Why is the expansion of the universe accelerating?

(talking about cosmic acceleration in 50 minutes or mission impossible)

$$G^a_b + \Lambda \delta^a_b = \kappa T^a_b$$

$$\frac{\ddot{a}(t)}{a(t)} = -4\pi \rho_{DE} \left(\frac{1}{3} + w\right)$$

$$S_{(5)} = \frac{1}{2} M^3_{(5)} \int d^4x\, dy \sqrt{-g_{(5)}} R_{(5)} + \frac{1}{2} M^2_{(4)} \int d^4x \sqrt{-g_{(4)}} R_{(4)} + S_{matter}$$

$$P_\kappa(l) = \frac{9}{4} H_o^4 \Omega_m^2 \int_0^{\chi_H} \frac{g^2(\chi)}{a^2(\chi)} P_{3D}\left(l / \sin_K(\chi), \chi\right) d\chi$$

Prof. Mustapha Ishak-Boushaki

Cosmology and Relativity Group

Department of Physics

The University of Texas at Dallas

The Nobel Prize in Physics 2011

Saul Perlmutter, Brian P. Schmidt, Adam G. Riess

The Nobel Prize in Physics 2011

Nobel Prize Award Ceremony

Saul Perlmutter

Brian P. Schmidt

Adam G. Riess

Photo: U. Montan

Saul Perlmutter

Photo: U. Montan

Brian P. Schmidt

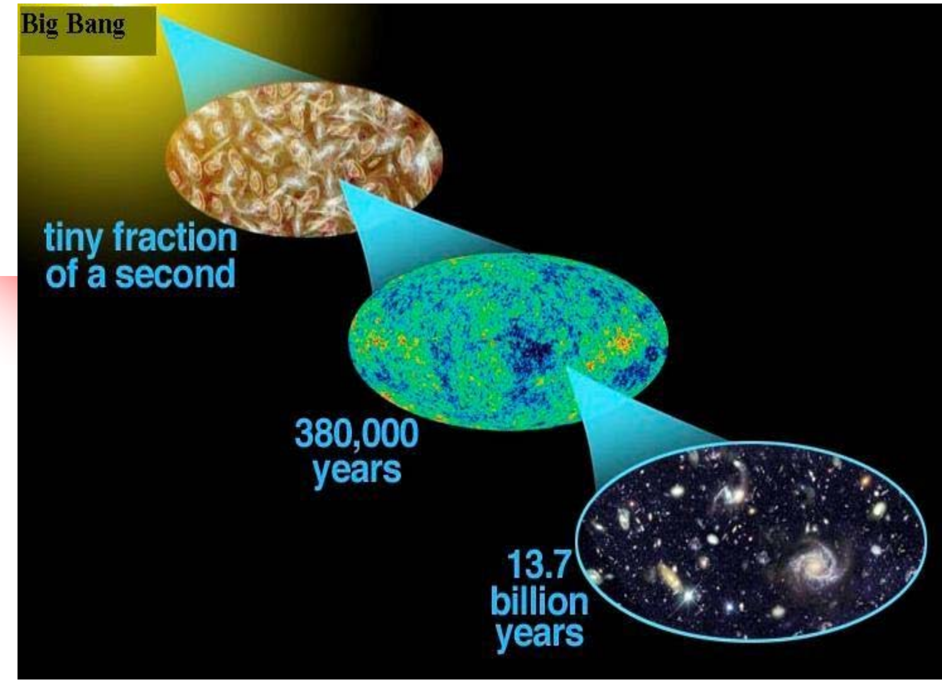
Photo: U. Montan

Adam G. Riess

The Nobel Prize in Physics 2011 was divided, one half awarded to Saul Perlmutter, the other half jointly to Brian P. Schmidt and Adam G. Riess *"for the discovery of the accelerating expansion of the Universe through observations of distant supernovae"*.

Summary:
Possible Causes to Cosmic Acceleration

- Proposed possibilities in thousands of scientific publications:
  - A dark energy component
  - General Relativity Cosmological Constant S
  - A modification to general relativity at cosmological scales; Higher dimensional physics
  - Apparent acceleration due to the fact that we live in a relativistic cosmological model more complex than FLRW
  - A completely unexpected explanation



UT Dallas Cosmology, Relativity, and Astrophysics Group

Faculty and students are involved in:
- theoretical projects
- numerical projects
- data re-use projects (from publicly available databases from NASA, SDSS, …)
For more info, contact
Dr. Ishak at mishak@utdallas.edu
Dr. Rindler at rindler@utdallas.edu
Dr. King at lxk111430@utdallas.edu

(Our group funded by NASA, DOE, and NSF)
http://utdallas.edu/~mxi054000/cosmogroup/

Research Interests include:
- Cosmology and General Relativity
- Cosmological models and probes
- Gravitational lensing
- Cosmic acceleration
- Computational cosmology
- Exact solutions to Einstein's equations and their applications
-Modified gravity models

My collaborators on the work discussed today are:
-Jacob Moldenhaeur (now assistant professor at Francis Marion University)
- Anthony Nwankwo (graduated in Dec. 2011)
- Jason Dossett (Ph.D. student)
- Austin Peel (Ph.D. student)
- Michael Troxel (Ph.D. student)
- David Spergel (Princeton)
- Anzhong Wang (Baylor)

The University of Texas at Dallas

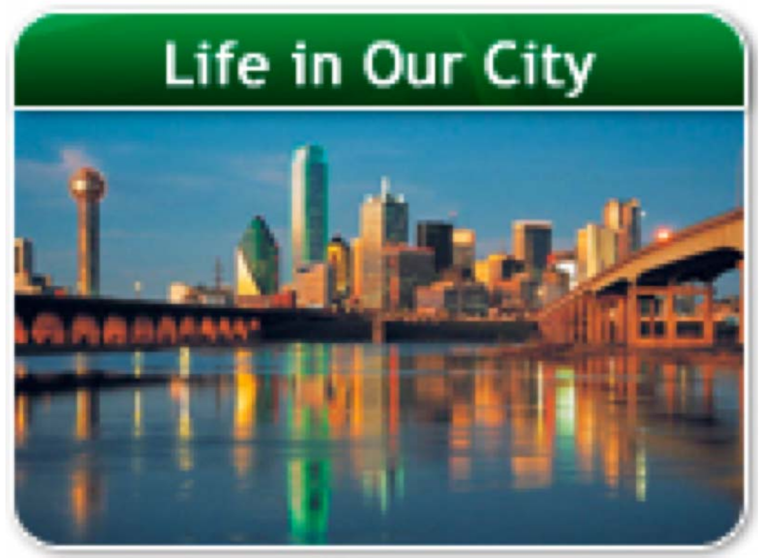
The University of Texas at Dallas is located a few miles north of Dallas

Our student body of about 19000 undergraduates and graduate students is culturally and ethnically diverse.

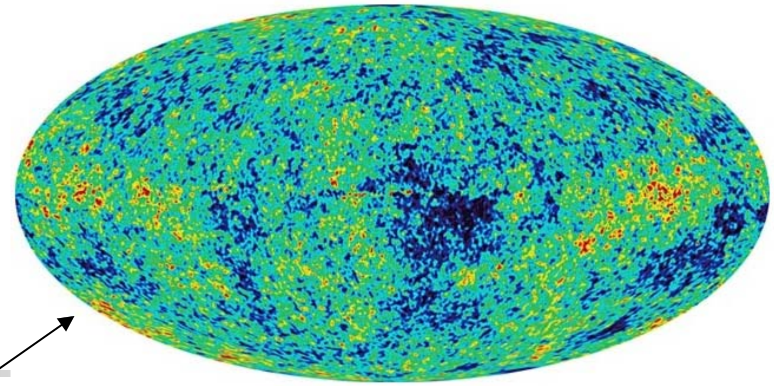
UTD is fully accredited with degrees in natural sciences, engineering, business, and humanities.

The UTD Department of Physics has 18 faculty members specializing in areas including

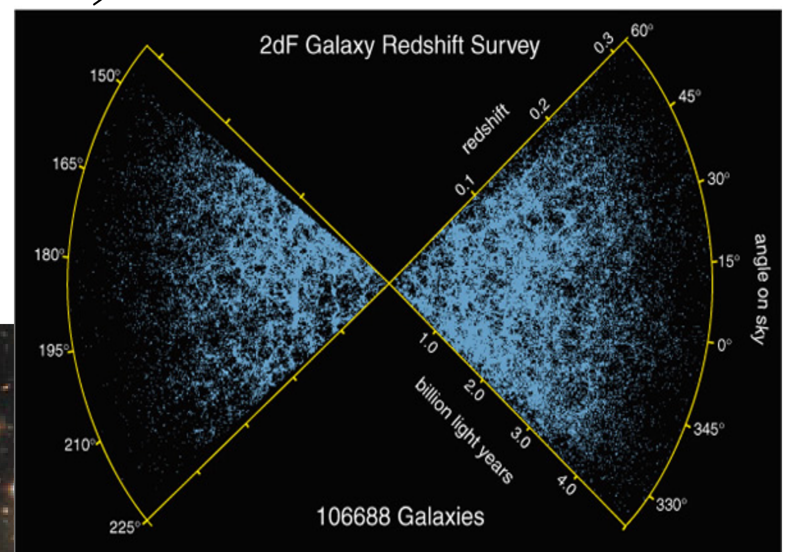
- **Space science**
- **Cosmology and Relativity**
- **Optical, magnetic and structural nanomaterials**
- **Condensed matter theory**
- **Biomaterials**

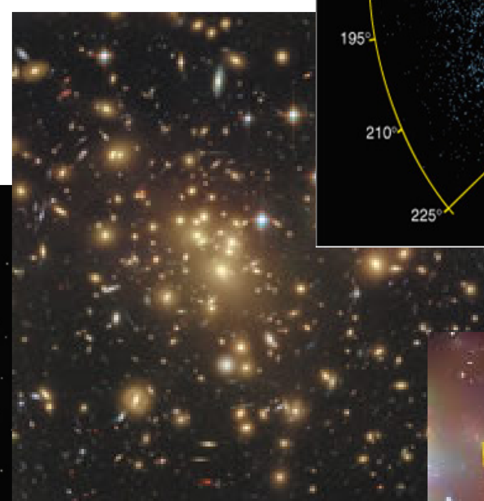


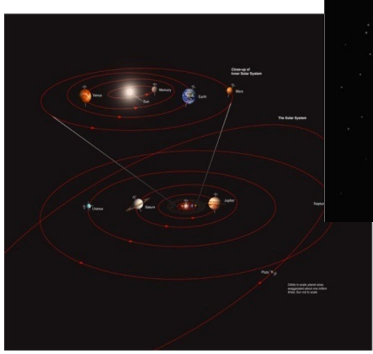
What is cosmology?

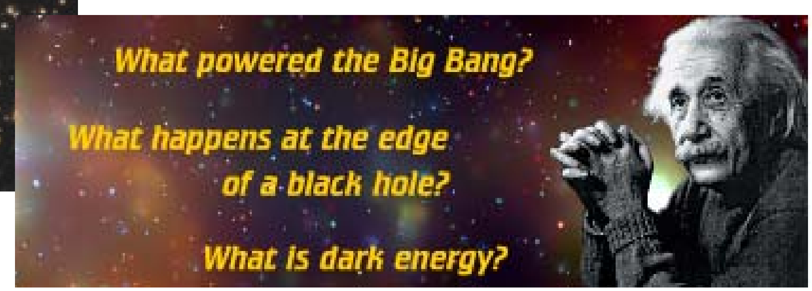


Cosmology is the science that studies the physics and astrophysics of the universe as a whole and also phenomena at very large scales of distance in the universe



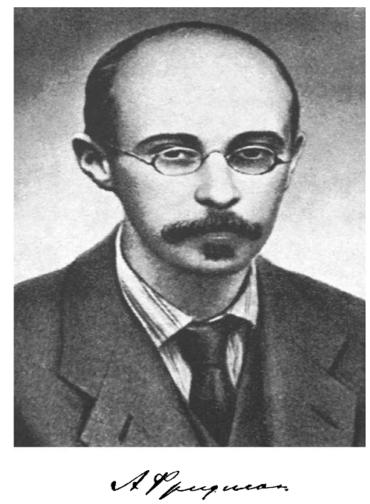






The standard model used in cosmology is called the Friedmann-Lemaitre-Robertson-Walker (FLRW) model

- Based on the General Relativity theory of Einstein, the model combines



- 1) The Big Bang ideas discussed by Friedmann and Lemaitre

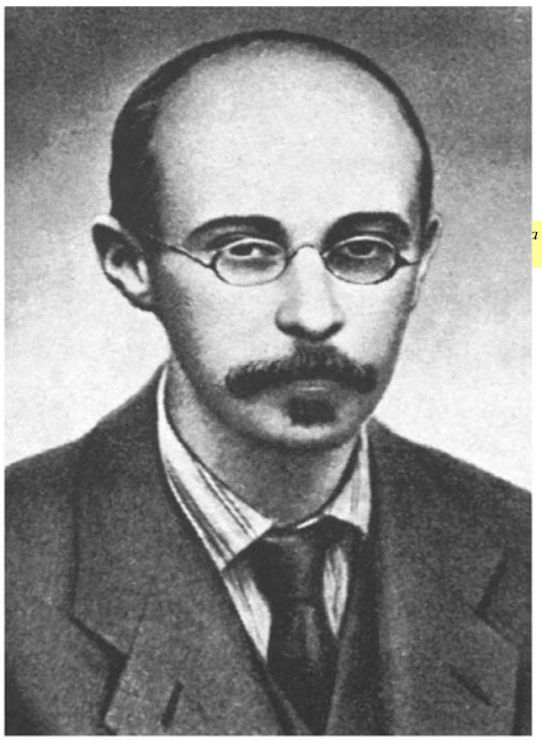
AND

- 2) A geometrical model represented by the metric of Robertson and Walker

Einstein's equations link the geometry of the universe to the matter and energy content of the universe

$$G_b^a = \kappa T_b^a \qquad G_b^a + \Lambda \delta_b^a = \kappa T_b^a$$

These give the Friedmann equations



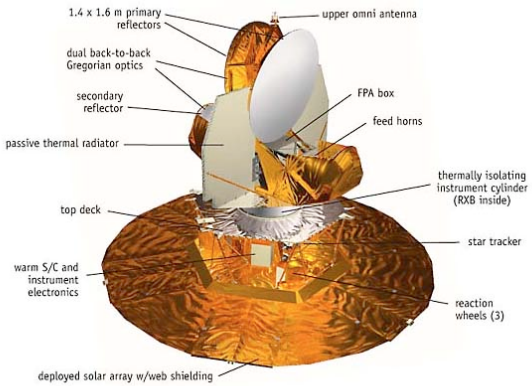
an expansion law for the universe

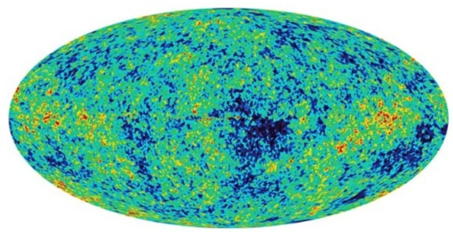
$$H^2(t) = \left(\frac{\dot{a}(t)}{a(t)} \right)^2 = \frac{8\pi\rho}{3} - \frac{k}{a(t)^2} + \frac{\Lambda}{3}$$

and an acceleration/deceleration law for the expansion

$$\frac{\ddot{a}(t)}{a(t)} = \frac{\Lambda}{3} - \frac{4\pi}{3}(\rho + 3p)$$

Great times for Cosmology with a plethora of complementary astronomical data



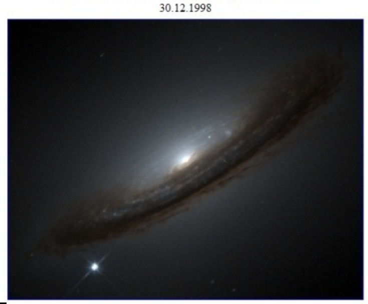


Cosmic Microwave Background (CMB)

