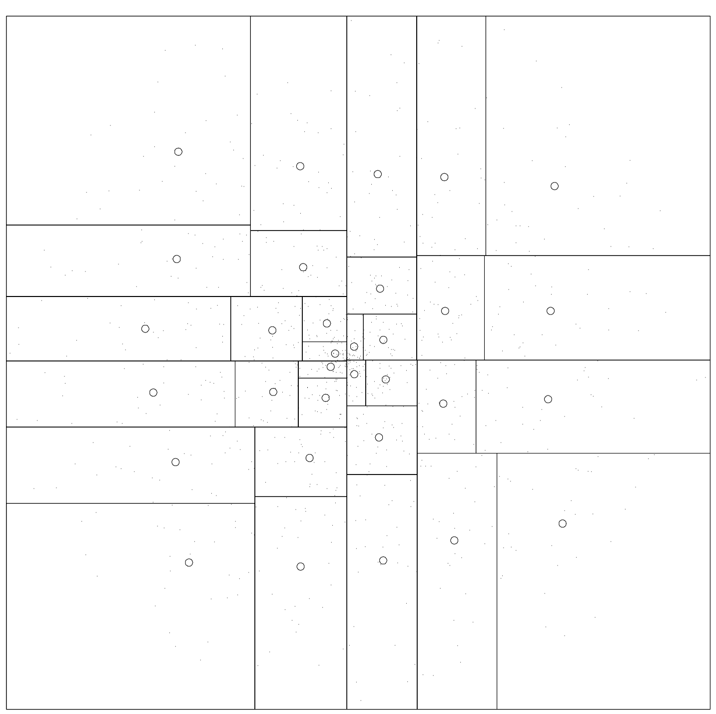


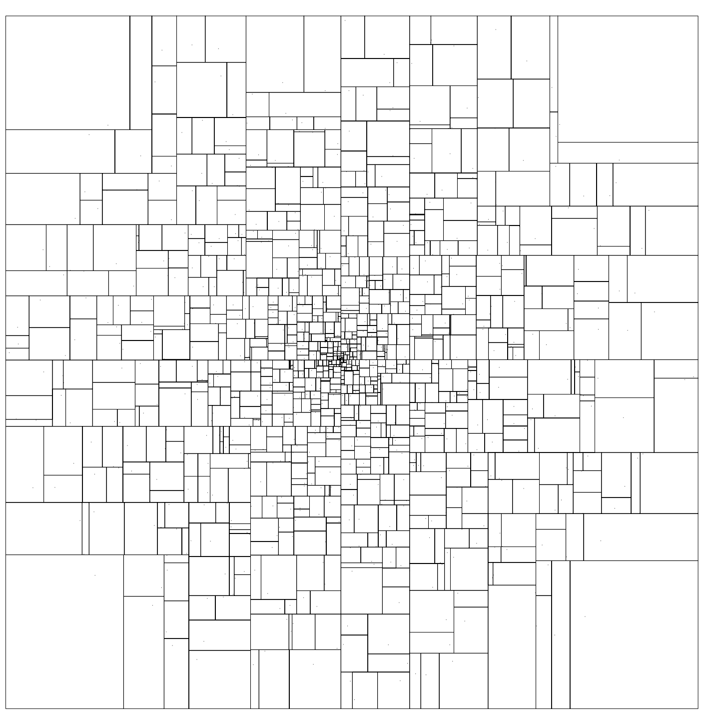
Why particle code ? II. Self-gravity

- Accurate in neighbors, and fast in remote
- Binary tree : Recursive Bisection Method (in parallelization)



$N \log N$ instead of N^2





FDM vs SPH

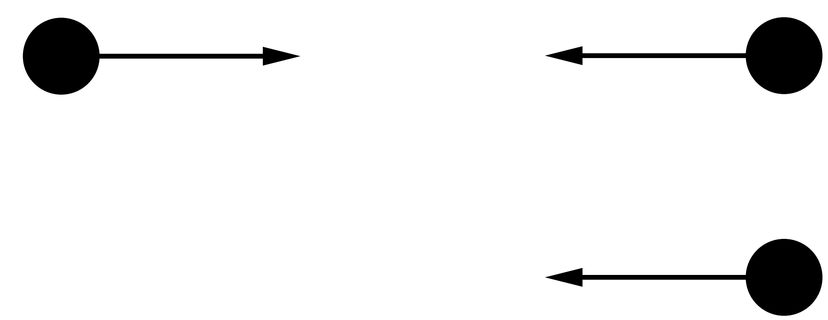
- Self-gravity, irregular geometry : SPH
- High energy explosion : FDM

Artificial viscosity

$$\dot{\boldsymbol{v}}_i = -\sum_j m_j \left(\frac{P_i}{\rho_i} + \frac{P_j}{\rho_j} + \Pi_{ij} \right) \boldsymbol{\nabla}_i W(\boldsymbol{r}_i - \boldsymbol{r}_j, h_{ij})$$

where $\Pi_{ij} = \begin{cases} \frac{-\alpha c_{ij}\mu_{ij} + \beta \mu_{ij}^2}{\rho_{ij}} & \text{if } \boldsymbol{v}_{ij} \cdot \boldsymbol{r}_{ij} \leq 0 \\ 0 & \text{elsewhere} \end{cases}.$

- essential for shocks
- turn on in approaching particles
- notorious side effects in a velocity shear (i.e. a keplerian disc)



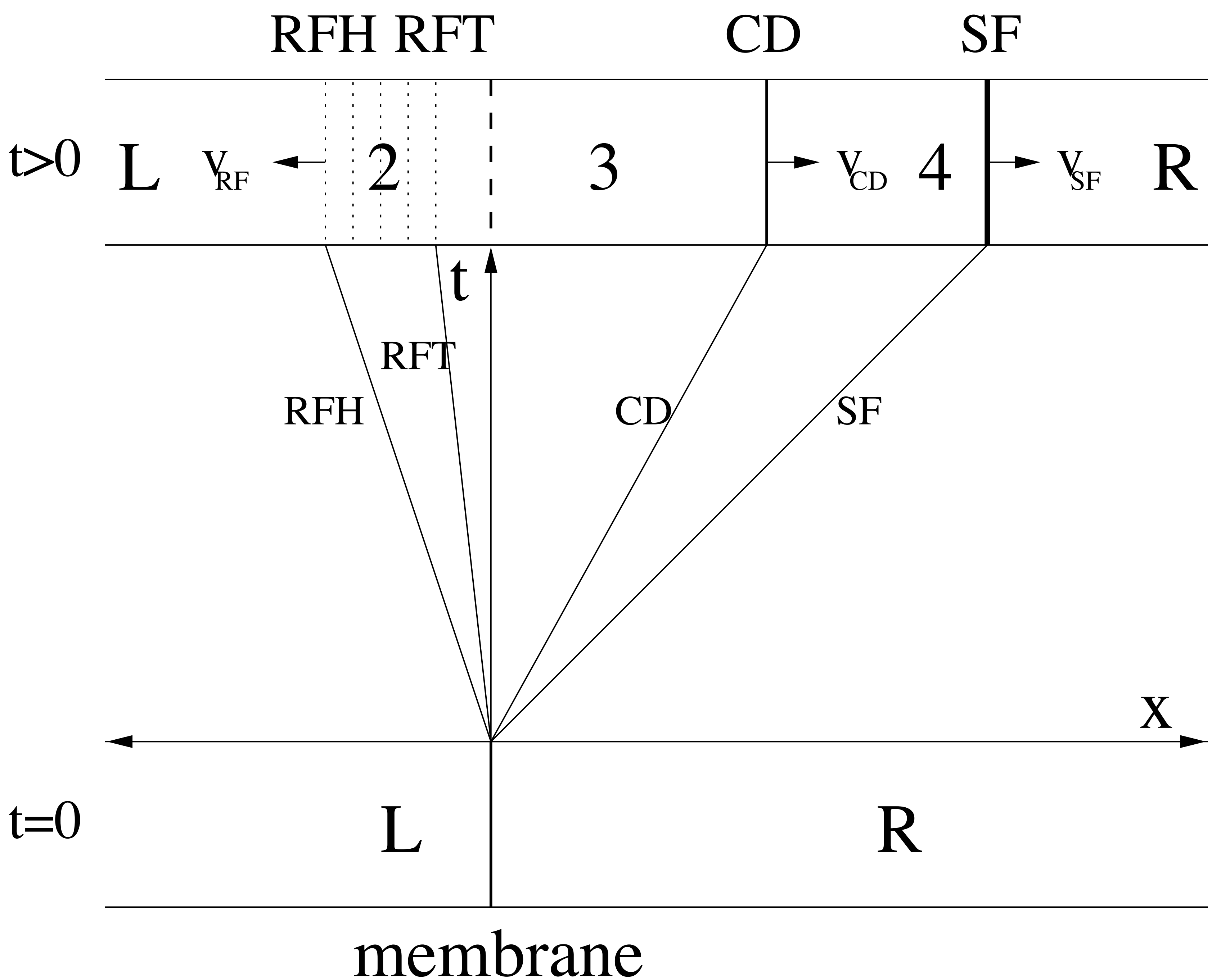
GodunovSPH

Riemann solver instead of the artificial viscosity

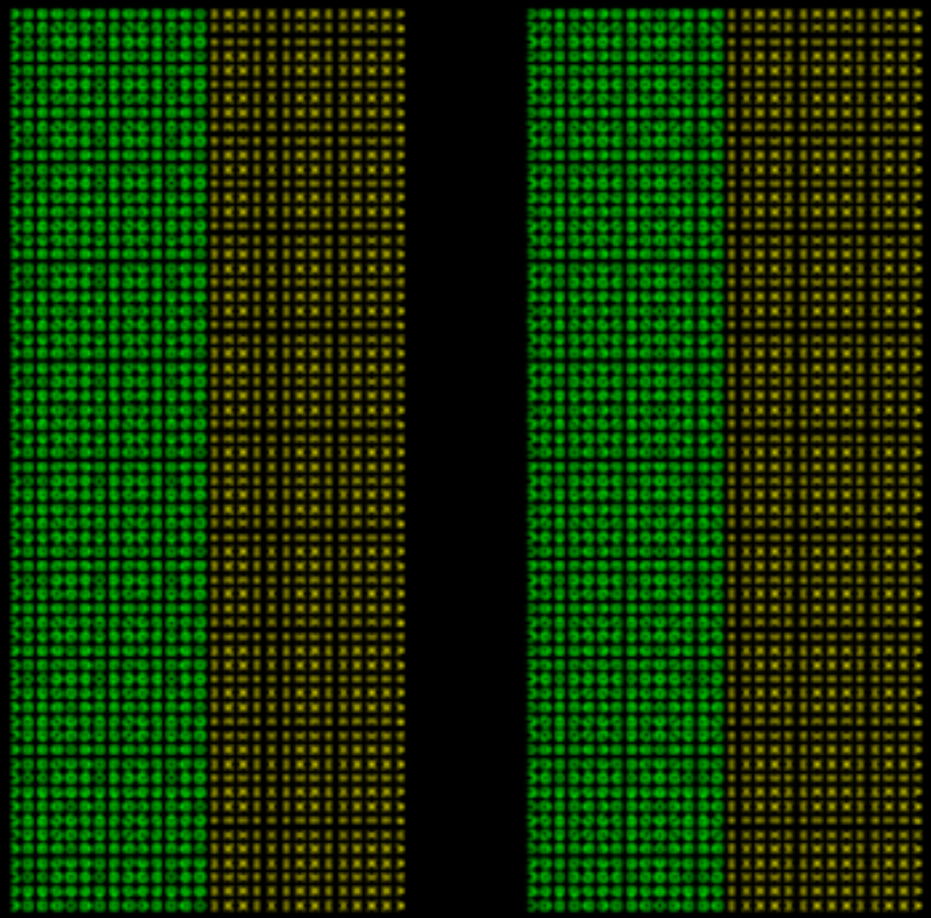
$$\dot{\boldsymbol{v}}_i = -\sum_j m_j \left(\frac{P_i}{\rho_i} + \frac{P_j}{\rho_j} \qquad \right) \boldsymbol{\nabla}_i W(\boldsymbol{r}_i - \boldsymbol{r}_j, h_{ij})$$

GSPH can describe (strong) shocks without the AV !