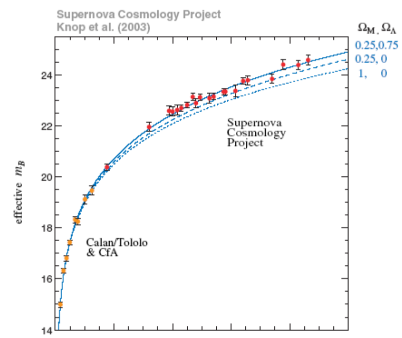
- Distance measurements to Supernovae

Supernova 1994D and the Unexpected Universe

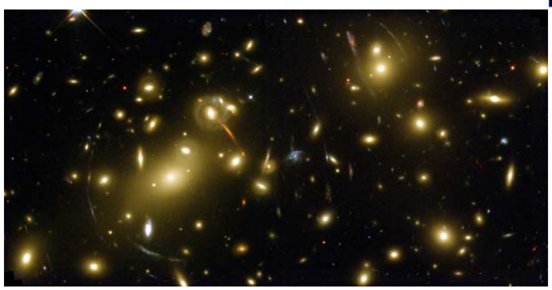
30.12.1998



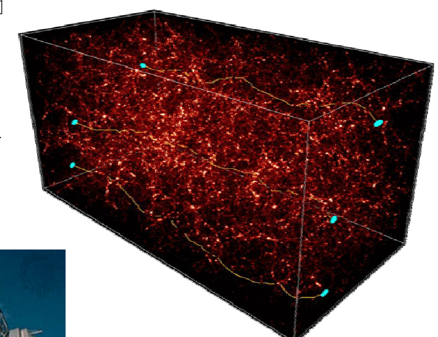
Credit: High-Z Supernova Search Team, HST, NASA



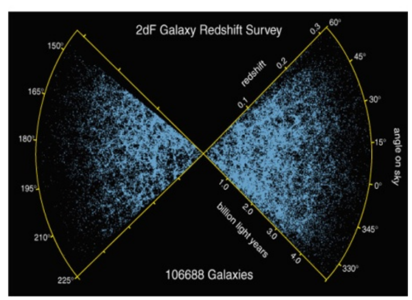
-



Gravitational lensing

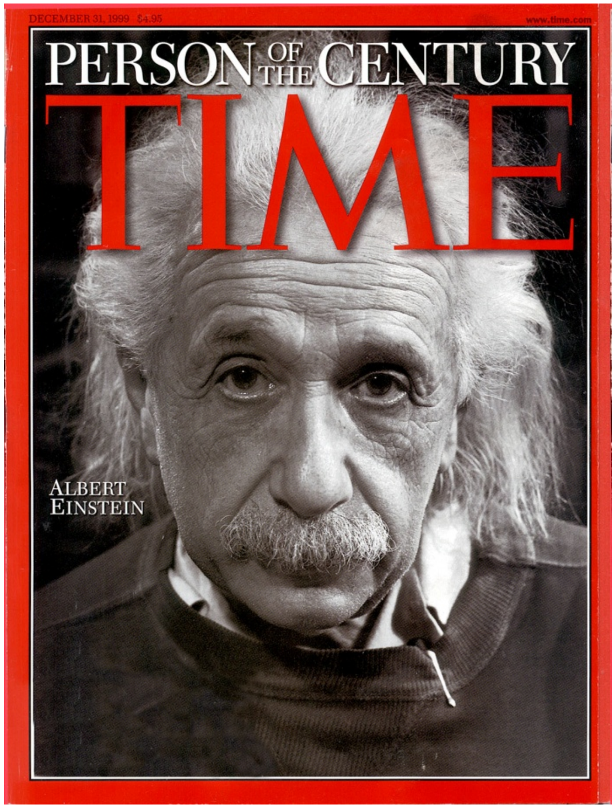


- Large scale structure measurements and surveys



Remarkable progress was achieved during the last century using the standard model

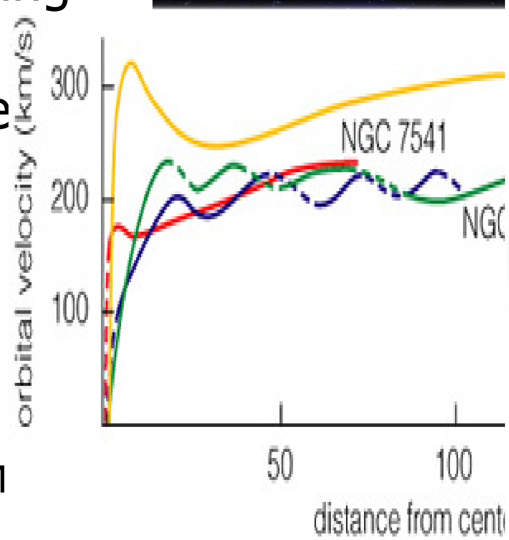
- Precision measurements of the expansion history of the universe
- Detection and precision measurements of the cosmic microwave background (CMB) radiation, a fossil radiations from very early stages of the universe
- A coherent history of structure formations in the universe
- Determination of the age of universe of about 13.7 billions years
- Spatial curvature of the universe is negligible (zero within 1% error)
- Concordance of results from independent cosmological data sets:
  - distances to supernovae
  - CMB
  - gravitational lensing
  - Baryon acoustic oscillations
  - galaxy clustering
  - galaxy cluster counts
  - ...

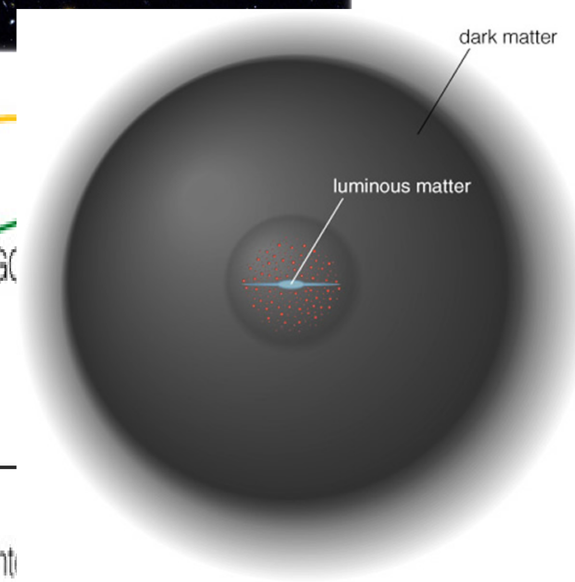


Remarkable puzzles have also been encountered and confirmed during the last century using the standard model

- Perhaps the two most puzzling questions are

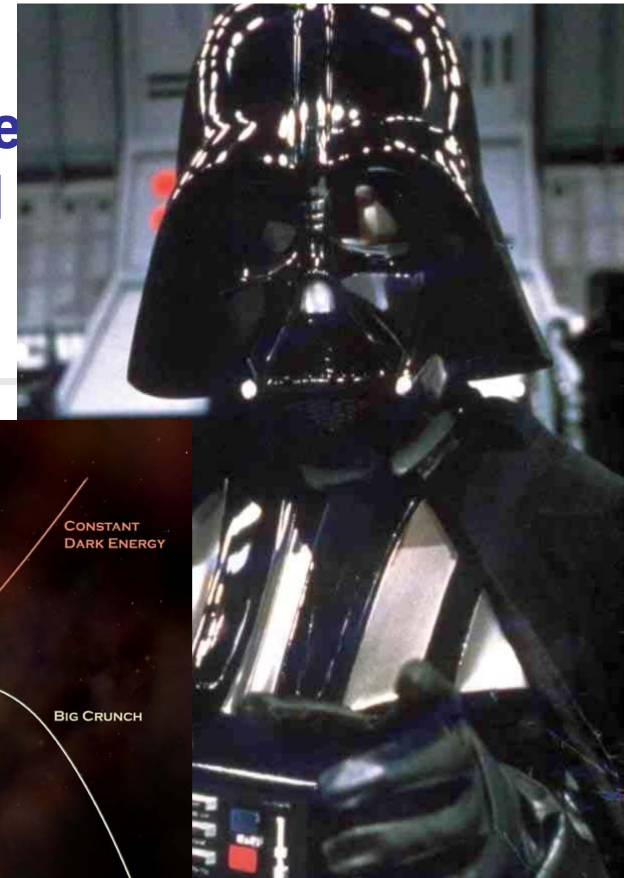
- 1) Dark Matter in galaxies and clusters of galaxies
  - 90% or more of the gravitating matter
  - It is gravitationally attractive like baryonic matter
  - No other interactions with photons or baryons



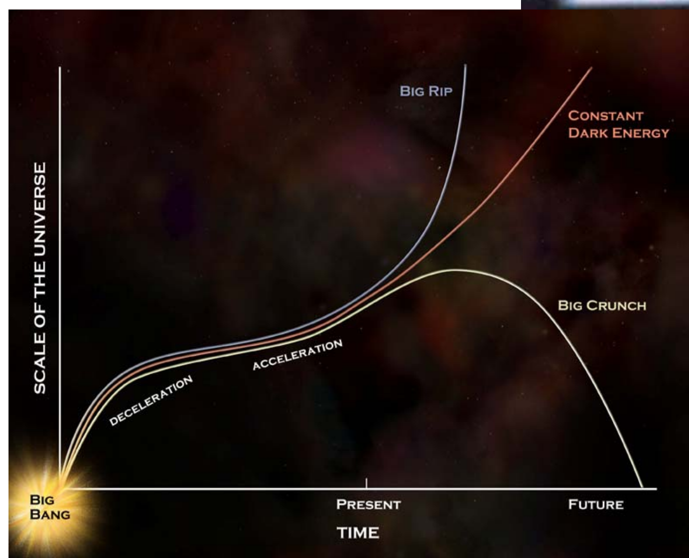


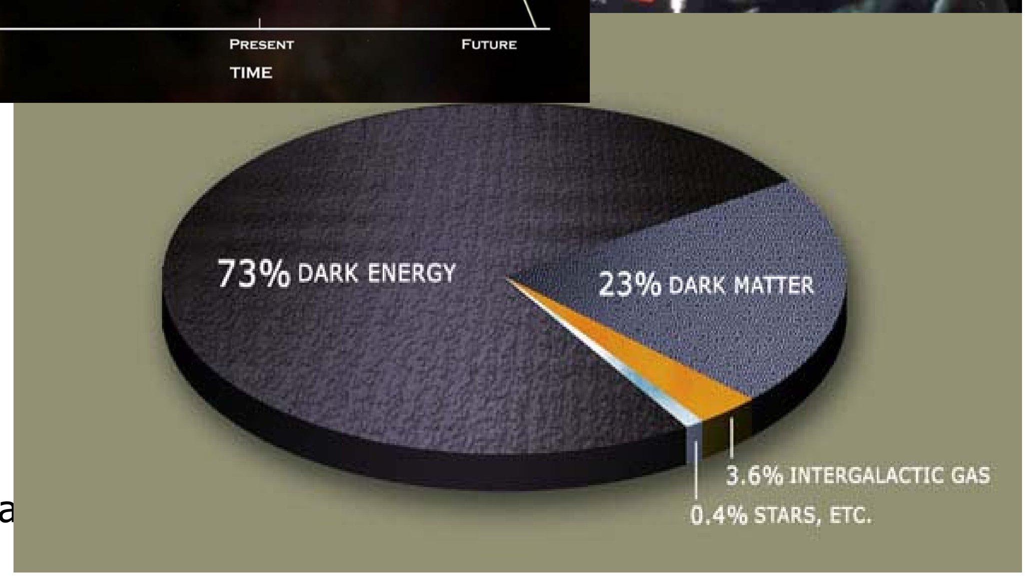
M

Remarkable puzzles have also been encountered and confirmed during the last century using the standard model

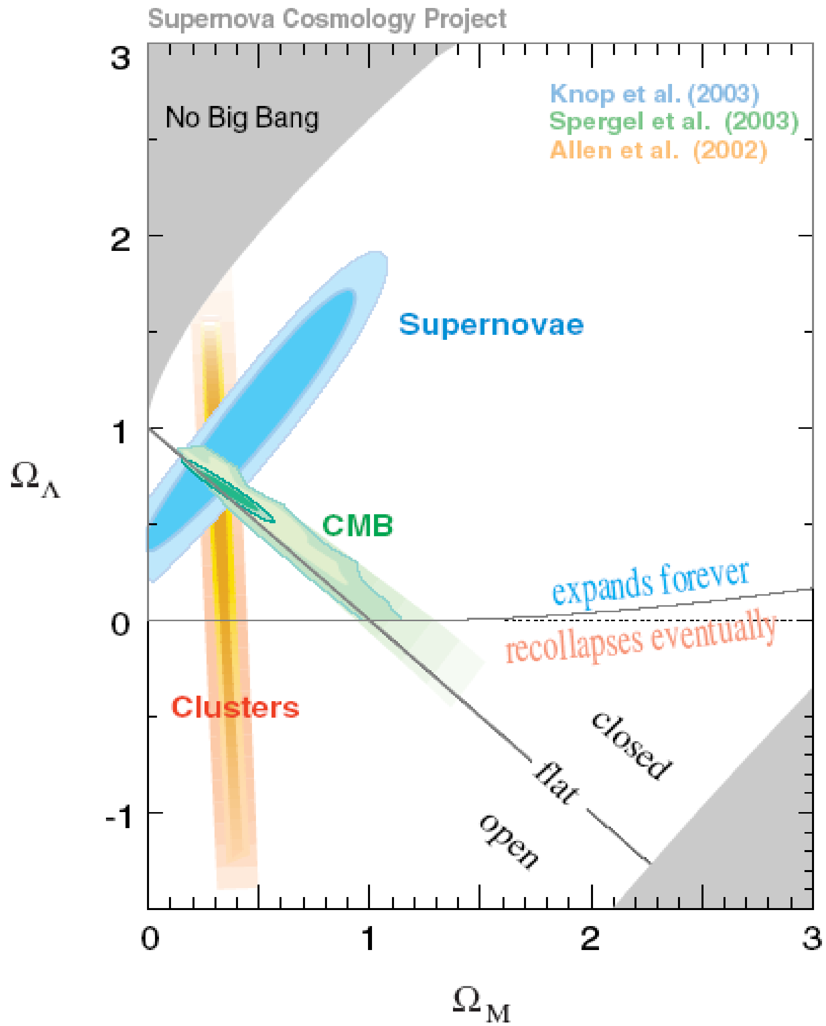


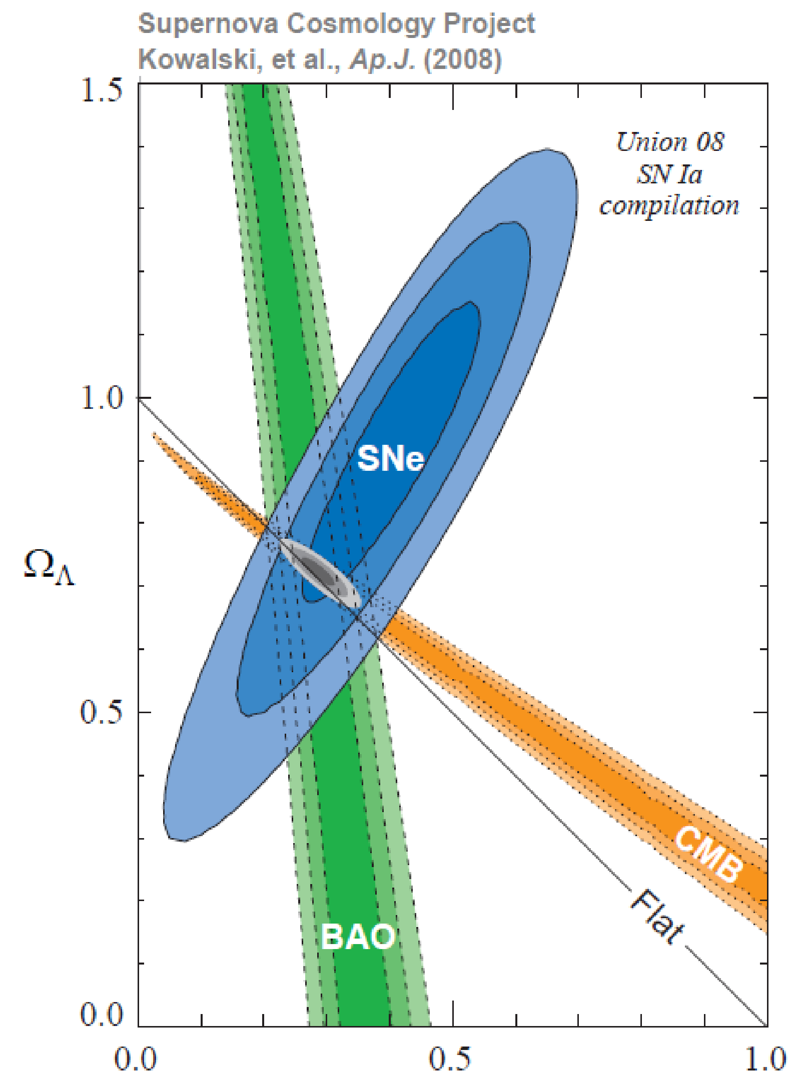
- 2) The expansion of the universe is speeding up
  - One would expect the expansion to be slowing down
  - Complementary data sets have been indicating this for more than a decade now (1998-2011)
  - Problem linked to other fields of physics beside cosmology (HEP, unification theories)





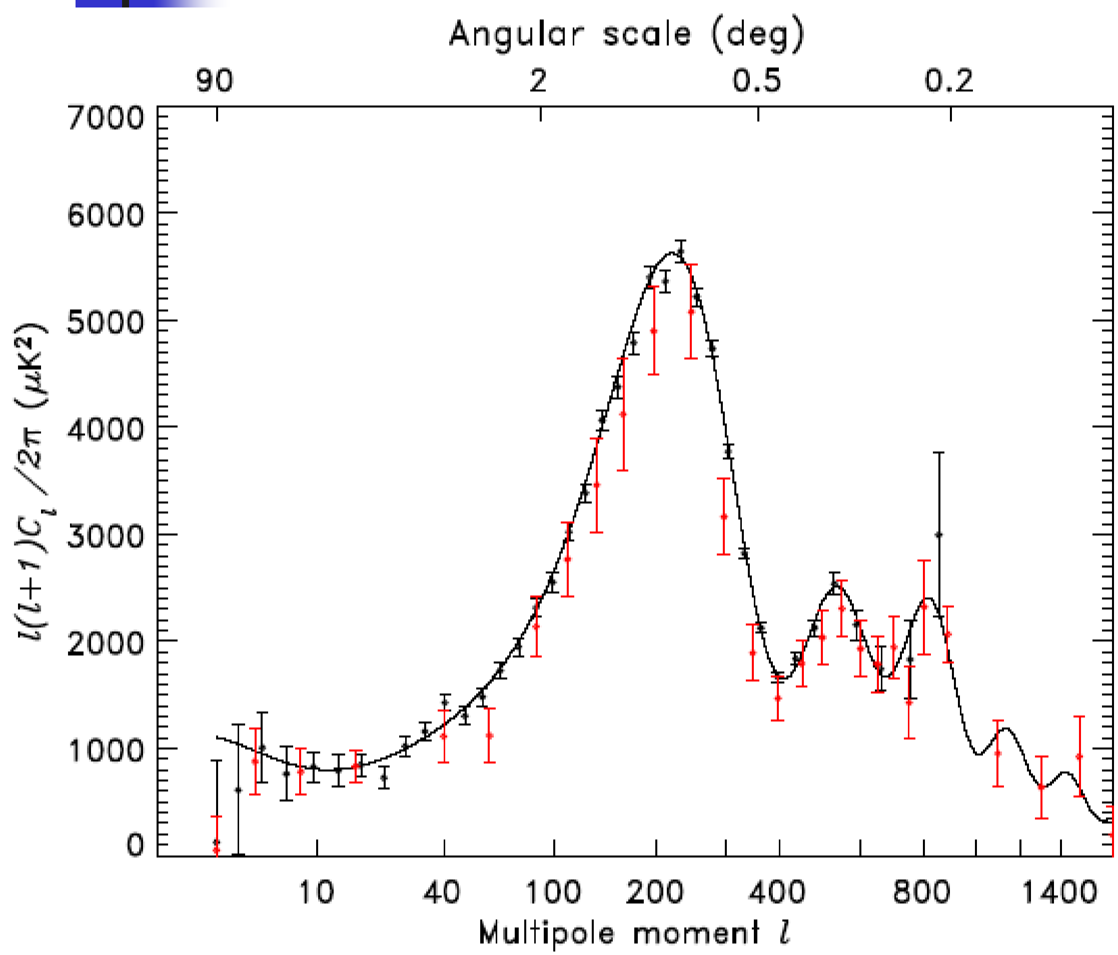
Complementary data sets all agree on the results





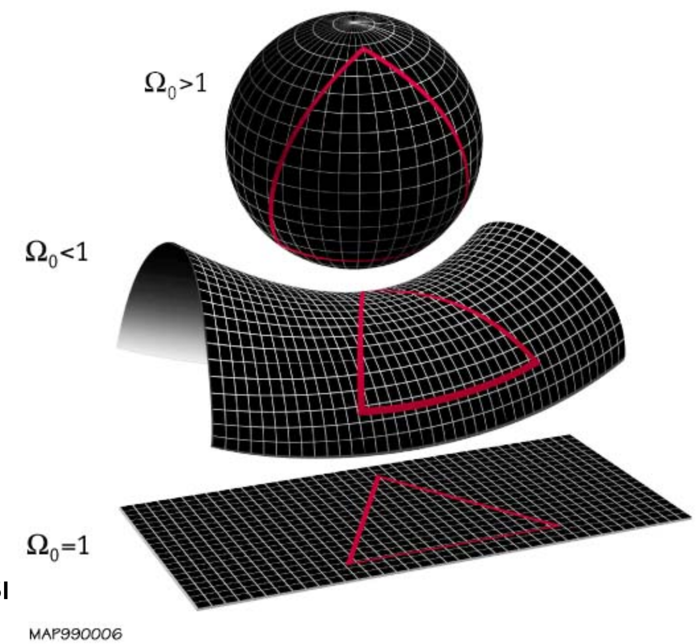
Skip: WMAP data combined with previous published data determined that the Universe is spatially flat with Ω_{total}=1.02 ±0.02, (i.e. negligible spatial curvature)

$$\Omega_B + \Omega_{DM} + \Omega_\Lambda = 1 - \Omega_k = \Omega_{Total}$$



The horizontal position of the peaks of the CMB power spectrum provides constraints on the distance to the surface of last scattering.

The distance found indicates a flat spatial geometry (i.e. negligible spatial curvature)



Why is the expansion of the universe accelerating?

- **Proposed possibilities in thousands of scientific publications:**

I. A dark energy component pervading the universe
  - Vacuum energy (recall QFT, Casimir plates)
  - A quintessence scalar field

II. A geometrical cosmological constant (as in General Relativity). But degenerate with I.

III. A modification to General Relativity at cosmological scales: e.g. higher order gravity models or higher dimensional physics

IV. An apparent acceleration due to an uneven expansion rate in an inhomogeneous cosmological model

V. Something we do not suspect al all.

Possibility I: Dark energy in the form of vacuum energy, cosmological constant, or quintessence field. This is mathematically possible within General Relativity!

(e.g. Upadhye, Ishak, Steinhardt, PRD 2005; Ishak, MNRAS 2005; Ishak, Found. of Physics 2008)

Can produce a cosmic acceleration because of their equation of state once put into Einstein's equations

The equation of state of the "cosmic fluid": $p = w\rho$

- for dust (= galaxies) (i.e. zero pressure) w=0
- for radiation w=1/3
- for a cosmological constant or vacuum energy w=-1

Other Dark Energy models can have w constant or w(t)

Negative w < -1/3 gives an accelerating expansion

$$\frac{\ddot{a}(t)}{a(t)} = -\frac{4\pi}{3}(\rho_{DE} + 3p_{DE})$$

$$\frac{\ddot{a}(t)}{a(t)} = -4\pi\rho_{DE}(\frac{1}{3} + w)$$

Possibility II:
A geometrical constant in the Einstein's equations

$$G_b^a = \kappa T_b^a$$

$$G_b^a + \Lambda \delta_b^a = \kappa T_b^a$$

These give the Friedmann equations with a cosmological constant

$$H^2(t) = \left(\frac{\dot{a}(t)}{a(t)} \right)^2 = \frac{8\pi\rho}{3} - \frac{k}{a(t)^2} + \frac{\Lambda}{3}$$

$$\frac{\ddot{a}(t)}{a(t)} = \frac{\Lambda}{3} - \frac{4\pi}{3}(\rho + 3p)$$

Λ is then just a constant of nature that we measure like Newton's constant, G. This is satisfactory for General Relativity but not for Quantum Field Theory and Unified theories of physics.

Possibility III: Example of modifications or extensions to General Relativity:

Higher order gravity models

- General Relativity is derived from variation of the Ricci scalar

$$S = \frac{M_p}{2}\int d^4x\sqrt{-g}R + \int d^4x\sqrt{-g}L_m$$

$$G_{\alpha\beta} = R_{\alpha\beta} - \frac{1}{2}R\,g_{\alpha\beta} = \frac{1}{M_p^2}T_{\alpha\beta}.$$

- Higher order gravity models are derived from functions of curvature invariants including the Ricci scalar but also other invariants (e.g. Carroll et al. PRD, 2003). Many papers looked at the so-called f(R) models

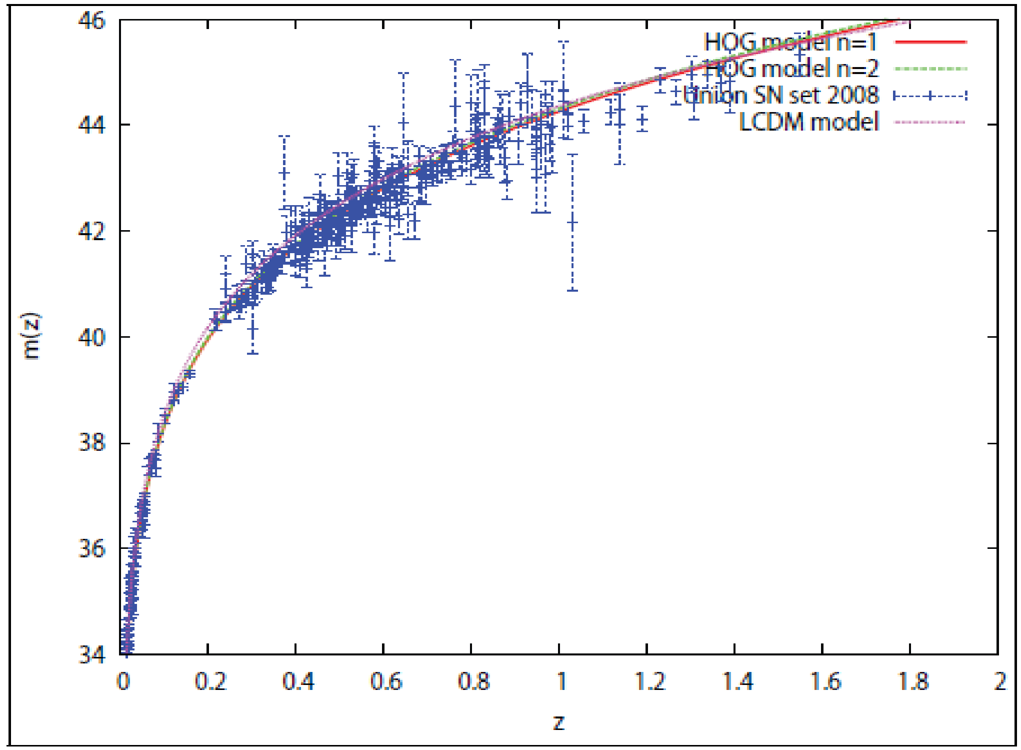
$$S = \frac{M_p}{2}\int d^4x\sqrt{-g}f(R, R^{\alpha\beta}R_{\alpha\beta}, R^{\alpha\beta\gamma\delta}R_{\alpha\beta\gamma\delta}, R^{\alpha\gamma}R_{\alpha\beta}R^{\beta}_{\gamma}, R^{\alpha\beta\mu\nu}R_{\alpha\beta\gamma\delta}R^{\gamma\delta}{}_{\mu\nu}, ...) + \int d^4x\sqrt{-g}L_m$$

- The field equations look like this (e.g. **Ishak and Moldenhauer, JCAP 2009a; Moldenhauer and Ishak, JCAP 2009b, 2010)**

- .
$$S^{\alpha\beta} - \frac{1}{4}g^{\alpha\beta}R - \frac{1}{2}g^{\alpha\beta}f + f_R S^{\alpha\beta} + \frac{1}{4}f_R g^{\alpha\beta}R + g^{\alpha\beta}f_{R;\gamma}{}^{\gamma} - f_{R;}{}^{\alpha\beta} + \frac{1}{2}f_{R1}S^{\alpha\gamma}S^{\beta}{}_{\gamma} + \frac{1}{8}f_{R1}S^{\alpha\beta}R$$
$$+\frac{1}{4}(f_{R1}S^{\alpha\beta})_{;\gamma}{}^{\gamma} + \frac{1}{4}g^{\alpha\beta}(f_{R1}S^{\gamma\delta})_{;\gamma\delta} - \frac{1}{4}(f_{R1}S^{\gamma\beta})_{;}{}^{\alpha}{}_{\gamma} - \frac{1}{4}(f_{R1}S^{\gamma\alpha})_{;}{}^{\beta}{}_{\gamma} = 8\pi G T^{\alpha\beta},$$