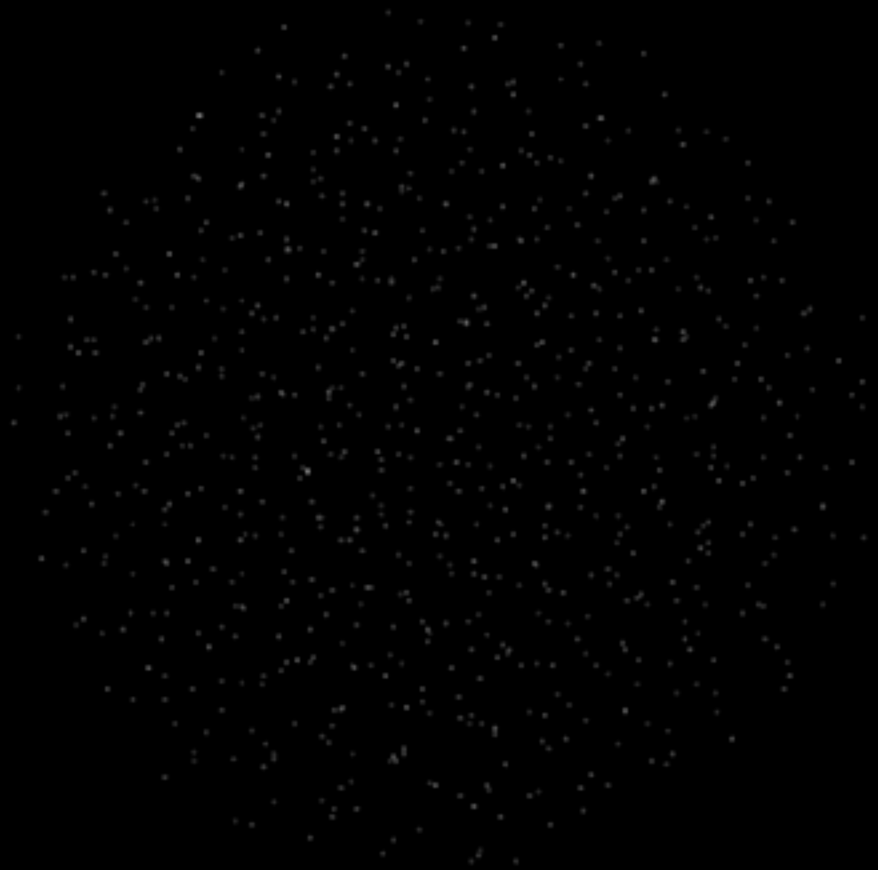
Riemann problem

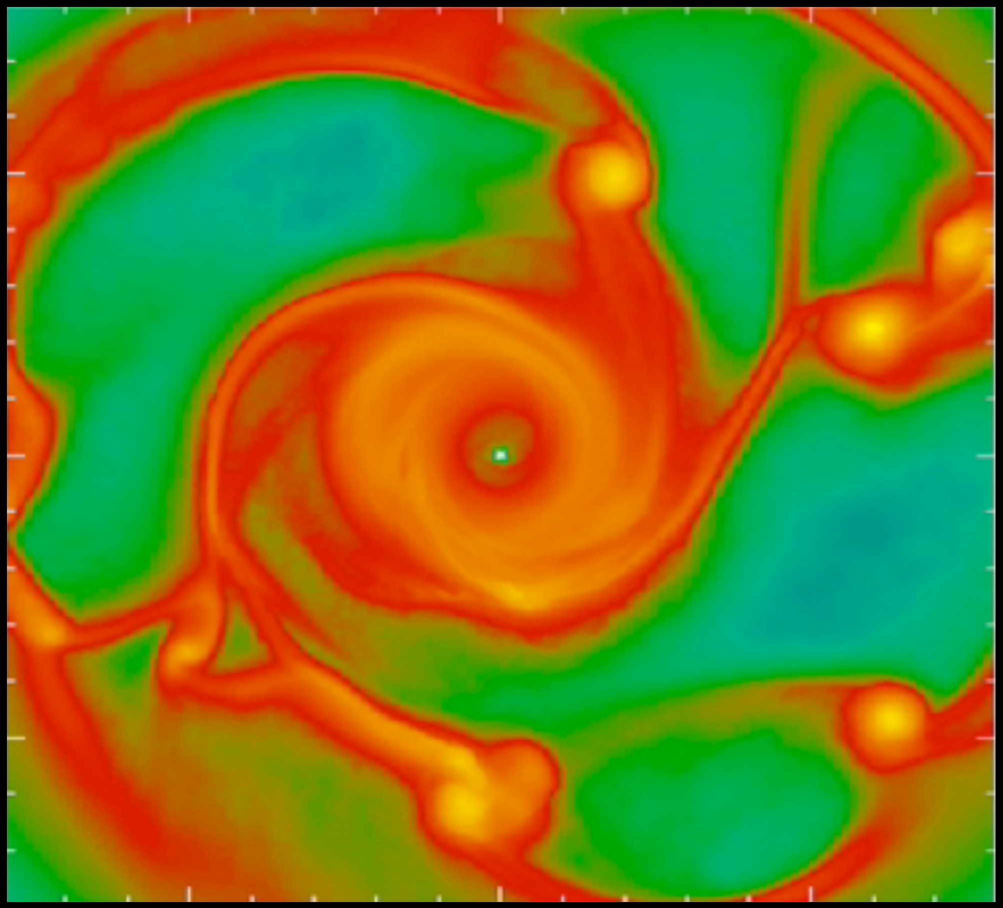


Side effect of AV



Can we believe these ?





Convergence

Lax-Richtmyer (1956)

and

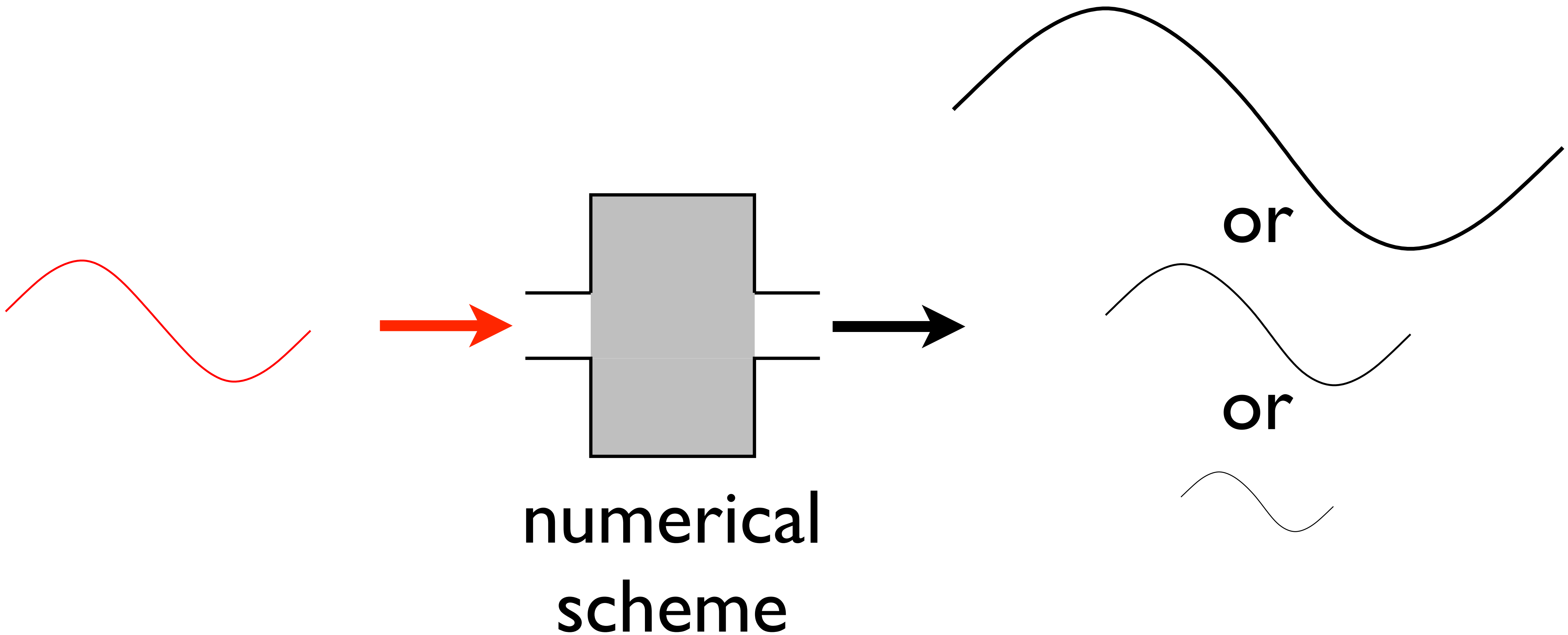
Lax-Wendroff (1960)

said that

“Conservation, Stability and Consistency

are essential for the Convergence”

Conservation



Growth or decay due to a numerical error → Amplitude Error

$$A_k \equiv A_k(t)$$

GSPH has no amplitude error, but..