Higher-order gravity models fit very well supernova, BAO, distance to CMB surface data

- Same dynamics as GR at galactic and sub-galactic scales
- Accelerate without the need for a dark energy component but because of a different coupling between spacetime geometry and matter-energy content
- With student, we proposed a systematic approach to higher order gravity models
- Figure and generalized Friedmann equation from Moldenhauer and Ishak, JCAP 2009b, 2010

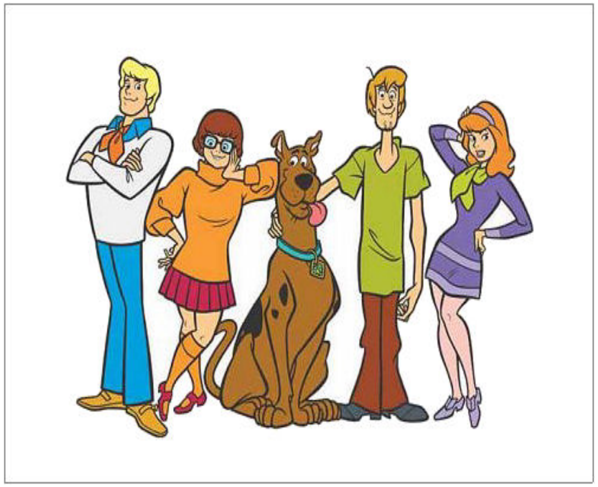


$$3H^2 - \frac{m^6}{6(6\beta\dot{H}^2 + 24\beta H^2\dot{H} + 24\beta H^4 - \dot{H}^2)^3}\Big(6048\beta^2 H^6\dot{H} + 1152\beta^2 H^8 - 240\beta H^2\dot{H}^3 - 360\beta H^4\dot{H}^2$$
$$-6H^2\dot{H}^3 + 5616\beta^2 H^4\dot{H}^2 + 3\dot{H}^4 + 864\beta^2 H^5\ddot{H} + 6H\dot{H}^2\ddot{H} + 1656\beta^2 H^2\dot{H}^3 - 144\beta H^3\dot{H}\ddot{H} + 216\beta^2 H\dot{H}^2\ddot{H}$$
$$-72\beta H\dot{H}^2\ddot{H} + 864\beta^2 H^3\dot{H}\ddot{H} - 36\beta\dot{H}^4 + 108\beta^2\dot{H}^4 + 48\beta H^5\ddot{H} + 144\beta H^6\dot{H}\Big) = 8\pi G\rho_m + 8\pi G\rho_r.$$

A big question: Distinguishing between possibility I: (dark energy) or possibility III (modified gravity) using cosmological data

- An important question is to distinguish between the two possibilities: Dark Energy or Modified gravity
- Comparing the growth rate of large scale structure (the rate of formation of clusters of galaxies) can be used to distinguish between the two competing alternatives
- Two methods have been proposed in literature so far:
  - 1) Looking for inconsistencies in the dark energy parameter spaces
  - 2) Constraining the growth of structure parameters

Trying to find clues from observations



Distinguishing between dark energy and modified gravity via inconsistencies in cosmological parameters

- The cosmic acceleration affects cosmology in two ways:
  - 1) It effects the expansion history of the universe
  - 2) It effects the growth rate of large scale structure in the universe (the rate at which clusters and super clusters of galaxies forms over the history of the universe)

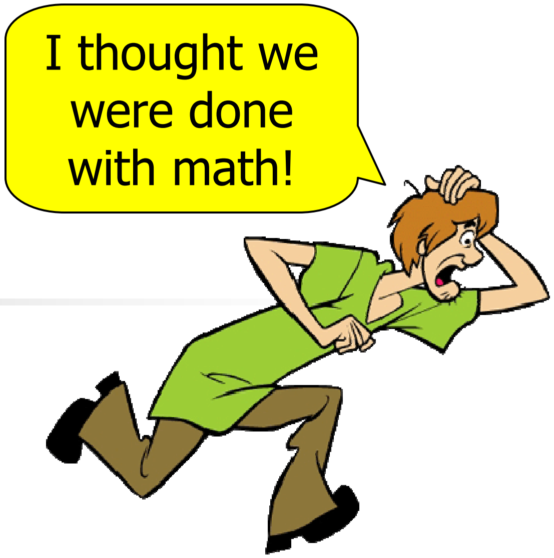
- The idea explored for method one is that, for dark energy models, these two effects must be consistent one with another because they are mathematically related by General Relativity equations

- The idea has been discussed by our group and others groups as well

- We proposed a procedure where the key step was to compare constraints on the expansion and the growth using different and specific pairs of cosmological probes in order to detect inconsistencies

- The presence of significant inconsistencies between the expansion history and the growth rate could be the indication of some problems with the underlying gravity theory

The consistency relation between the expansion history and the growth rate of large scale structure (Ishak, Upadhye, and Spergel, PRD 2006)



- For the standard FLRW model with k=0 and a Dark Energy component, the expansion history is expressed by the Hubble function and is given by

$$H(z) = Ho\sqrt{(1-\Omega_{de})(1+z)^3 + \Omega_{de}\,\varepsilon(z)} \qquad (1)$$

- And the growth rate G(a=1/(1+z)) is given by integrating the ODE:

$$G'' + \left[\frac{7}{2} - \frac{3}{2}\frac{w(a)}{1+X(a)}\right]\frac{G'}{a} + \frac{3}{2}\frac{1-w(a)}{1+X(a)}\frac{G}{a^2} = 0; \quad G(a) = \frac{D(a)}{a}; \quad D(a) = \frac{\delta(a)}{\delta(1)} \qquad (2)$$

- For Modified Gravity DGP models and k=0, the expansion history is given by

$$H(z) = Ho\left[\frac{1}{2}(1-\Omega_m) + \sqrt{\frac{1}{4}(1-\Omega_m)^2 + \Omega_m(1+z)^3}\right] \qquad (3)$$

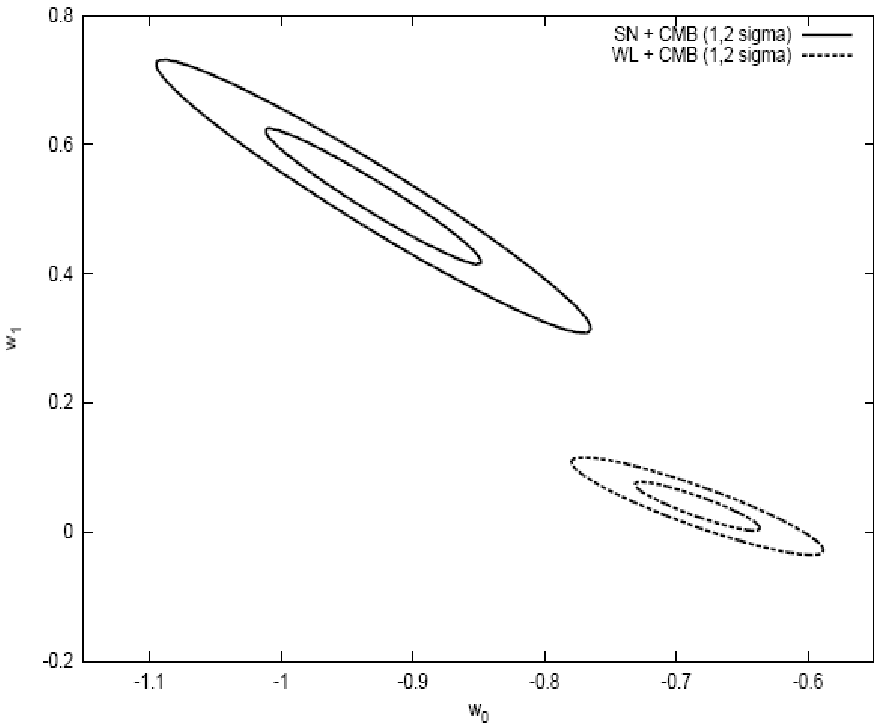
- And the growth rate of function is given by

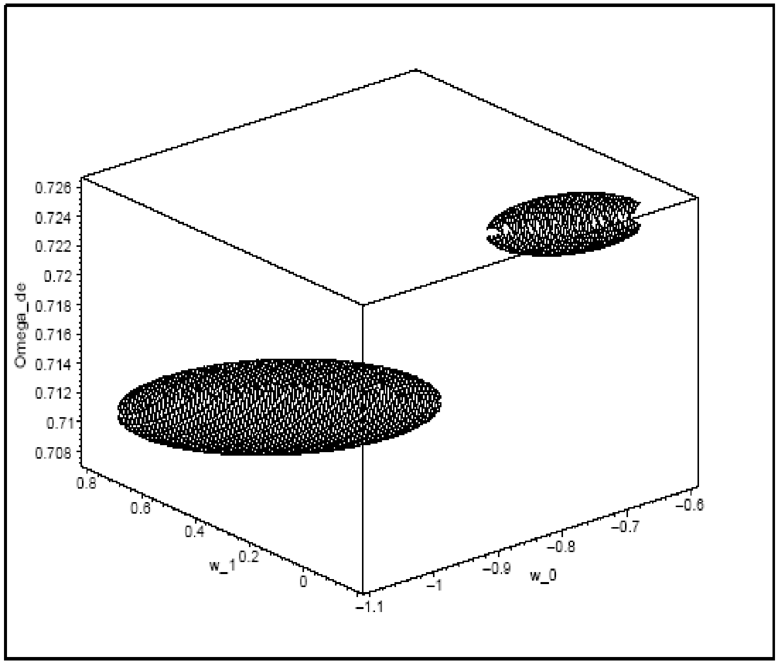
$$\ddot{\delta} + 2H\dot{\delta} - 4\pi G\rho\left(1 + \frac{1}{3\beta}\right)\delta = 0 \qquad \beta = 1 - 2r_c H\left(1 + \frac{\dot{H}}{3H^2}\right) \qquad (4)$$

- Equation (1) and (2) must be mathematically consistent one with another via General Relativity. Similarly, equation (3) and (4) must be consistent one with another via DGP theory

- Our approach uses cosmological probes in order to detect inconsistencies between equations (1) and (2).

Results: Equations of state found using two different combinations of simulated data sets. Solid contours are for fits to the [Supernova + CMB] data combination, while dashed contours are for fits to [Weak Lensing + CMB] data combination.
(MI, Upadhye, and Spergel, Phys.Rev. D74 (2006) 043513)





The significant difference (inconsistency) between the equations of state found using these two combinations is a due to the DGP model in the simulated data.

In this simulated case, The inconsistency tells us that we are in presence of modified gravity rather than GR+Dark Energy.

Method two: is based on parameterization of the Growth rate of large scale structure

Gong, Ishak, Wang 2009; Ishak, Dossett, 2009;

Dosset, Ishak, Moldenhauer, Gong, Wang, 2010)

- large scale matter density perturbation, $\delta = \Delta\rho_m / \rho_m$, satisfies the ODE:

$$\ddot{\delta} + 2H\dot{\delta} - 4\pi G_{eff}\,\rho_m \delta = 0$$

- The ODE can be written in terms of the logarithmic growth rate $f = d\ln\delta / d\ln a$ as:

$$f' + f^2 + \left(\frac{\dot{H}}{H^2} + 2\right) f = \frac{3}{2}\frac{G_{eff}}{G}\Omega_m$$

where the underlying gravity theory is expressed via the expression for G_{eff}, *H(z), and* $\Omega_m(z)$.

A constant growth rate index parameter

- The growth function f *can be* approximated using the ansatz

$$f = \Omega_m^{\gamma}$$

where γ is the growth index parameter

- It was found there that

$$f(z) = \Omega_m^{0.6} \qquad f = \Omega_m^{4/7}$$

were good approximations for matter dominated models.

The growth index parameter as a discriminator for Gravity Theories

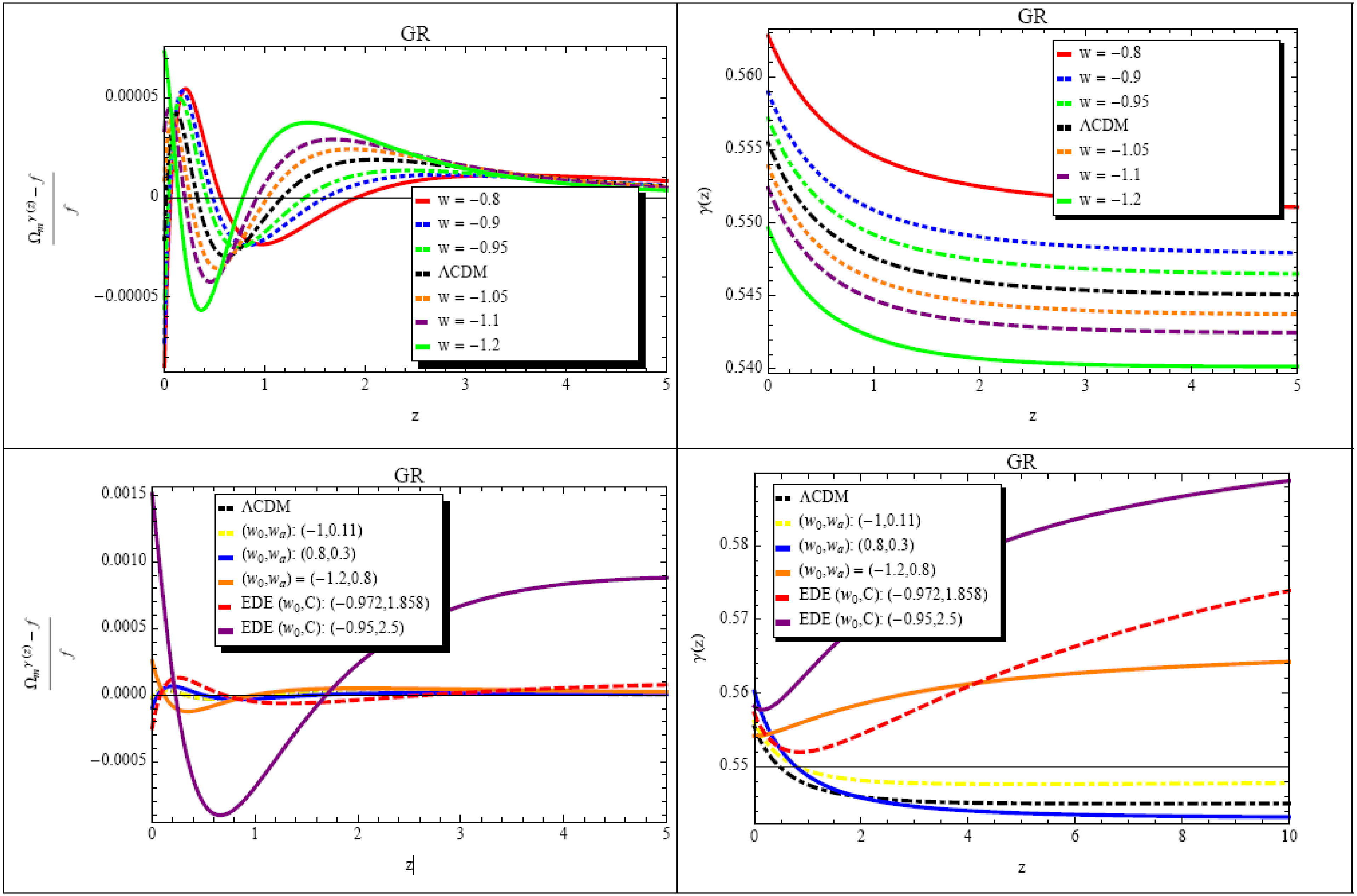
- The asymptotic constant growth index parameter takes distinctive value for distinct gravity theories