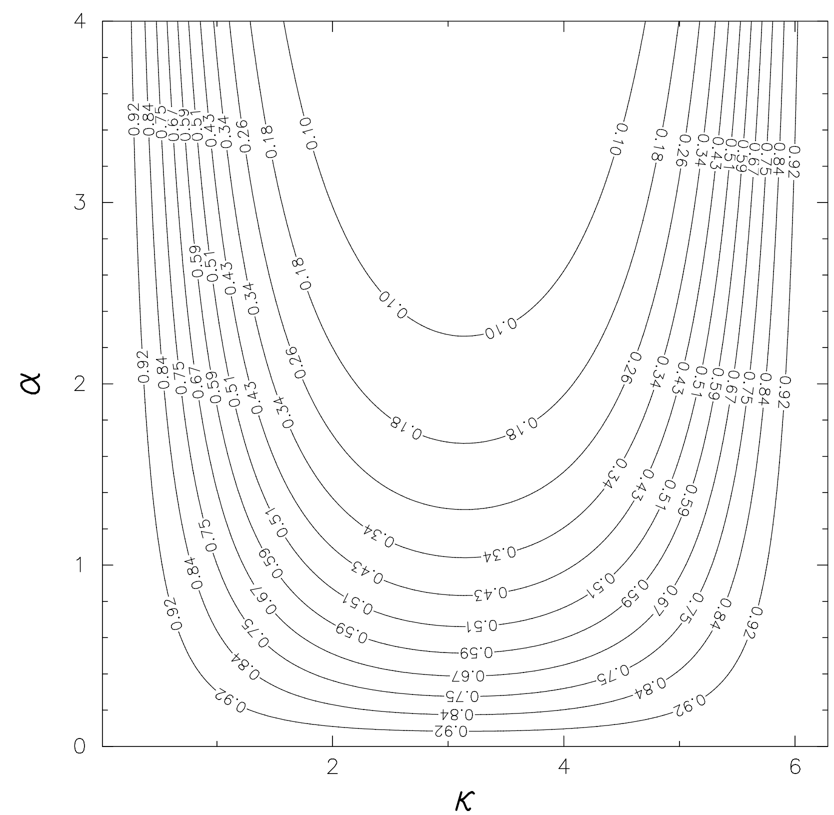
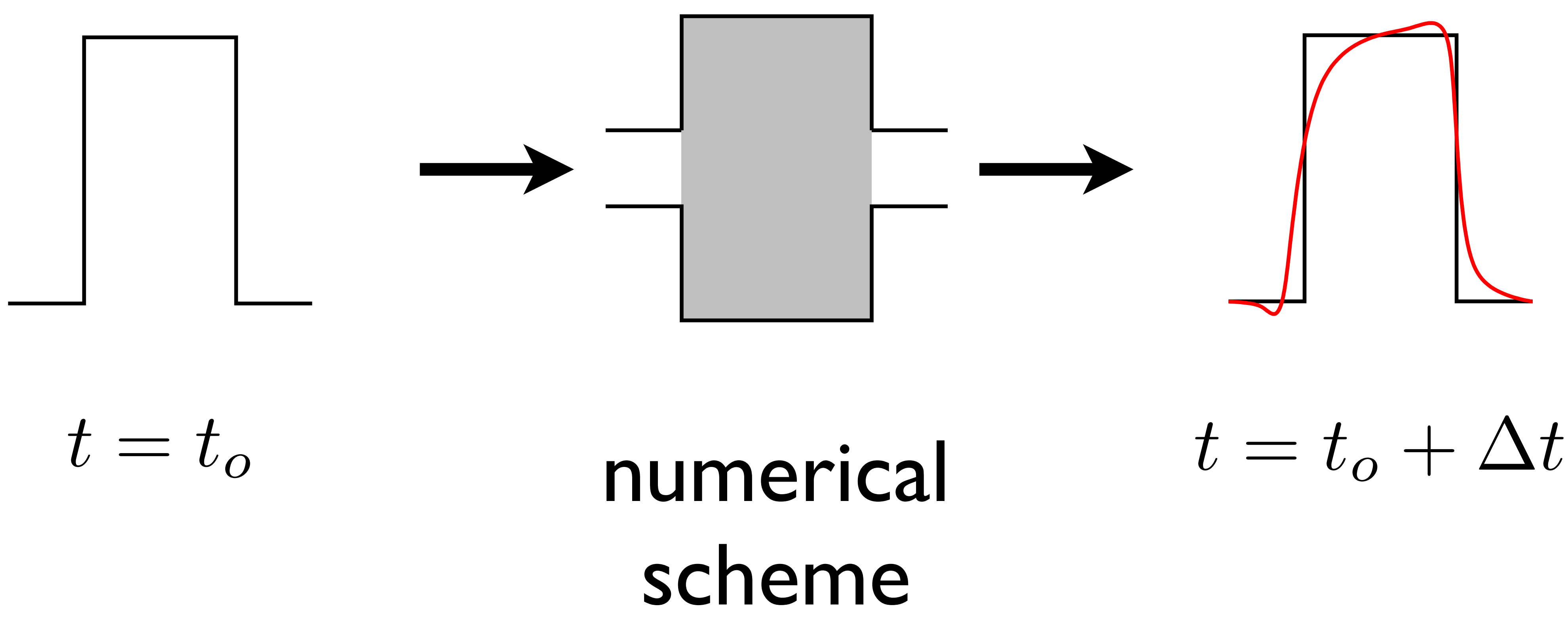


Figure 3.6: Amplitude error contours for isothermal SPH with artificial viscosity on the $\kappa - \alpha$ plane. $b_{SPH,AV}$ is always negative, so the amplitude of the initial perturbation decays with time. The contours show the values at the time, $t = 1$, i.e. just after one period of the wave. The amplitude error depends very weakly on α in the stable region. This means that provided α is set in the stable region, the value of α is not very critical.

Stability

Phase Error, $v \equiv v(k)$



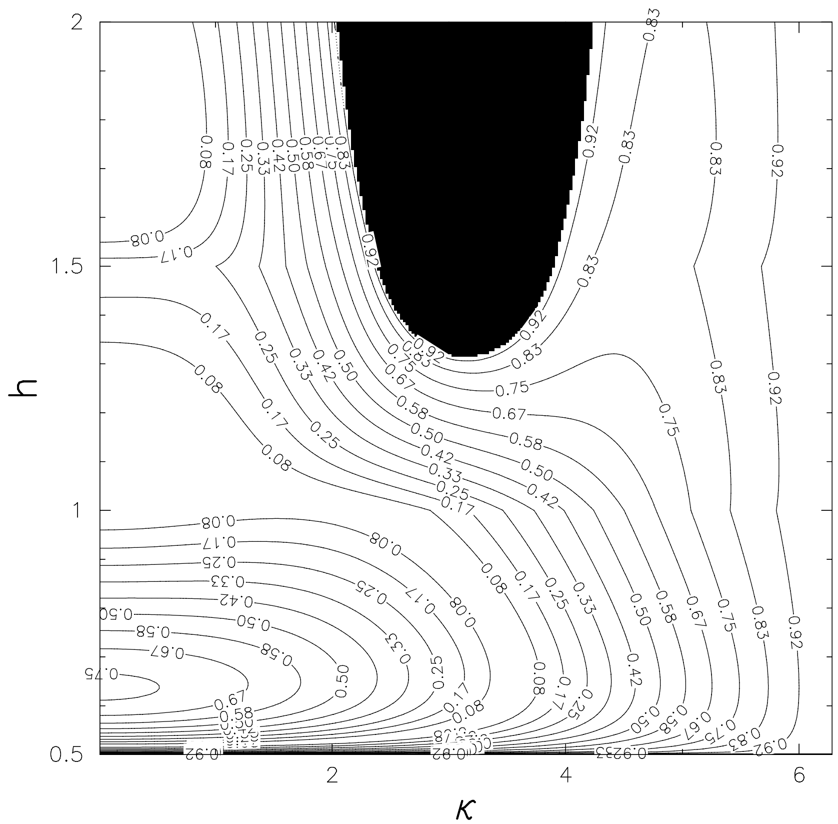


Figure 3.5: Phase error contours for isothermal SPH with artificial viscosity. α is set to 1. The region filled by black in the upper-middle area is the unstable region. In this region, $\omega^2_{SPH,AV}$ is smaller than 0.

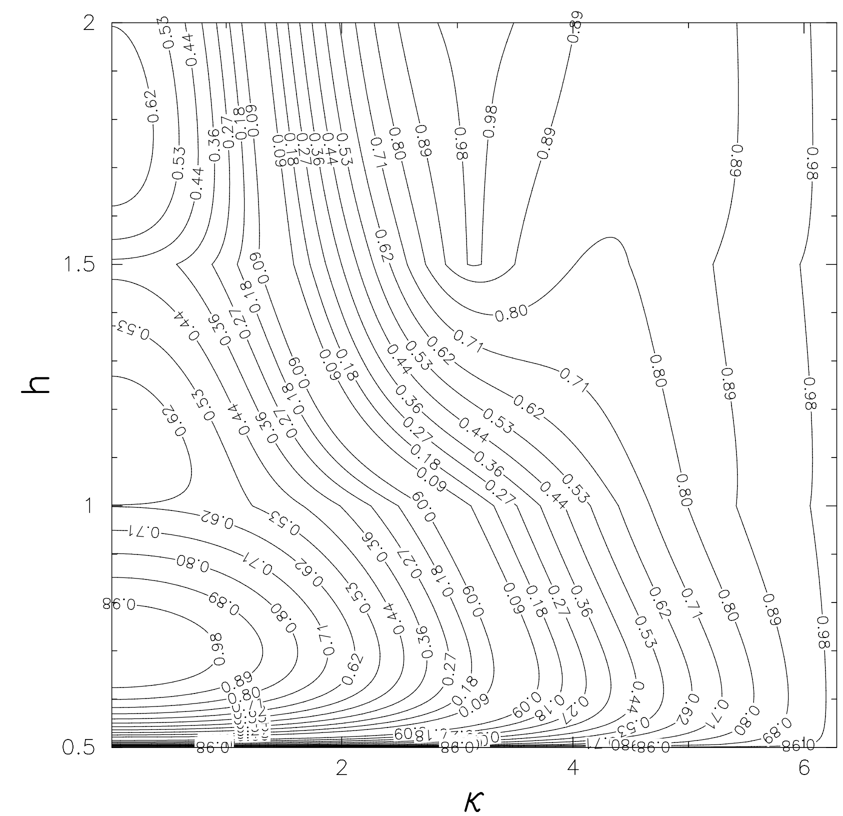
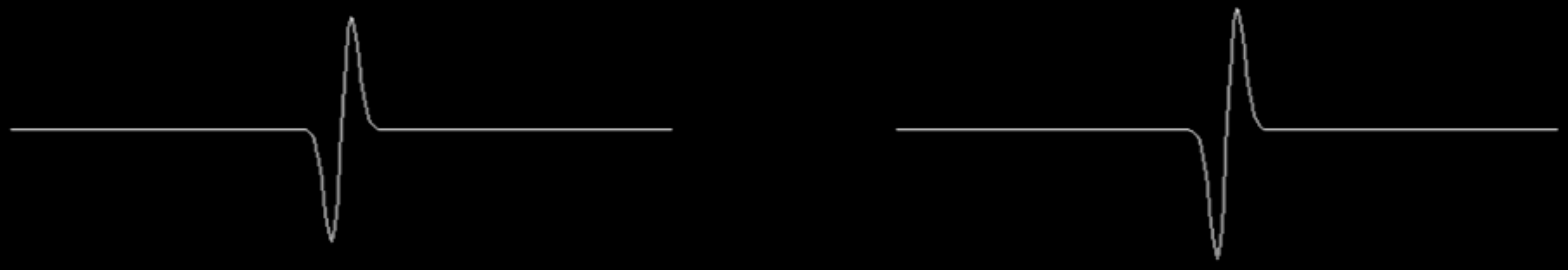


Figure 3.7: Phase error contours for isothermal GPH. At all wavelengths, ω^2_{GPH} is positive, and the general trend is very similar to the case of SPH without artificial viscosity.