- Thus, can be used to probe the underlying gravity theory and the cause of cosmic acceleration

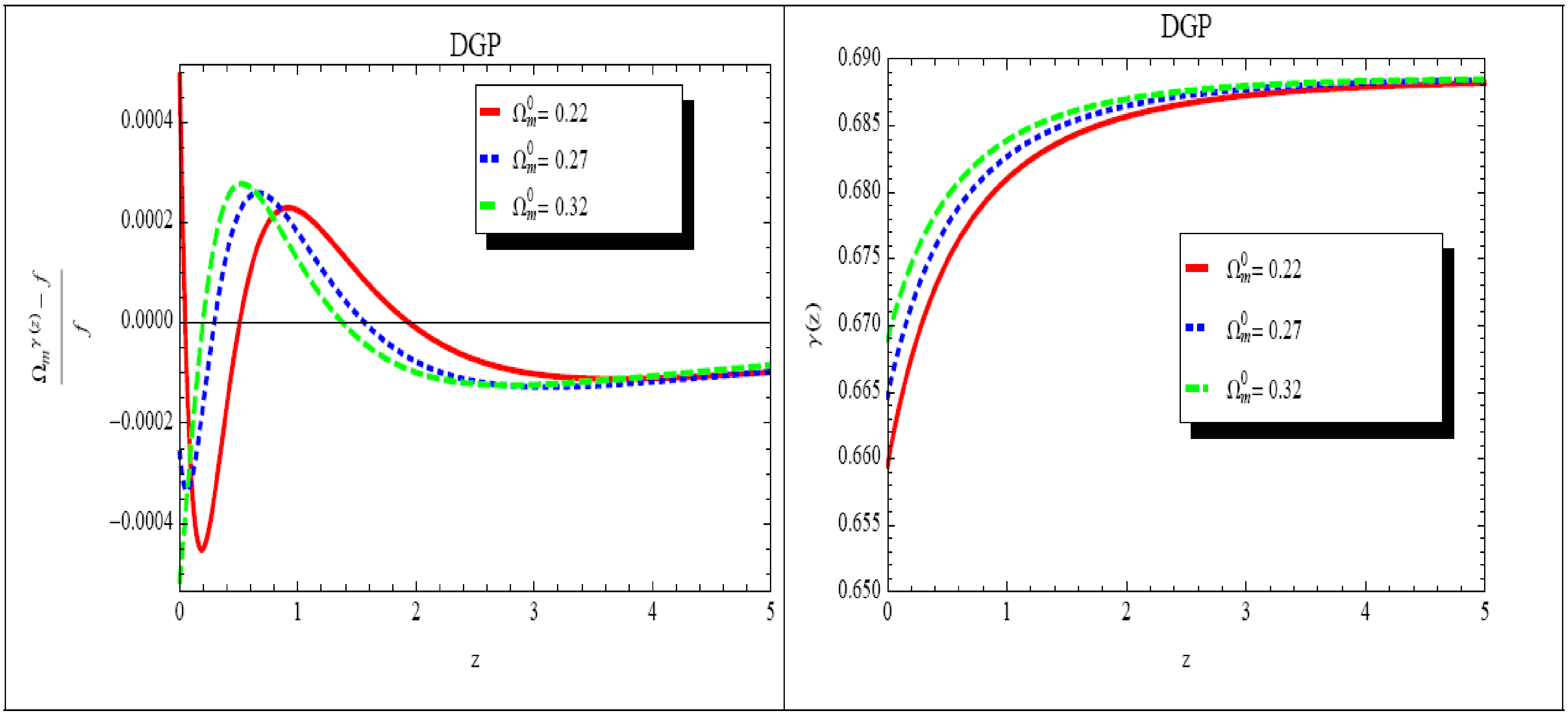
- γ=6/11=0.545 for the Lambda-Cold-Dark-Matter model. (i.e. for w=-1), i.e. General Relativistic Models.

- γ=11/16=0.687 for the flat DGP modified gravity model [e.g. Linder and Cahn, 2007; Gong 2008].

Growth index parameter for GR + Dark Energy models. LEFT: Very precise parameterization. RIGHT: Very little dispersion around the γ=6/11=0.545



Growth index parameter for DGP models.
LEFT: Very precise parameterization.
RIGHT: Very little dispersion around the γ=11/16=0.687



Method II: Modified growth parameters (MG parameters).

- MG parameters, P, Q, and D, take value 1 in GR but deviate from it in modified gravity models.

$$k^2\phi = -4\pi G a^2 \sum_i \rho_i \Delta_i Q$$

$$k^2(\psi - R\phi) = -12\pi G a^2 \sum_i \rho_i (1+w_i)\sigma_i Q,$$

$$k^2(\psi+\phi) = -8\pi G a^2 \sum_i \rho_i \Delta_i \mathcal{D} - 12\pi G a^2 \sum_i \rho_i (1+w_i)\sigma_i Q.$$

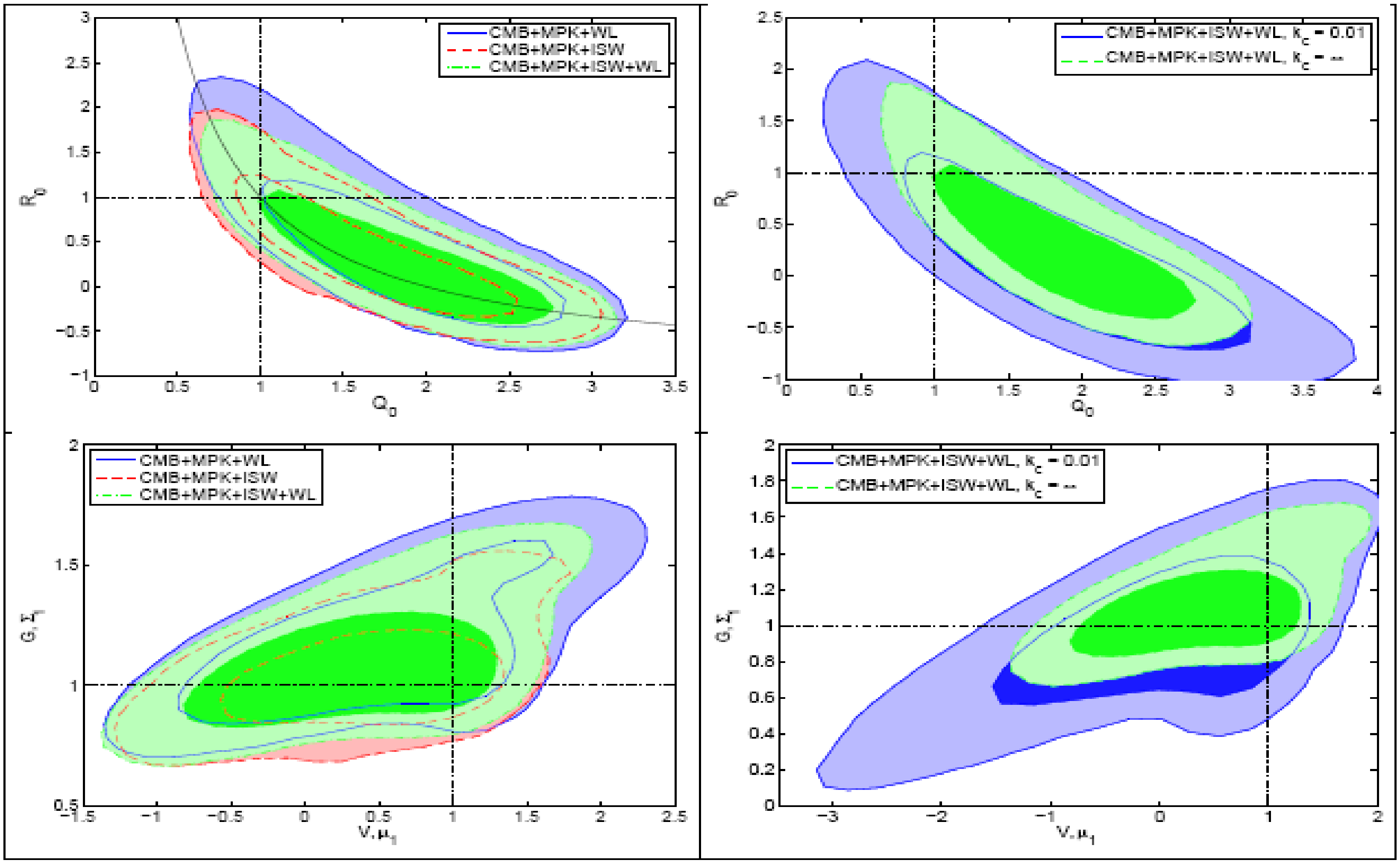
- Dossett, Ishak, Moldenhauer, PRD, 2011a, 2011b; Dossett, Ishak, PRD submitted 2012)

- See also IsitGR software package at http://www.utdallas.edu/~jdossett/isitgr/, used by at least 4 other groups in the world working on the question (UK, Italy, Portugal, Romania)

Using the latest cosmological data sets including refined COSMOS 3D weak lensing (Jason Dossett, Jacob Moldenhauer, Mustapha Ishak)

Phys.Rev.D84:023012,2011

No apparent deviation from GR using current data. More precise data coming.



ISiTGR:
Integrated Software in Testing General Relativity

Version 1.1

Developed by **Jason Dossett**, **Mustapha Ishak**, and **Jacob Moldenhauer**.

What is ISiTGR?

ISiTGR is an integrated set of modified modules for the software package CosmoMC for use in testing whether observational data is consistent with general relativity on cosmological scales. This latest version of the code has been updated to allow for the consideration of non-flat universes. It incorporates modifications to the codes: CAMB, CosmoMC, the ISW-galaxy cross correlation likelihood code of Ho et al, and our own weak lensing likelihood code for the refined COSMOS 3D weak lensing tomography of Schrabback et al to test general relativity.

A detailed explanation of the modifications made to these codes allowing one to test general relativity are described in our papers: arXiv:1109.4583 and arXiv:1205.2422.

How to get ISiTGR

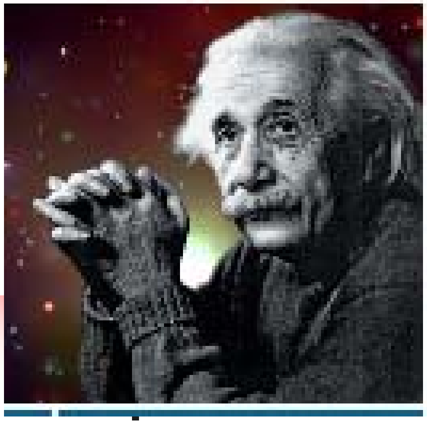
Two versions of ISiTGR are available. The normal version of ISiTGR uses a functional form to evolve the parameters used to test general relativity and is available here. ISiTGR_BIN, on the other hand, gives you two options to evovle the parameters used to test general relativity. The first option is to bin the parameters in two redshift and two scale bins, alternatively one can use the hybrid evolution method, as seen in our paper, where scale dependence evolves monotonically, but redshift dependence is binned. That code can be downloaded here.

Download Here: ISiTGR ISiTGR_BIN

The original (flat only) verison of ISiTGR as well as builds for other versions of CosmoMC are available here (**this version is for CosmoMC 01/2012**).

Possible Causes of Cosmic Acceleration

- Proposed possibilities in thousands of scientific publications:
  - A dark energy component
  - GR cosmological constant
  - A modification to general relativity at cosmological scales; Higher dimensional physics
  - → Apparent acceleration due to the fact that we live in a relativistic cosmological model more complex than FLRW

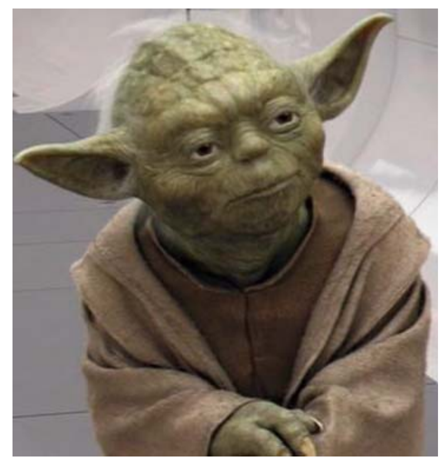


Possibility IV:
"May General Relativity Be With You"
(Jedi Einstein)

- **A fourth possibility: Apparent acceleration due to the fact that we live in a relativistic cosmological model more complex than FLRW**

- GR history is full of surprises: starting from the prediction of a non-static expanding universe which already encountered some resistance

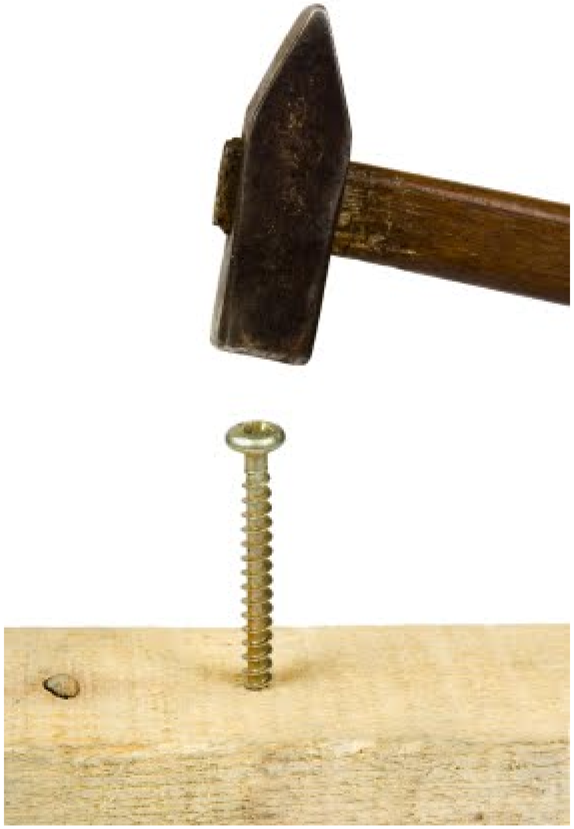
"May the force be with you" , (Jedi Yoda)



Today: Dark Side times (Dark Energy, Dark Matter, Cosmological constant, Modified Gravity models...)