Sound wave



Phase error-free scheme is important in inkjet printers

$$q_i^{n+1} = q_i^n - \frac{\boldsymbol{u}}{4}\frac{\Delta t}{\Delta x}\left(q_{i+1}^n + 3q_i^n - 5q_{i-1}^n + q_{i-2}^n\right) - \frac{\boldsymbol{u}^2}{4}\frac{\Delta t^2}{\Delta x^2}\left(q_{i+1}^n - q_i^n - q_{i-1}^n + q_{i-2}^n\right)$$

Fromm's scheme (1984)

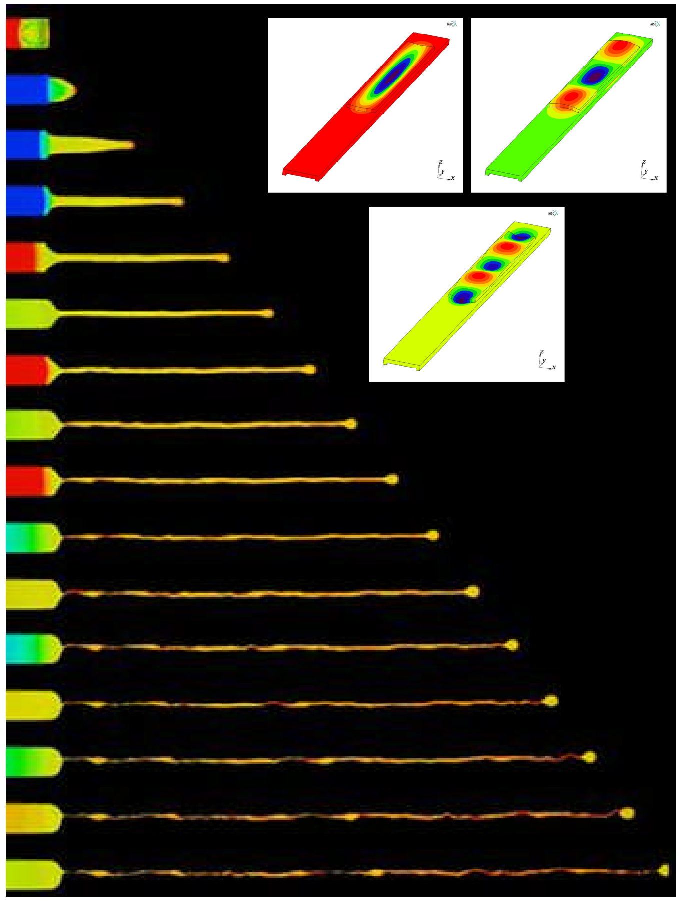


Figure 2. Inter-disciplinary simulation of piezoelectric inkjet formation. Time interval is 6% of firing cycle.

Zeng (2009)

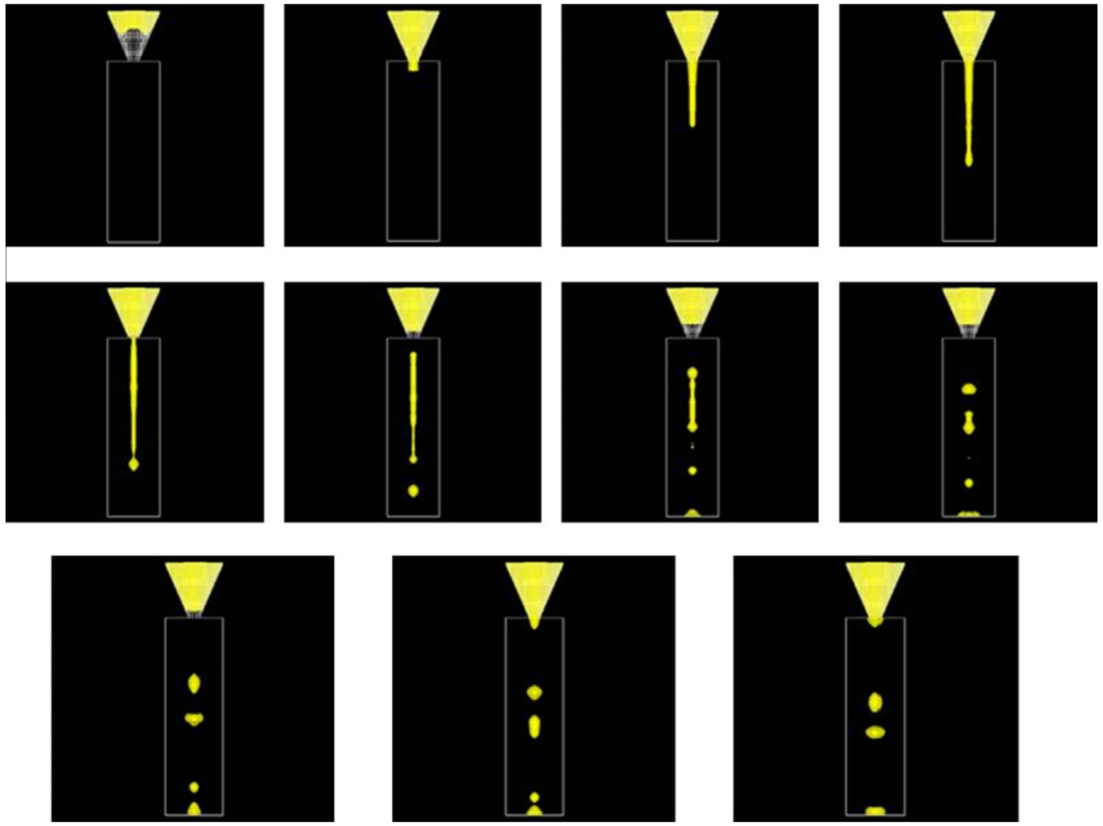


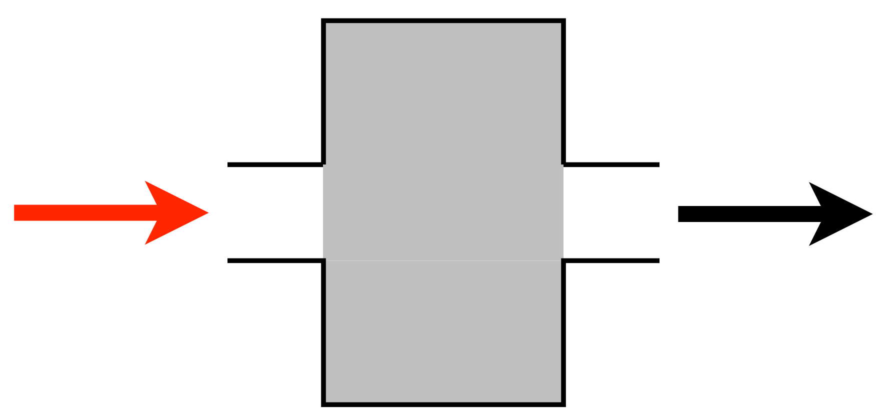
Fig. 9. Numerical results of column droplet.

Chang (2011)

Consistency

- Resemblance of PDE and the numerical equations
- Analysis of the truncation error
- Order of accuracy

$$\begin{cases} C_o \\ C_o + C_1 x \\ C_o + C_1 x + C_2 x^2 \\ \vdots \end{cases}$$

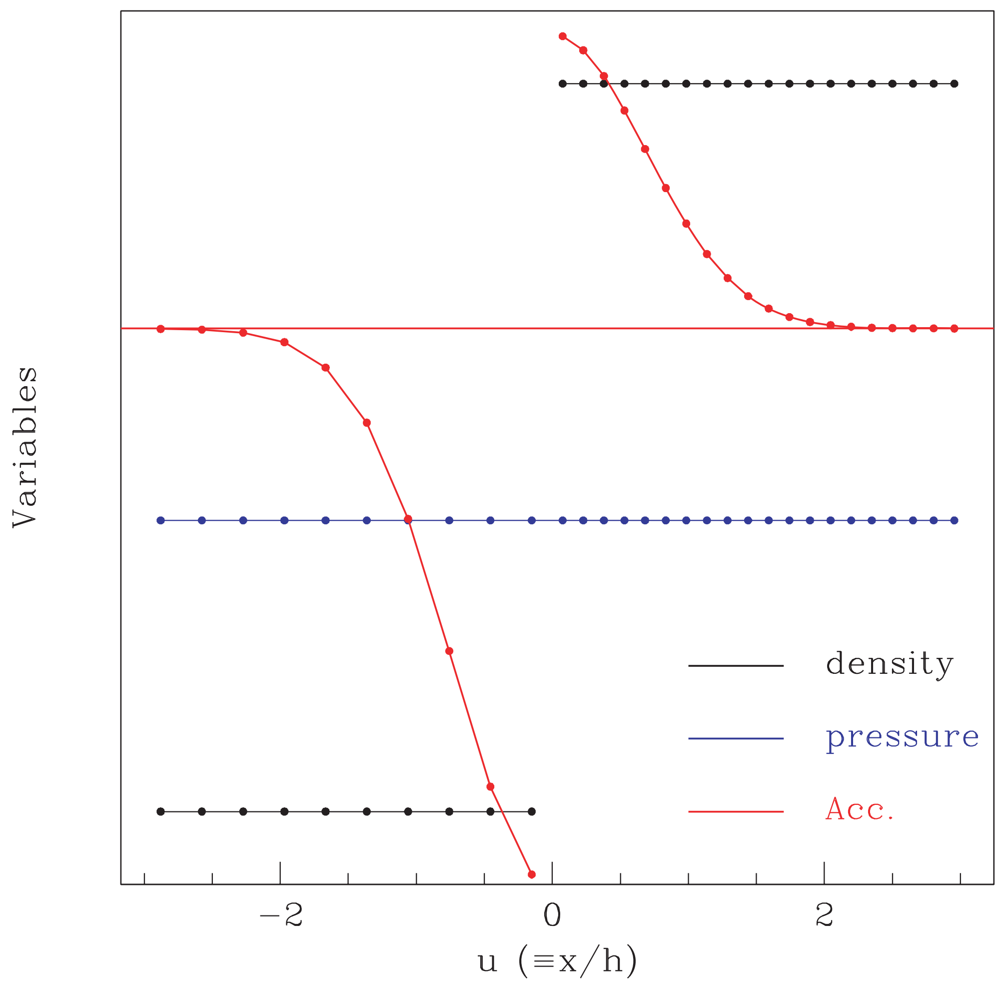


numerical
scheme

$$\begin{cases} C_o \\ C_o + C_1 x \\ C_o + C_1 x + C_2 x^2 \\ \vdots \end{cases}$$

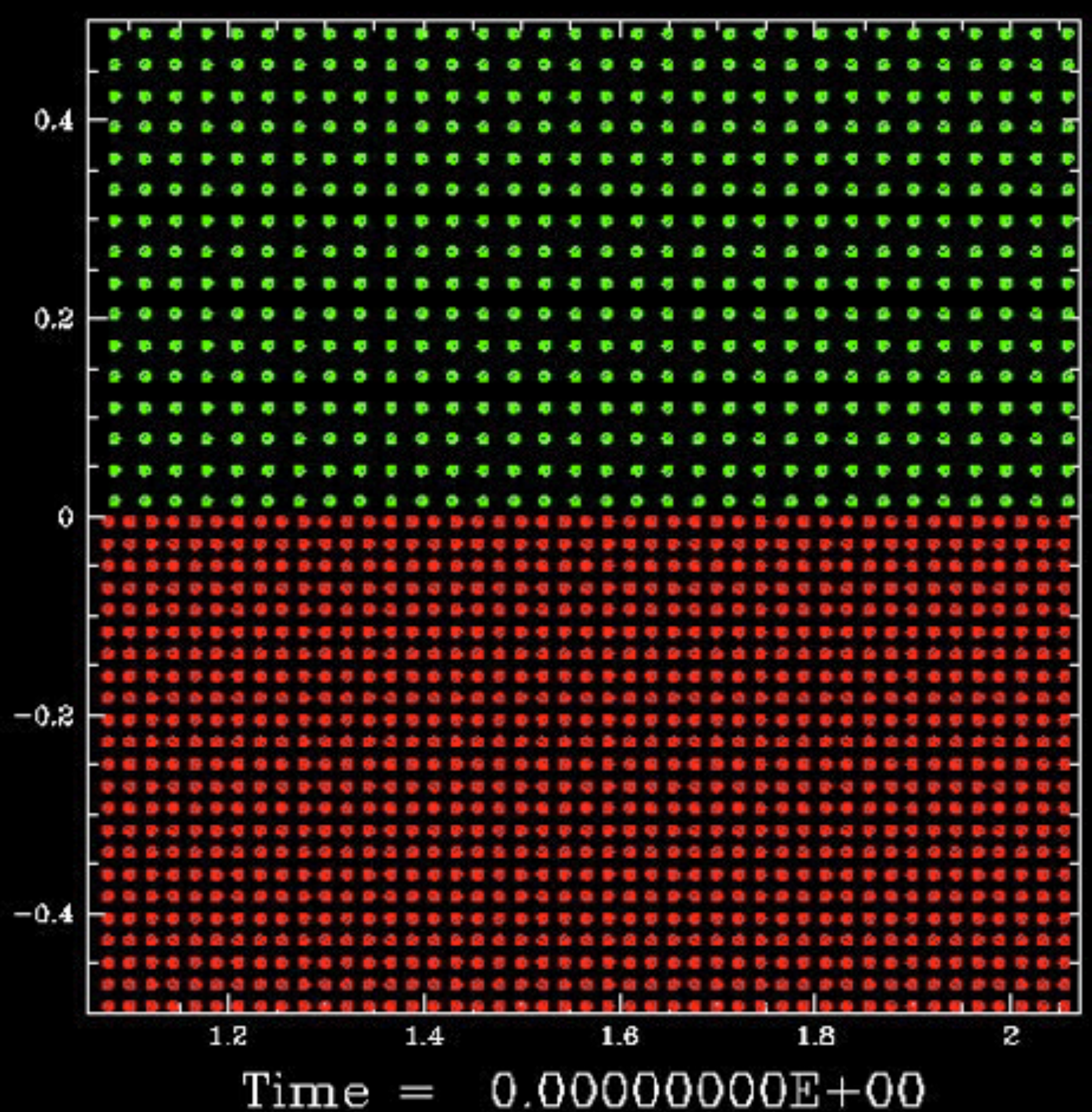
Numerical Surface tension

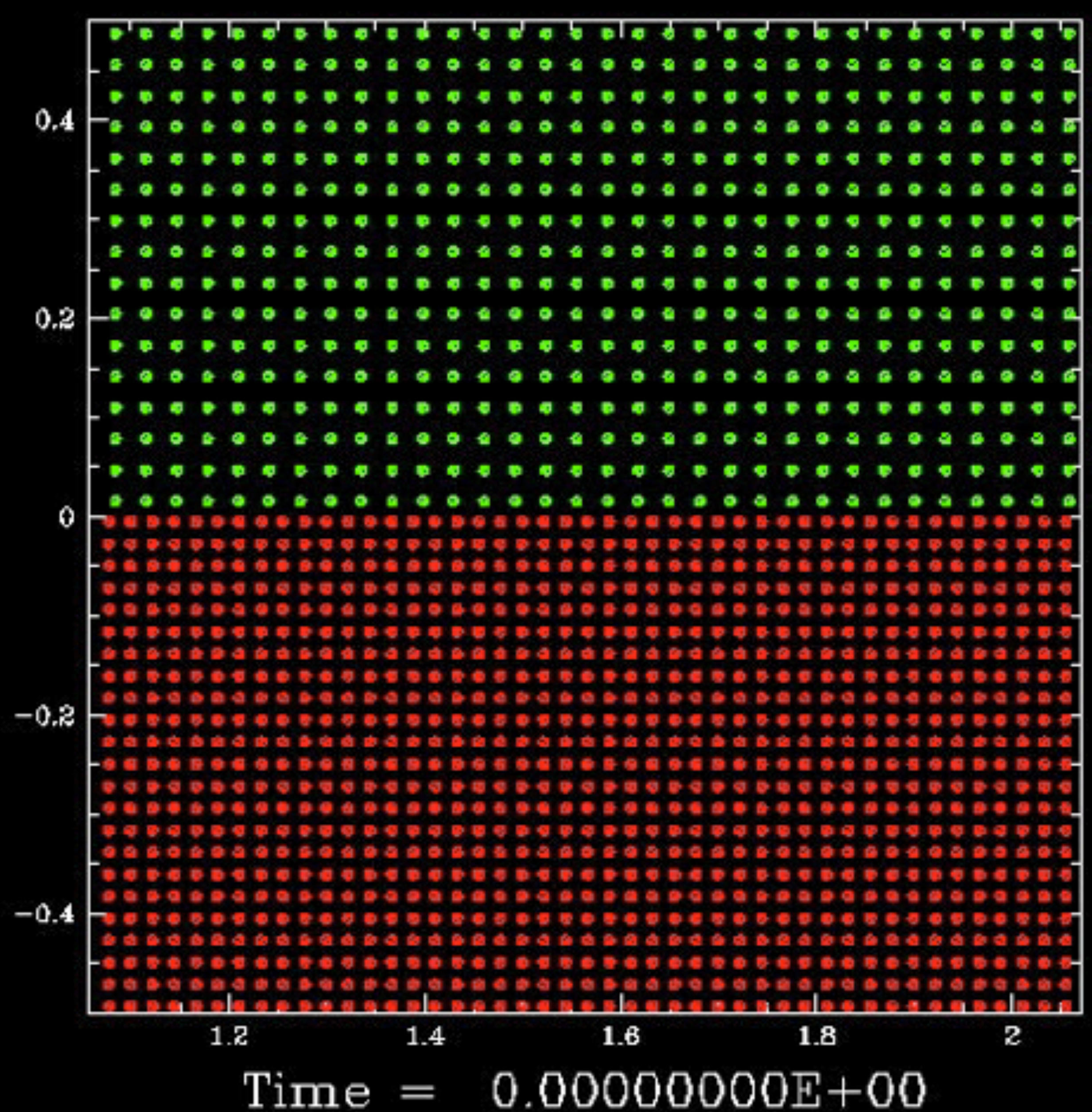
- SPH is inconsistent.
- The numerical surface tension appears due to the inconsistency.