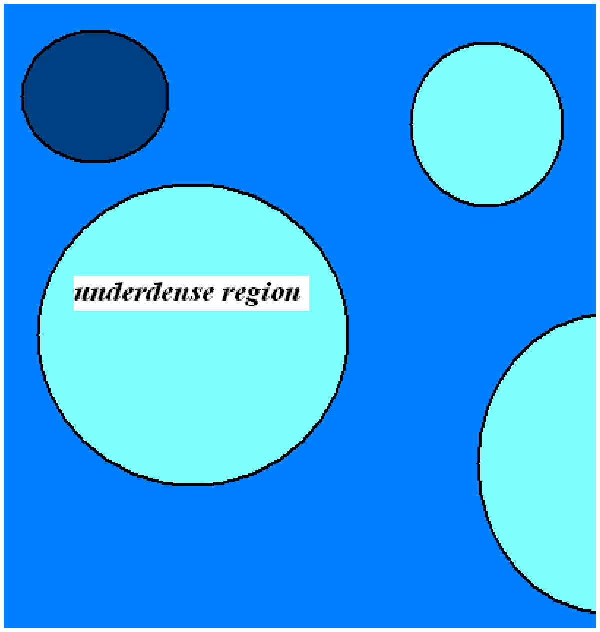
Do we have the right model in hands?

- We can't explain ~70% (or~95%) of the observed dynamics
- Observations of the expansion rate of Supernovae can have different interpretations in FLRW versus an Inhomogeneous model
- Do we live in a complex and subtle general relativistic cosmological model?
- Is the FLRW model limiting our ability to interpret observations?
- Well motivated questions in view of the non-linearity of GR, and the unsolved averaging problem in cosmology



Apparent acceleration seen from one of the under-dense regions in the universe

- Apparent acceleration can result from the Hubble parameter, H_0, being larger inside the under-dense region than outside of that region
- In FLRW, H(t) is a function of time only but in inhomogeneous models H(t,**r**) is a function of time and space
- Supernova observations imply a larger H_0 at low redshifts then at higher redshifts
- In FLRW models this implies acceleration while in inhomogeneous models different values of H are possible without acceleration



Apparent acceleration using the Szekeres-Szafron inhomogeneous models

- Several interesting papers explored the question using the Lemaitre-Tolman-Bondi (LTB) models
- However, because of the spherical symmetry of LTB, the results can be viewed as a proof of concept unless we sacrifice the cosmological/Copernican principle
- It is desirable to explore the question of apparent acceleration using more general models than LTB
- Derived by Szekeres (1975) with no-symmetries (no killing vector fields) with a dust source. Generalized to perfect fluids by Szafron (1977). Studied by a number of authors.
- Regarded as good models to study our inhomogeneous universe (GFR Ellis)
- Have a flexible geometrical structure that can fit cosmological constraints and observations at various scales

The approach is consistent with the Copernican Principle

- Clarification: we are not proposing the Szekeres model as the true model of the universe
- In this scenario, apparent acceleration is due to the fact that we happen to live in **one of the many** under-dense regions of the universe.
- No need to be close to the center of the under-dense region. In fact, there is no exact definition of a center in these models since not spherically symmetric

So this is not inconsistent with the Copernican Principle (Nicholas Copernicus) or the cosmological principle

Apparent acceleration using the Szekeres-Szafron inhomogeneous models

- The Szekeres metric in KH coordinates

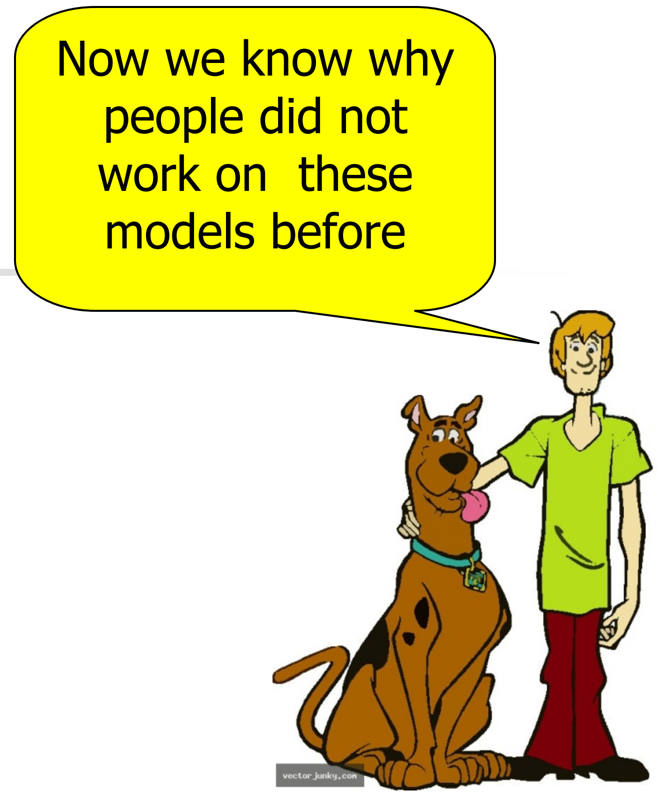
$$ds^2 = c^2 dt^2 - \frac{\left(R'(t,r) - R(t,r) \frac{E'(r,p,q}{E(r,p,q)} \right)^2}{\varepsilon - k(r)} dr^2 - R(t,r)^2 \frac{dp^2 + dq^2}{E(r,p,q)^2}$$

- There are sub-cases and we explored one of them but plan to look into the other cases as well
- hyperbolic (*$k(r)<0$*), parabolic (*$k(r)=0$*), and elliptic (*$k(r)>0$*)
- The function *$E(r,p,q)$* and the constant *$\varepsilon=0,+1,$or -1 also define further sub-cases and* mapping of various hyper-surfaces.

Observations in inhomogeneous models and the null geodesic equations

- The null geodesic equations describe the motion of light rays arriving to us from astronomical objects
- It is necessary to solve these equations it in order to derive observable functions, such as the luminosity-distance to supernovae
- This equation is easily solved in the FLRW but not in the Szekeres models, and here we employ an analytical and numerical approach to the problem

Observations in inhomogeneous models and the null geodesic equations (not radial)



$$\frac{d^2t}{d\lambda^2}+\frac{R_{,tr}-R_{,t}E_{,r}}{1-k}\left(R_{,r}-\frac{RE_{,r}}{E}\right)\left(\frac{dr}{d\lambda}\right)^2+\frac{RR_{,t}}{E^2}\left[\left(\frac{dp}{d\lambda}\right)^2+\left(\frac{dq}{d\lambda}\right)^2\right]=0$$

$$\frac{d^2r}{d\lambda^2}+\left(2\frac{R_{,tr}-\frac{R_{,t}E_{,r}}{E}}{R_{,r}-\frac{RE_{,r}}{E}}\right)\left(\frac{dt}{d\lambda}\frac{dr}{d\lambda}\right)+\left(\frac{R_{,rr}-\frac{R_{,r}E_{,r}}{E}-\frac{RE_{,rr}}{E}+R\left(\frac{E_{,r}}{E}\right)^2}{R_{,r}-\frac{RE_{,r}}{E}}+\frac{k_{,r}}{2(1-k)}\right)\left(\frac{dr}{d\lambda}\right)^2$$

$$+\left(2\frac{R}{E^2}\frac{E_{,r}E_{,p}-EE_{,pr}}{R_{,r}-\frac{RE_{,r}}{E}}\right)\left(\frac{dr}{d\lambda}\frac{dp}{d\lambda}\right)+\left(2\frac{R}{E^2}\frac{E_{,r}E_{,q}-EE_{,qr}}{R_{,r}-\frac{RE_{,r}}{E}}\right)\left(\frac{dr}{d\lambda}\frac{dq}{d\lambda}\right)-\frac{R}{E^2}\frac{1-k}{R_{,r}-\frac{RE_{,r}}{E}}\left[\left(\frac{dp}{d\lambda}\right)^2+\left(\frac{dq}{d\lambda}\right)^2\right]=0$$

$$\frac{d^2p}{d\lambda^2}+2\frac{R_{,t}}{R}\left(\frac{dt}{d\lambda}\frac{dp}{d\lambda}\right)-\left(\frac{R_{,r}-\frac{RE_{,r}}{E}}{R(1-k)}\left(E_{,r}E_{,p}-EE_{,pr}\right)\right)\left(\frac{dr}{d\lambda}\right)^2+2\left(\frac{R_{,r}}{R}-\frac{E_{,r}}{E}\right)\left(\frac{dr}{d\lambda}\frac{dp}{d\lambda}\right)-\frac{E_{,p}}{E}\left(\frac{dp}{d\lambda}\right)^2-2\frac{E_{,q}}{E}\left(\frac{dp}{d\lambda}\frac{dq}{d\lambda}\right)+\frac{E_{,p}}{E}\left(\frac{dq}{d\lambda}\right)^2=0$$

$$\frac{d^2q}{d\lambda^2}+2\frac{R_{,t}}{R}\left(\frac{dt}{d\lambda}\frac{dq}{d\lambda}\right)-\left(\frac{R_{,r}-\frac{RE_{,r}}{E}}{R(1-k)}\left(E_{,r}E_{,q}-EE_{,qr}\right)\right)\left(\frac{dr}{d\lambda}\right)^2+2\left(\frac{R_{,r}}{R}-\frac{E_{,r}}{E}\right)\left(\frac{dr}{d\lambda}\frac{dq}{d\lambda}\right)+\frac{E_{,q}}{E}\left(\frac{dp}{d\lambda}\right)^2-2\frac{E_{,q}}{E}\left(\frac{dp}{d\lambda}\frac{dq}{d\lambda}\right)-\frac{E_{,q}}{E}\left(\frac{dq}{d\lambda}\right)^2=0$$

Observations in inhomogeneous models and the null geodesic equations: numerical integration

- We introduced the redshift in the equations

$$\frac{d(ln(1+z))}{d\lambda} = -\frac{1}{\frac{dt}{d\lambda}}\left(\left(\frac{\dot{R}'R + R\dot{R}(\frac{E'}{E})^2 - (R'\dot{R} + R\dot{R}')\frac{E'}{E}}{1-k}\right) \times \left(\frac{dr}{d\lambda}\right)^2 + \frac{R\dot{R}}{E^2}\left(\left(\frac{dp}{d\lambda}\right)^2 + \left(\frac{dq}{d\lambda}\right)^2\right)\right). \quad (26)$$

- The system can be regarded as a second order ODE system with the parameters given by the Einstein Field Equations

- Further, we used the Runge-Kutta method with the following vectors in order to separate the 4 second order ODEs to 8 first order ODEs

$$y = \left\{t, r, p, q, \frac{dt}{dl}, \frac{dr}{dl}, \frac{dp}{dl}, \frac{dq}{dl}\right\} \quad (27$$

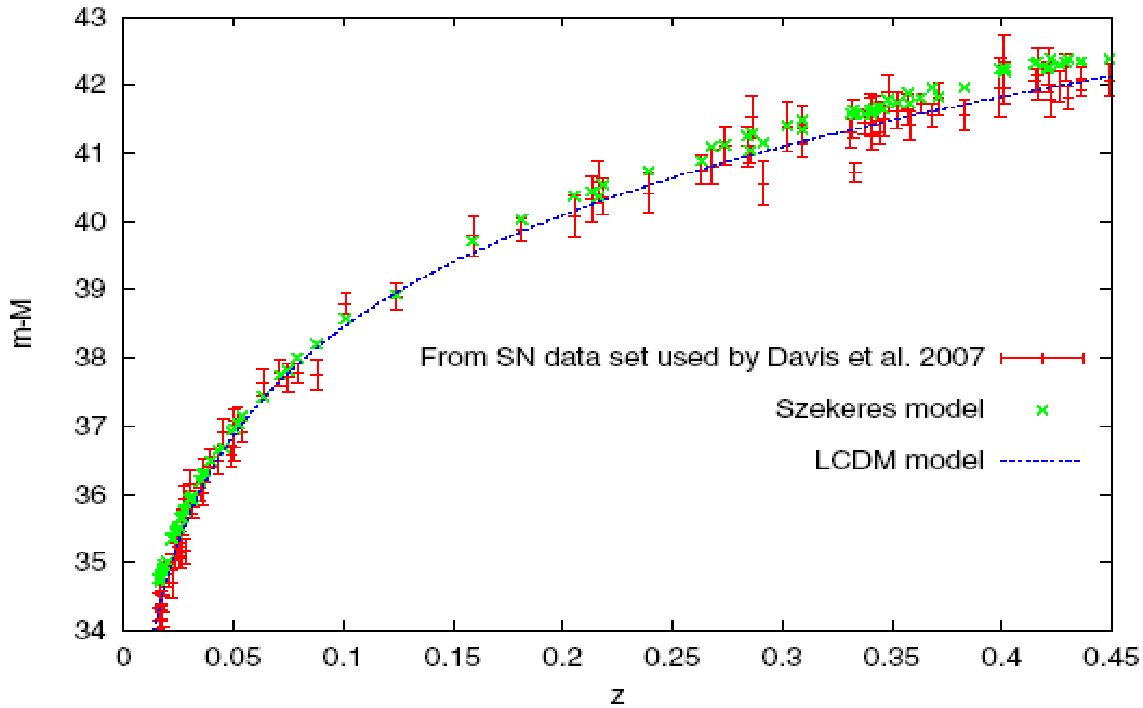
and

$$\frac{dy}{dl} = \left\{\frac{dt}{dl}, \frac{dr}{dl}, \frac{dp}{dl}, \frac{dq}{dl}, \frac{d^2t}{dl^2}, \frac{d^2r}{dl^2}, \frac{d^2p}{dl^2}, \frac{d^2q}{dl^2}\right\} \quad (28$$

Hubble diagram for the Szekeres models

- The luminosity-distance is found numerically using
- $$d_L(z) = (1+z)^2 \frac{R(t,r)}{\tilde{E}(r,p,q)}$$
- It dependends on r, p, and q (or similarly on r, theta, and phi)
- Next, the magnitude is given by $m(z) - M = 5\log_{10}(d_L) + 25$
- We used Supernova Combined Data Set as in Davis et al 2007, Wood-Vasey et al 2007, and Riess et al 2007.