Cha et al. 2010

Numerical surface tension

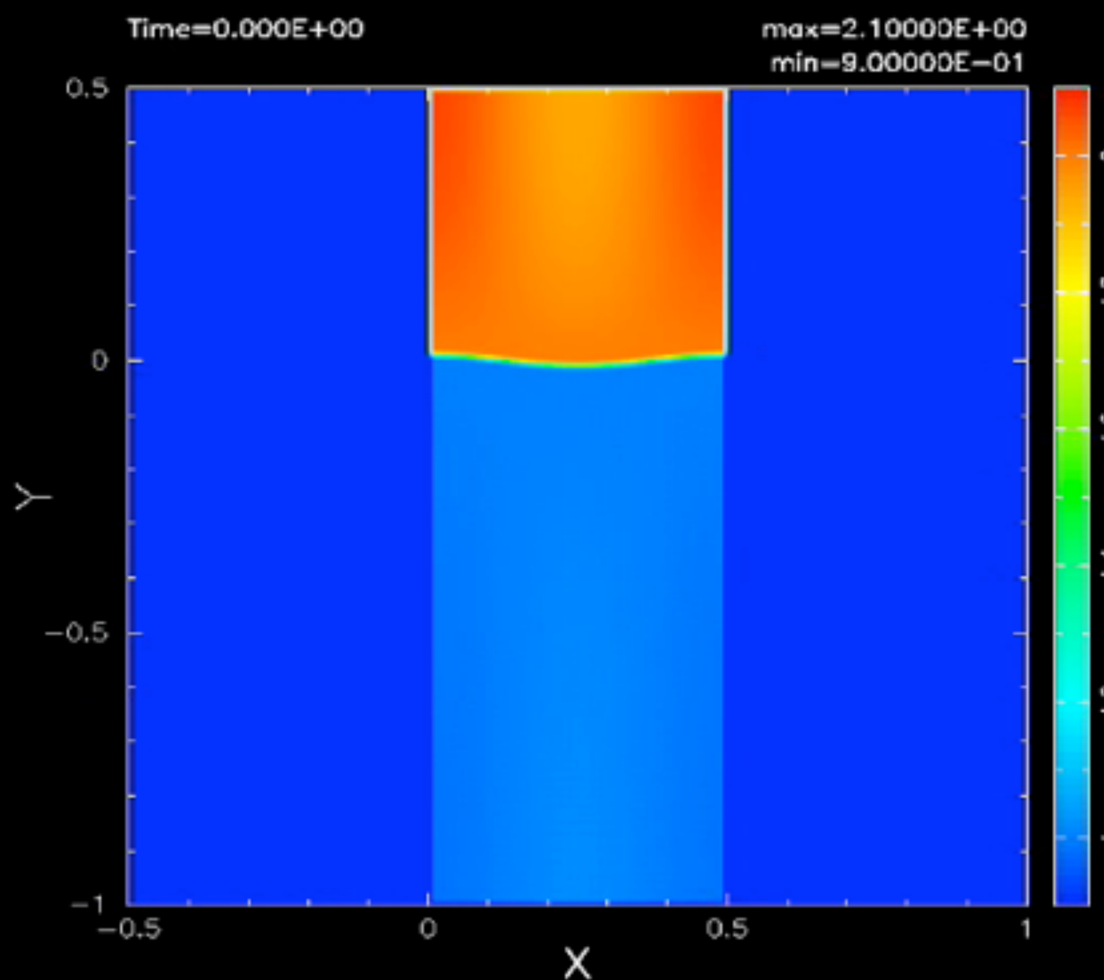


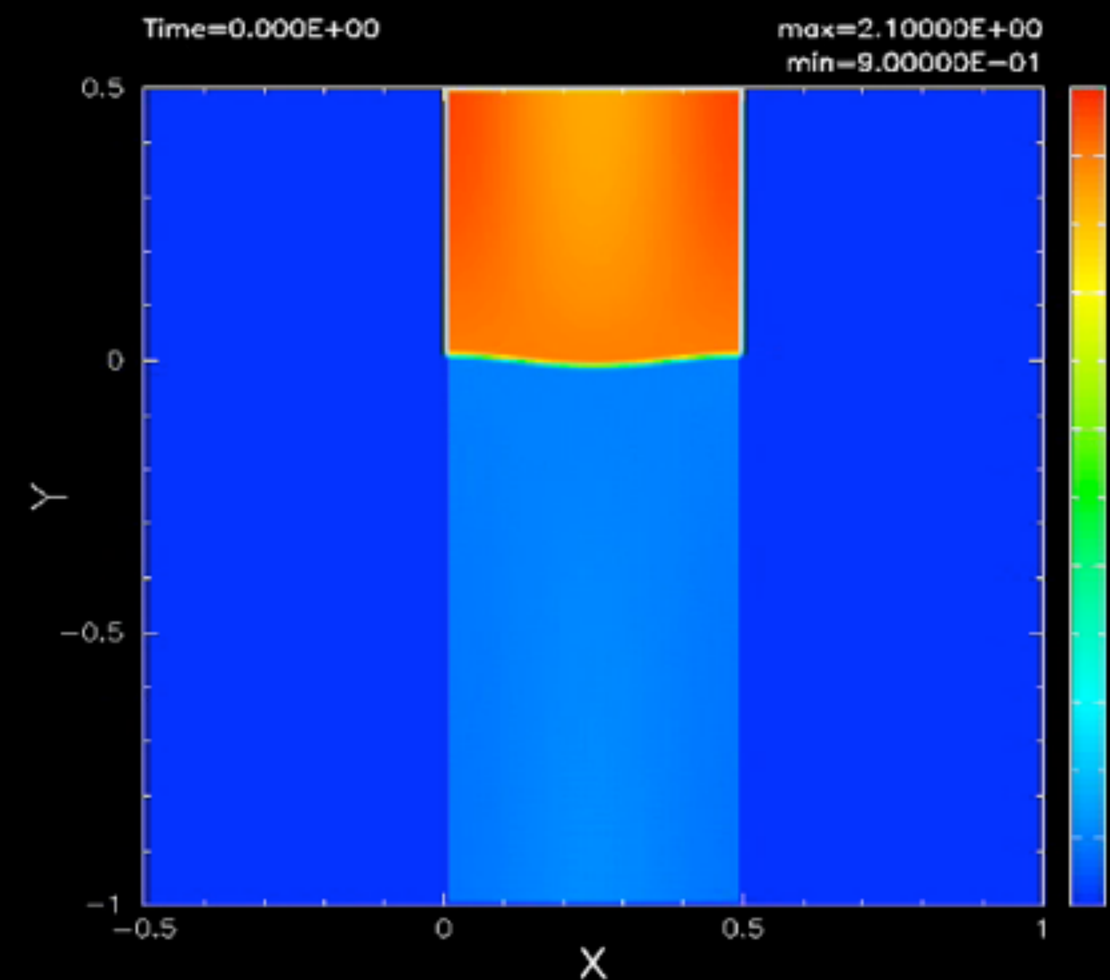


GSPH .vs. SPH

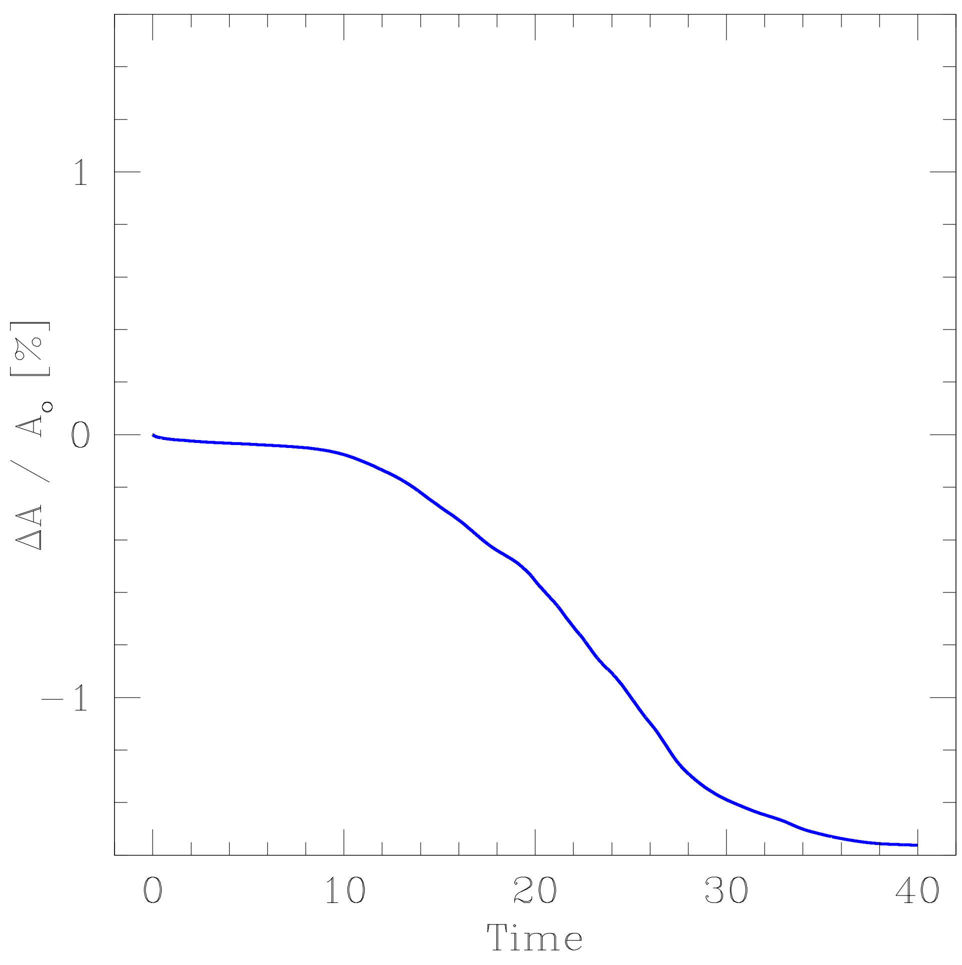
	GSPH	SPH
Conservation	O	X
Stability	O	X
Consistency	First-ordered	Zeroth-Ordered

Rayleigh-Taylor Inst.



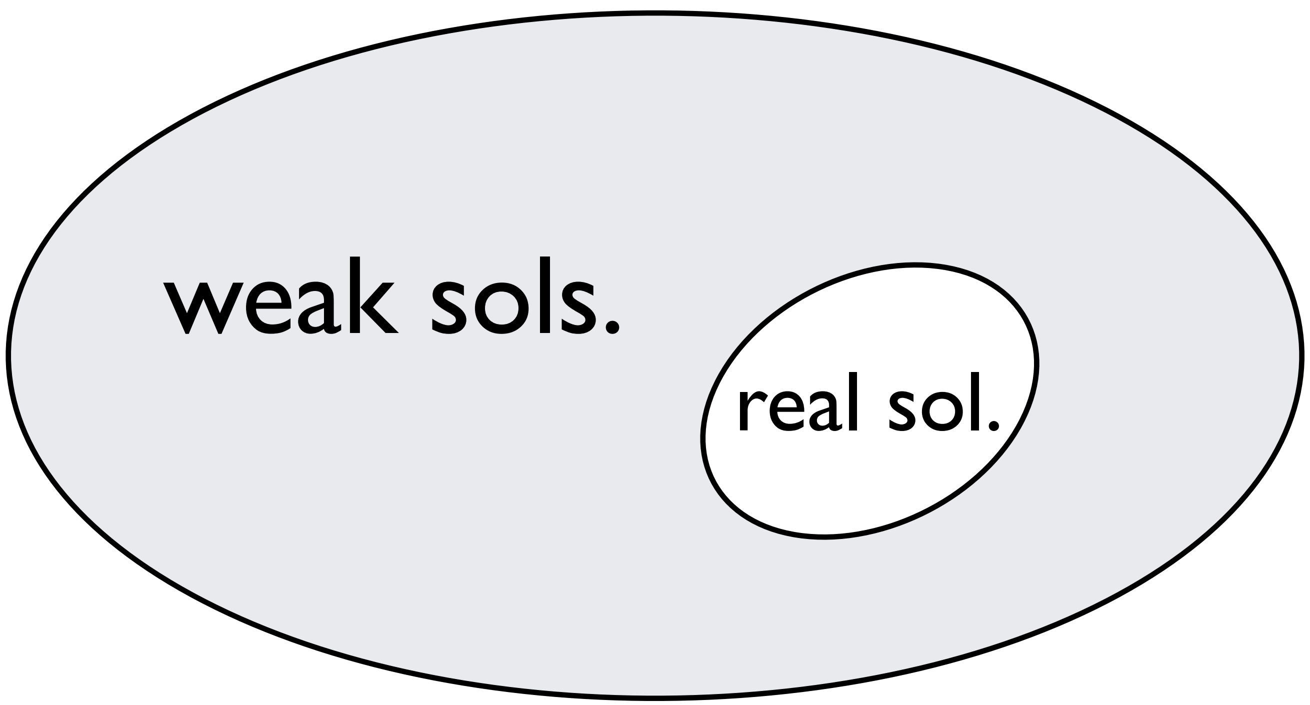


What was wrong ?



- Lax-Richtmyer and Lax-Wendroff said necessary conditions.
- Convergence to a weak solution !

Weak solutions



Entropy condition

(Olenik, 1963)

$$\frac{ds}{dt} \geq 0$$

Thermal compatibility

the entropy of a fluid, $S = c_v \ln\left(\frac{P}{\rho^\gamma}\right)$

$$\frac{Du}{Dt} = -\frac{P}{\rho}\boldsymbol{\nabla}\cdot\boldsymbol{v}$$

$$\frac{D\rho}{Dt} = -\rho\boldsymbol{\nabla}\cdot\boldsymbol{v}$$

$$\therefore \frac{Du}{Dt} = -P\frac{DV}{Dt}$$

Density estimation

$$\rho_i = \sum_j \frac{m_j}{V_i} \int \frac{1}{\rho(\boldsymbol{x})} W((\boldsymbol{x} - \boldsymbol{x}_i), h_i) W((\boldsymbol{x} - \boldsymbol{x}_j), h_j) d\boldsymbol{x}$$

tcGSPH

Assume $\frac{1}{V_i} \simeq \rho(\boldsymbol{x}) \simeq \rho_i$, then

GSPH

$$\rho_i = \sum_j m_j \int W((\boldsymbol{x} - \boldsymbol{x}_i), h_i) W((\boldsymbol{x} - \boldsymbol{x}_j), h_j) d\boldsymbol{x}$$

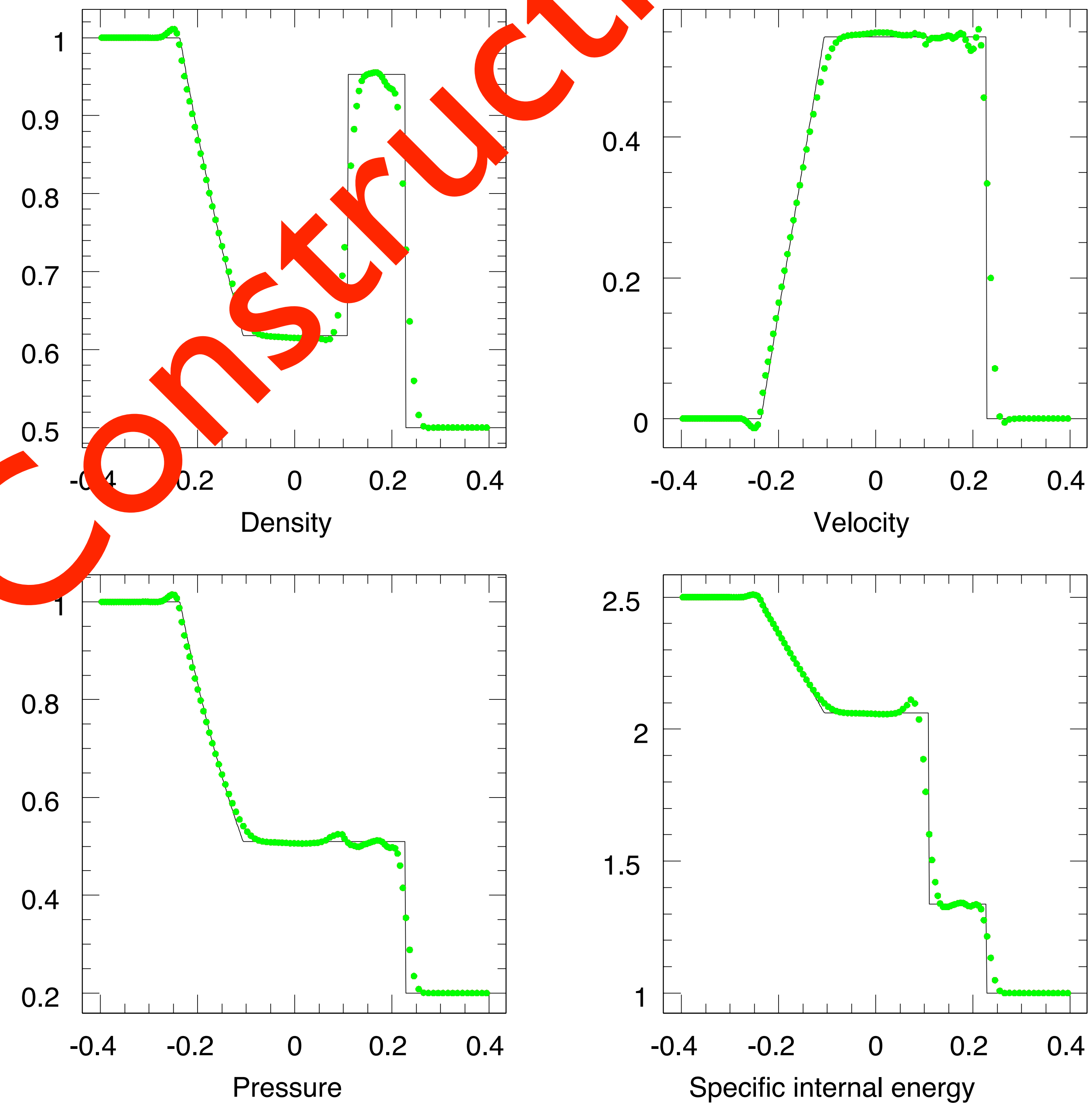
Assume $\frac{1}{V_i} \simeq \rho(\boldsymbol{x}) \simeq \rho_i \simeq \rho_j$, then

SPH

$$\rho_i = \sum_j m_j W(\boldsymbol{x}_i - \boldsymbol{x}_j, h_{ij})$$

Shock tube test

- The interactions between the two different fluid.
- It is called Sod test (Sod 1978)
- A good test problem



Under Construction !

SPH, GSPH & tcGSPH

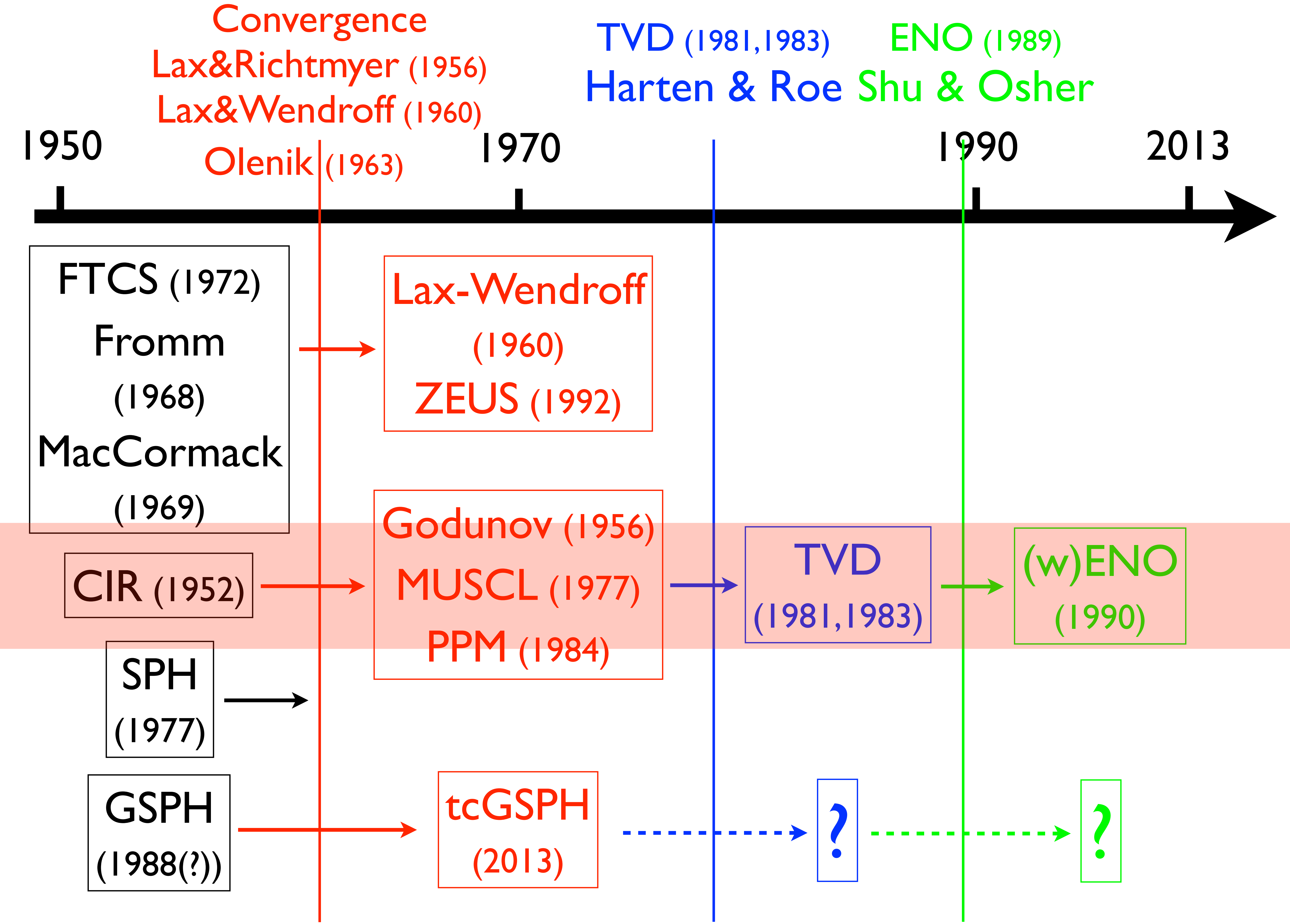
	SPH	GSPH	tcGSPH
Conservation	X	O	O
Stability	X	O	O
Consistency	Zeroth-Ordered	First-ordered	First-ordered
Entropy condition	X	X	O

Why are the results different ?

They didn't obey the rules.

Can I believe my simulation ?

Yes I can.



Convergence
Lax&Richtmyer (1956)
Lax&Wendroff (1960)
Olenik (1963)
TVD (1981,1983)
Harten & Roe
ENO (1989)
Shu & Osher
1950
1970
1990
2013
FTCS (1972)
Fromm (1968)
MacCormack (1969)
Lax-Wendroff (1960)
ZEUS (1992)
CIR (1952)
Godunov (1956)
MUSCL (1977)
PPM (1984)
TVD (1981,1983)
(w)ENO (1990)
SPH (1977)
GSPH (1988(?))
tcGSPH (2013)
?
?