Results: Ishak et al. Phys. Rev. D 78, 123531 (2008)



- **The data is 94 Supernova (up to $1+z=1.449$) from Davis et al 2007, Wood-Vasey et al 2007, and Riess et al 2007**
- **The Szekeres model fits the data with a chi^2=112. This is close to the chi^2=105 of the LCDM concordance FLRW model.**
- **Because of the possible systematic uncertainties in the supernova data, it is not clear that the difference between the two chi^2 and fits is significant. And we did not explore all the Szekeres models**
- **The Szekeres model used is also consistent with the requirement of spatial flatness at CMB scales.**

Title: Luminosity distance and redshift in the Szekeres inhomogeneous cosmological models

Nwankwo, Ishak, Thompson JCAP 1105:028, (2011)

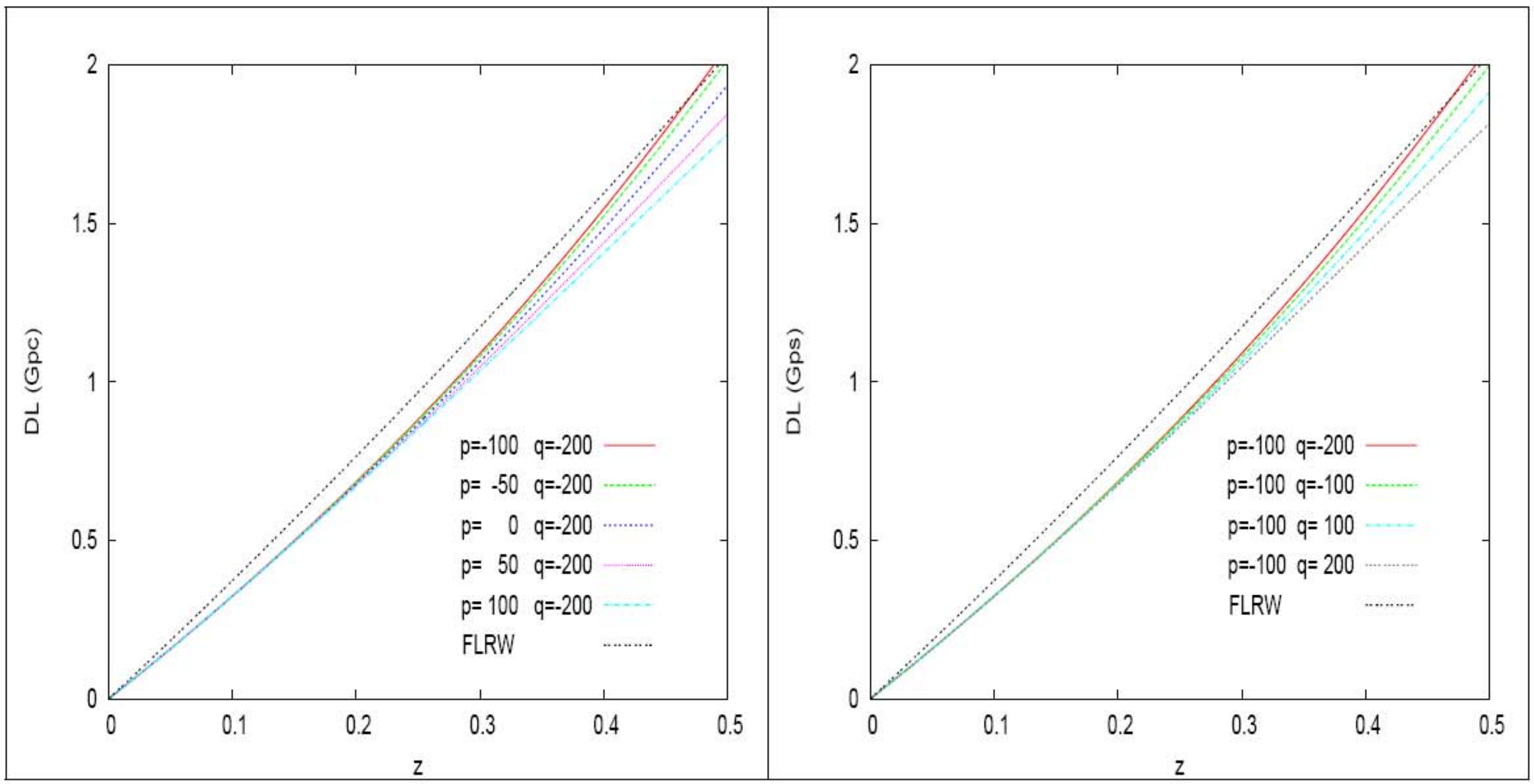
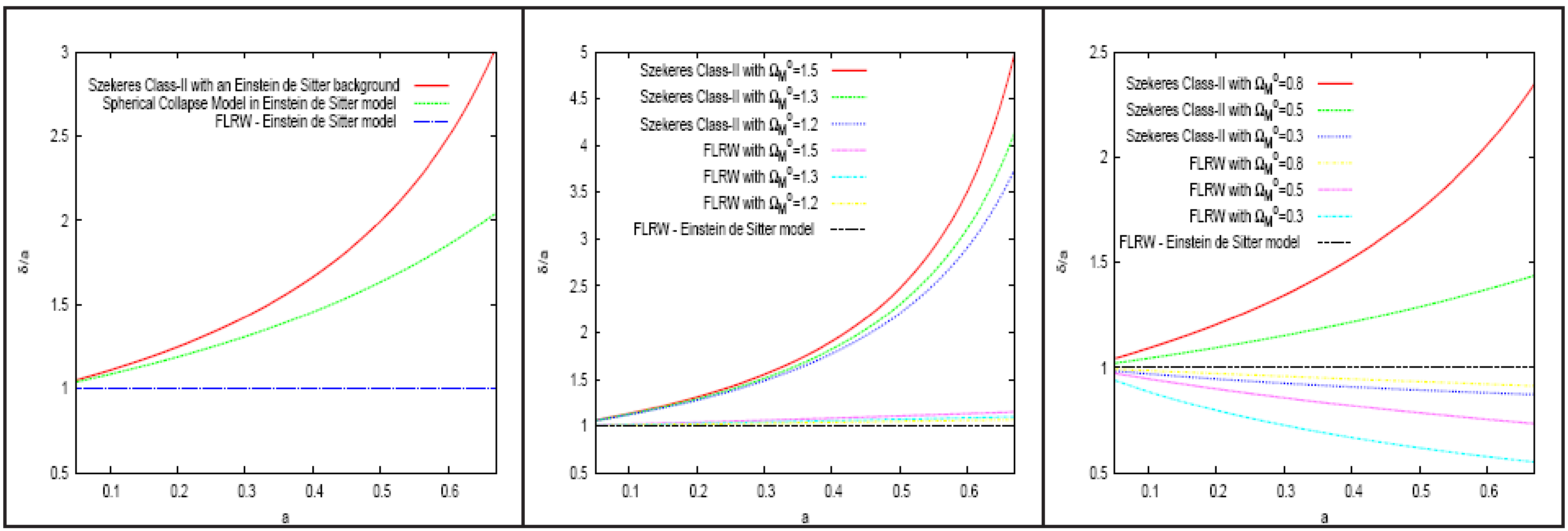


FIG. 1: Luminosity distances for a Szekeres model that is not axially or spherically symmetric. To the left, the value of q is fixed to -200 while p is varied by taking the values $-100, -50, 0, 50, 100$. To the right, the value of p is fixed to -100 while p is varied by taking the values $-200, -100, 0, 100, 200$. The Szekeres inhomogeneous model used here is for illustration purposes only and is specified in section V-A. The luminosity distance for an open FLRW model is plotted as well.

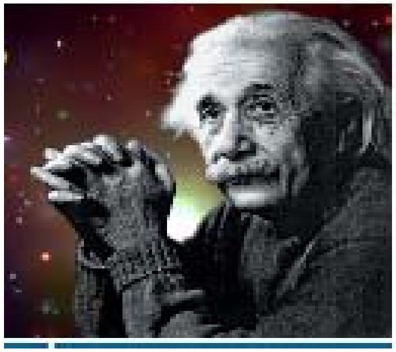
Exploring the growth of large scale structure using Szekeres models.

Ishak, Peel, PRD 2012; Peel, Ishak, Troxel, PRD submitted 2012; more to come

$$G'' + \left(4 - \frac{\Omega_M(a)}{2}\right)\frac{G'}{a} + 2(1-\Omega_M(a))\frac{G}{a^2} - \frac{2}{a}\frac{(G+aG')^2}{1+aG} - \frac{3}{2}\Omega_M(a)\frac{G^2}{a} = 0.$$



Conclusions



- We learned a lot about our universe as a whole (model, expansion, age, …)
- There is a great concordance between different and independent cosmological observations that led to a concordance standard cosmological model
- The discovered acceleration of the cosmic expansion is one of the most important problems in cosmology and all physics
- A lot of efforts are made in order to constrain the equation of state
- In addition to constraining the equation of state, it is necessary to have consistency tests based on comparisons of the expansion to the growth rate of structure
- Two methods are possible and will be conclusive with future experiments
- More work is also required to investigate the possibility of apparent acceleration due more subtle relativistic models
- The Szekeres model fits current supernova data almost as well as the LCDM model and are also consistent with the spatial flatness required by the CMB; dark energy is not needed in this case.
- Approach can be consistent with the Copernican Principle
- Cosmology is booming with new data and that should help to solve some these outstanding questions

Work in progress

Mustapha Ishak. Physics. UTD.

Summary:
Possible Causes to Cosmic Acceleration

- Proposed possibilities in thousands of scientific publications:
  - A dark energy component
  - General Relativity cosmological constant
  - A modification to general relativity at cosmological scales; Higher dimensional physics
  - Apparent acceleration due to the fact that we live in a relativistic cosmological model more complex than FLRW
  - A completely unexpected explanation

The luminosity-distance depends on the cosmological parameters

$D_L \equiv H_0 d_L / c$ is the dimensionless luminosity distance

$$D_L(z) = (1+z) \int_0^z \frac{1}{\sqrt{(1-\Omega_\Lambda)(1+z')^3 + \Omega_\Lambda \mathcal{E}(z')}} dz',$$

$$\mathcal{E}(z) \equiv \begin{cases} (1+z)^{3(1+w_0-w_1)}\, e^{3w_1 z} & \text{if } z < 1, \\ (1+z)^{3(1+w_0+w_1)}\, e^{3w_1(1-2\ln 2)} & \text{if } z \geq 1. \end{cases}$$

$$m = 5\, \log_{10}(D_L) + \mathcal{M}.$$

$\mathcal{E}(z)$ is a function of the redshift and contains the dark energy parameters $\Omega_{\Lambda\,(\text{or de})}$, w_0 and w_1

The information on the magnification and distortion of images is contained in the convergence power spectrum

$$P_\kappa(l) = \frac{9}{4} H_o^4 \Omega_m^2 \int_0^{\chi_H} \frac{g^2(\chi)}{a^2(\chi)} P_{3D}\left(l/\sin_K(\chi), \chi\right) d\chi$$

$$g(\chi) = \int_\chi^{\chi_H} n(\chi') \frac{\sin_K(\chi' - \chi)}{\sin_K(\chi')} d\chi'$$

The power spectrum is sensitive to several cosmological parameters

Weak lensing captures the effect of Dark Energy on the expansion history and its effect on the growth factor of large-scale structure

$$W(H_o, \Omega_m, \chi(z), H(z))$$

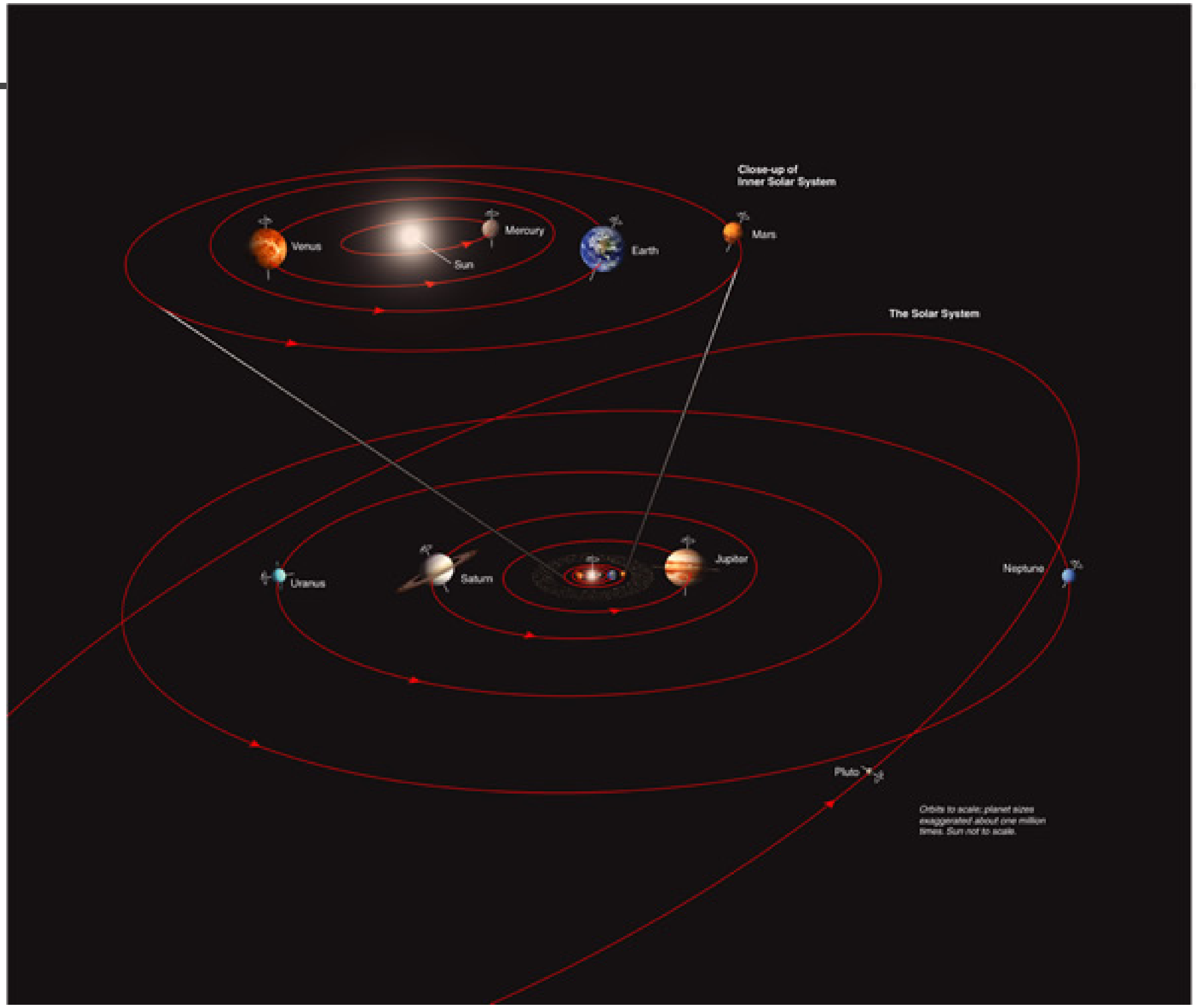
$$P_{3D}(k, z) \propto P_{3D}^{prim}(k) T^2(k, z) \left[\frac{G(z)}{G(0)}\right]^2 NLM(k, z)$$