Future

- Nonlinear, nonisothermal stability analysis
- Total Variation Diminish (TVD)
- Non-oscillatory property
- Different parallelization (i.e. GPU)

tcGSPH is

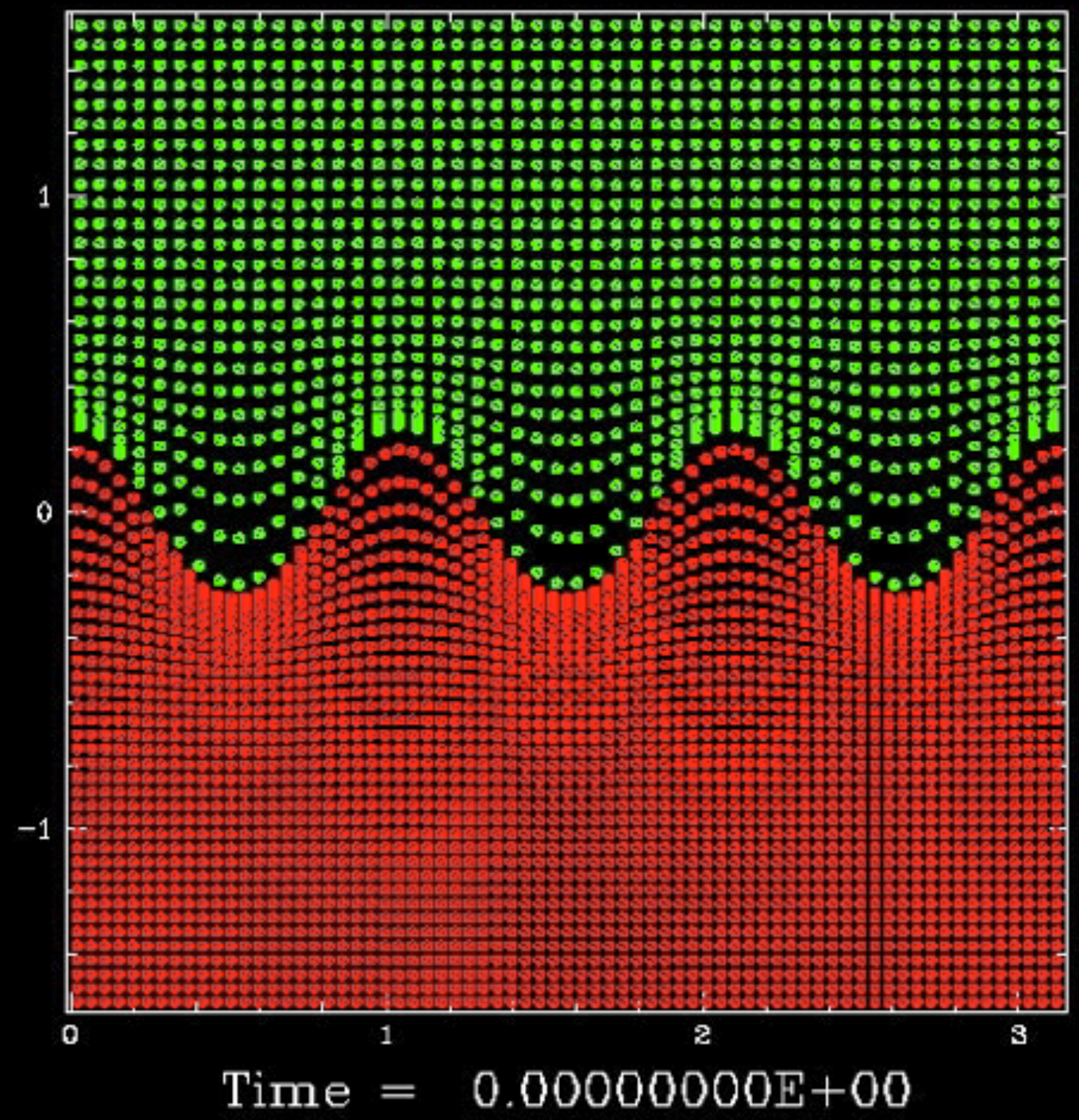
- working.
- better.

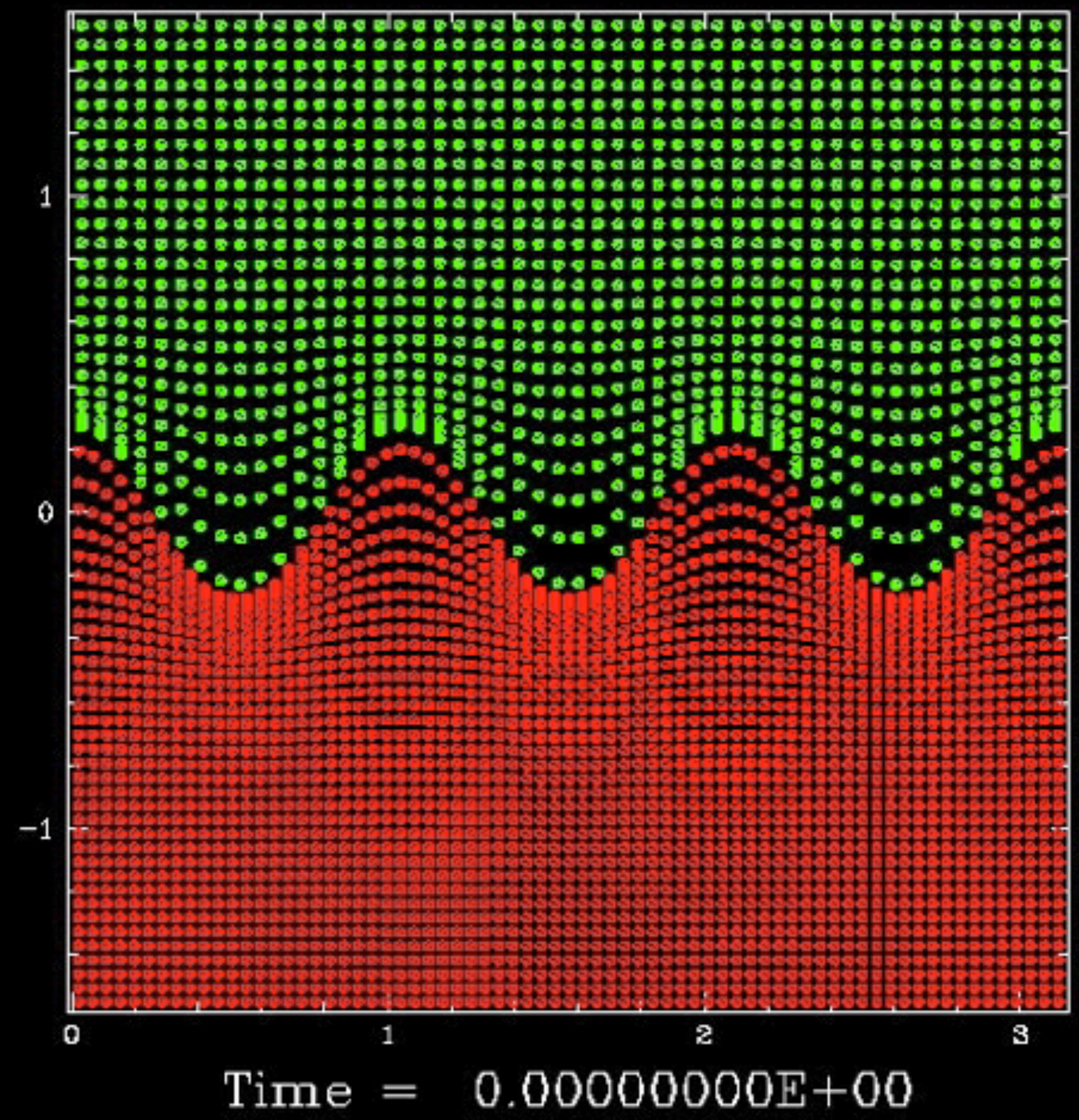
Public code for CVs

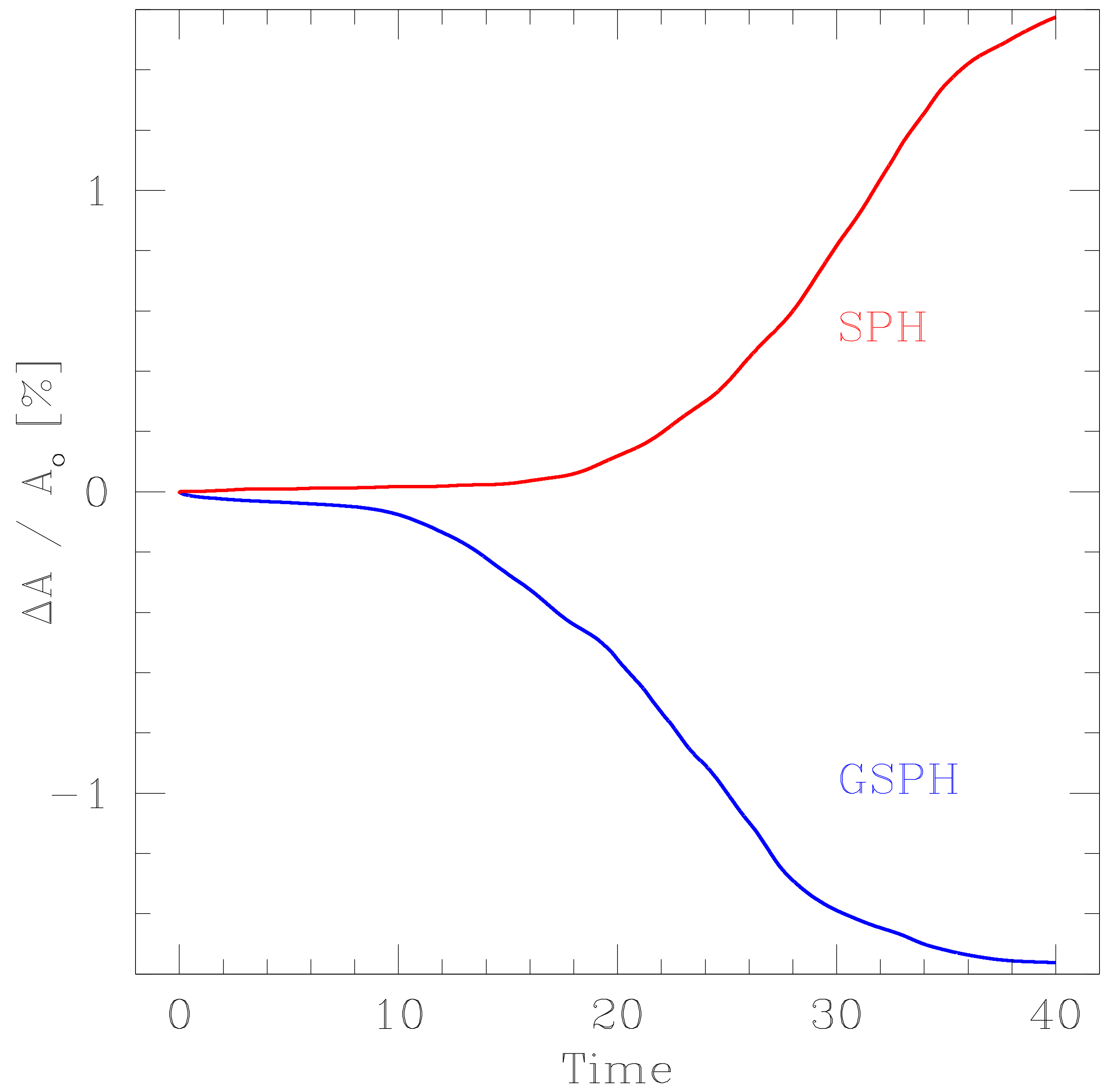
- Convergence scheme : high resolution scheme
- Radiation HD by the diffusion approximation
- Fully 3-dimensional self-gravity code
- Parallelization with MPI2-libraries
- Magneto hydrodynamics (optional)
- Documentation : HTML, (La)TeX, UNIX man...
- Scientific visualization

Any participation should be welcomed.

Solenoidal field







1
0
−1
0
10
20
30
40
Time
ΔA / A∘ [%]
SPH
GSPH