$$D(a) = \frac{\delta(a)}{\delta(1)}$$

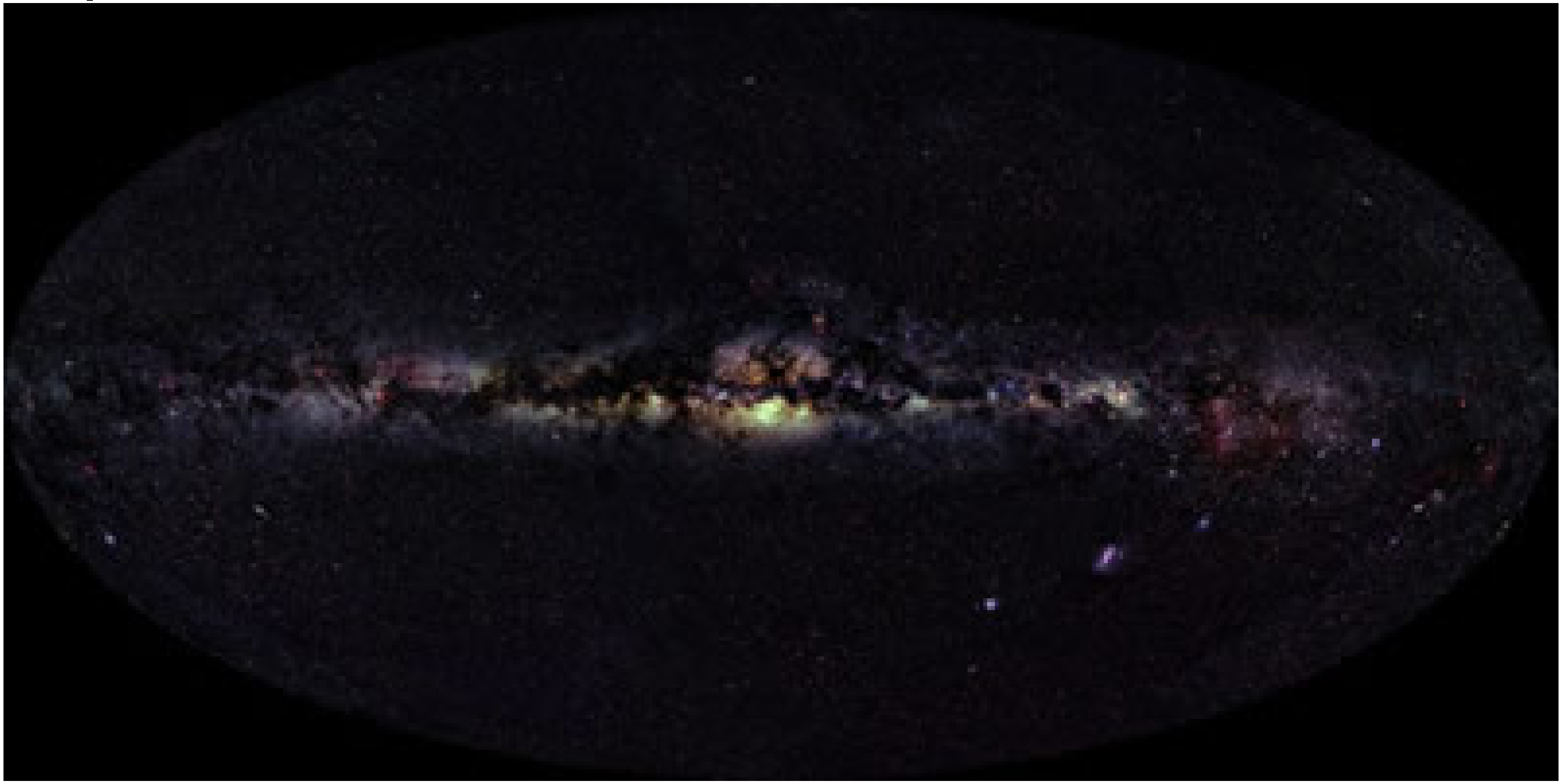
$$G(a) = \frac{D(a)}{a}$$

$$G'' + \left[\frac{7}{2} - \frac{3}{2}\frac{w(a)}{1 + X(a)}\right]\frac{G'}{a} + \frac{3}{2}\frac{1 - w(a)}{1 + X(a)}\frac{G}{a^2} = 0$$

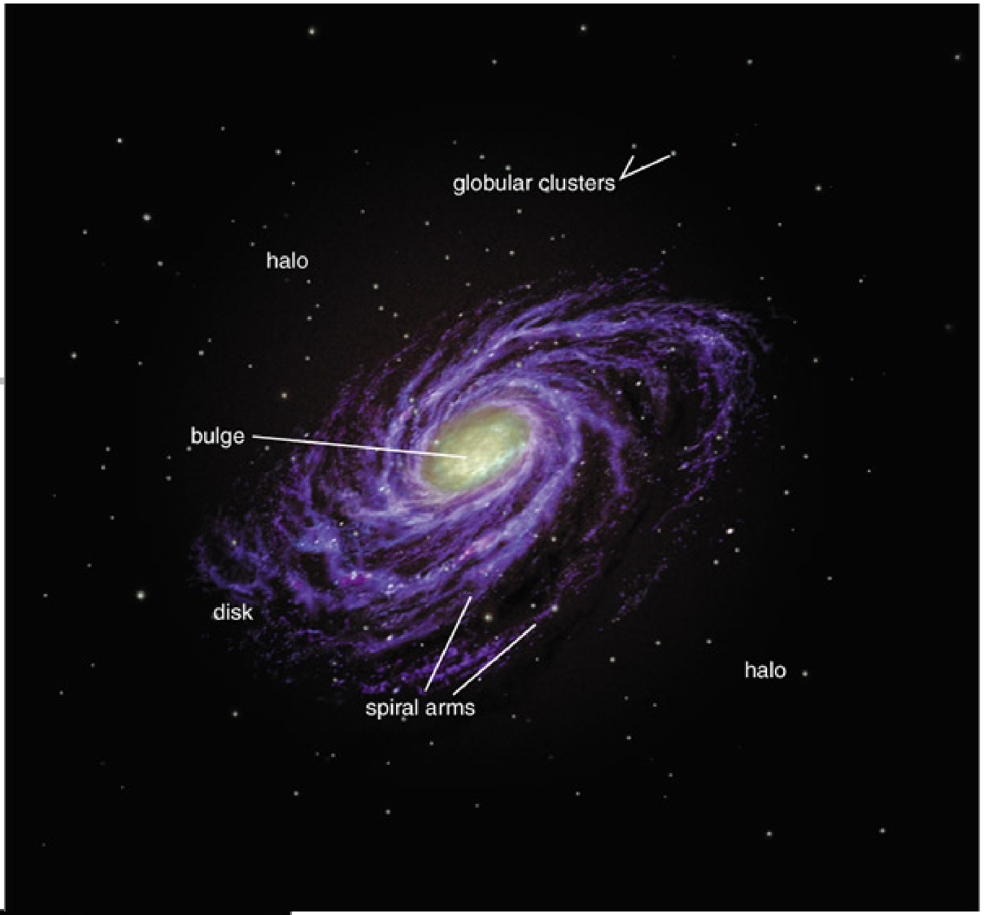
Our solar system

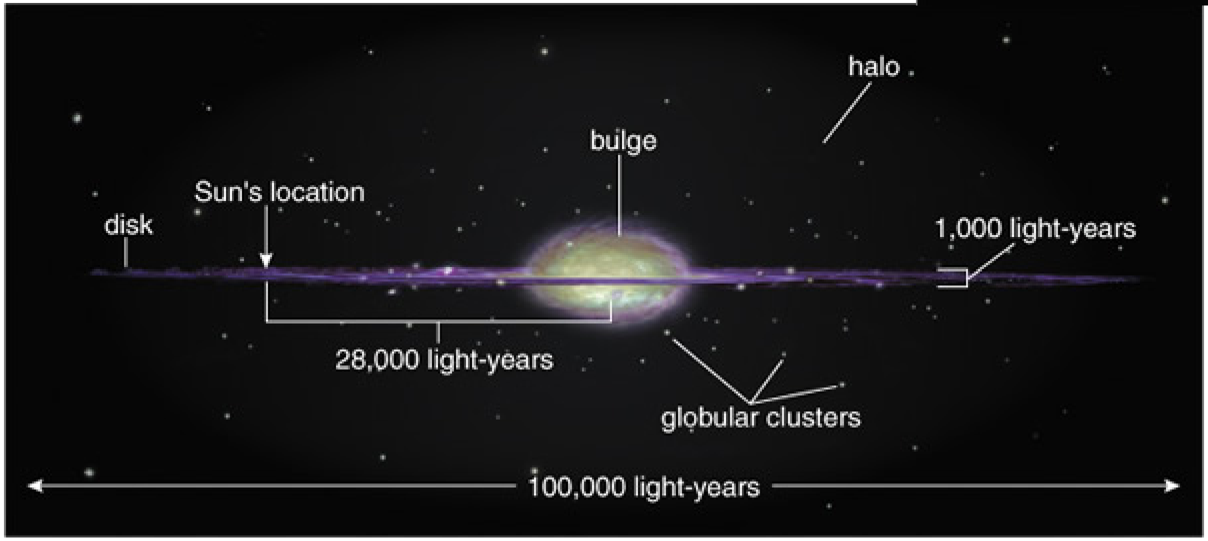


Our galaxy: The milky Way (or the Galaxy)

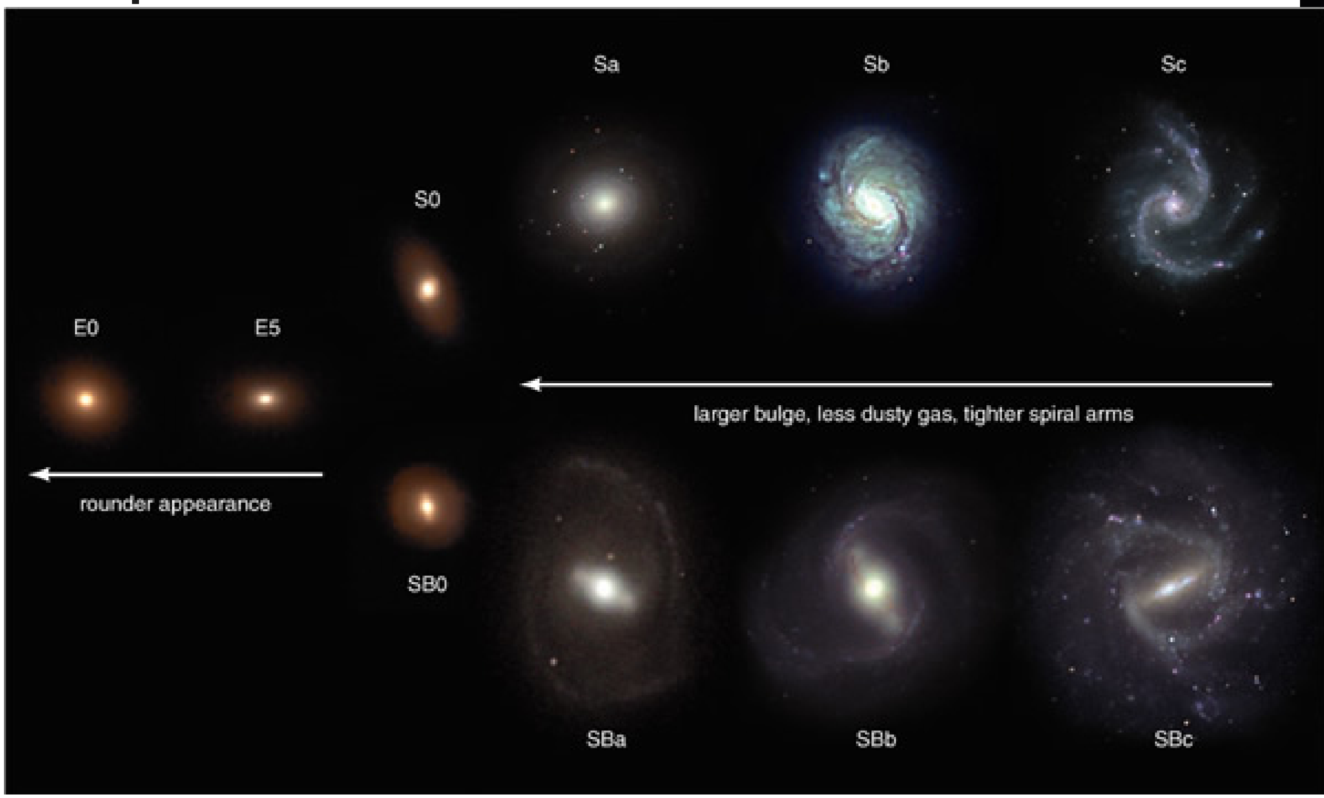


Our galaxy: The milky Way (or the Galaxy)

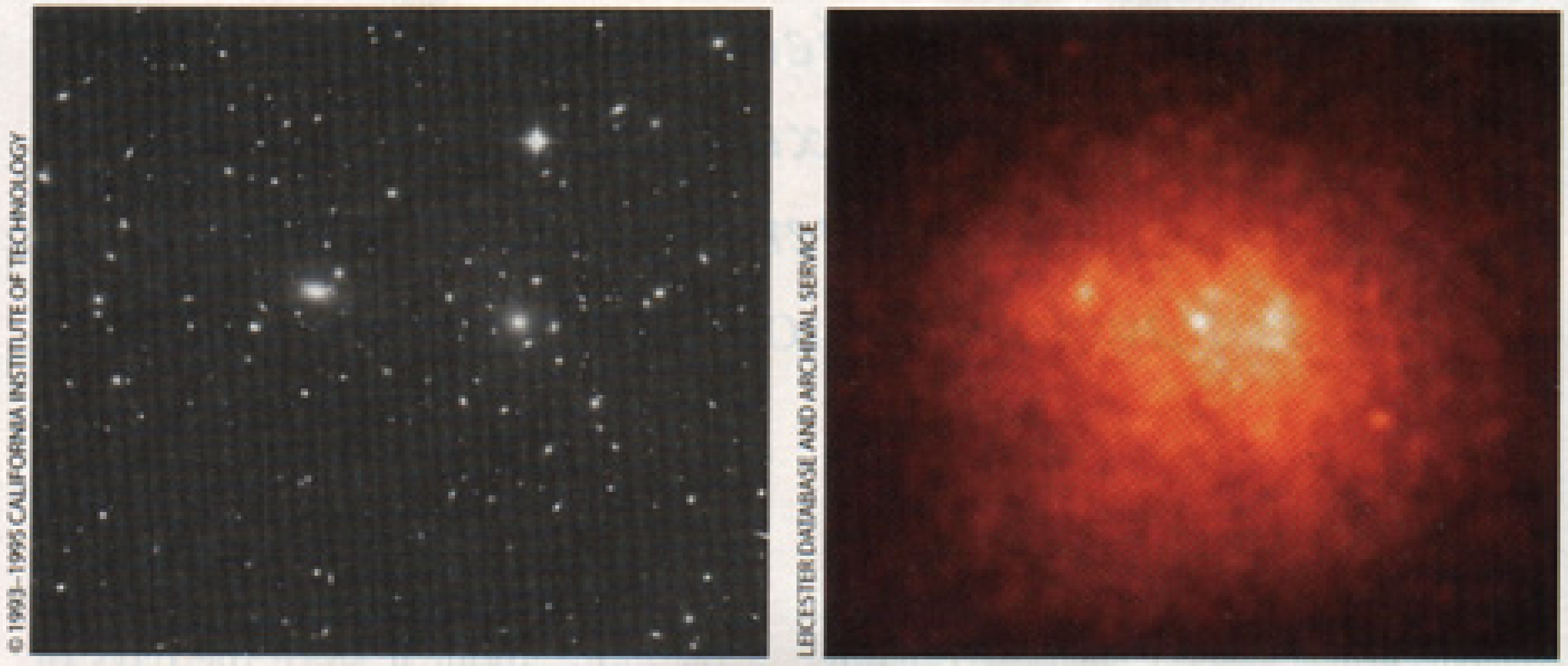




Classification of types of galaxies



Clusters and Super-clusters of galaxies

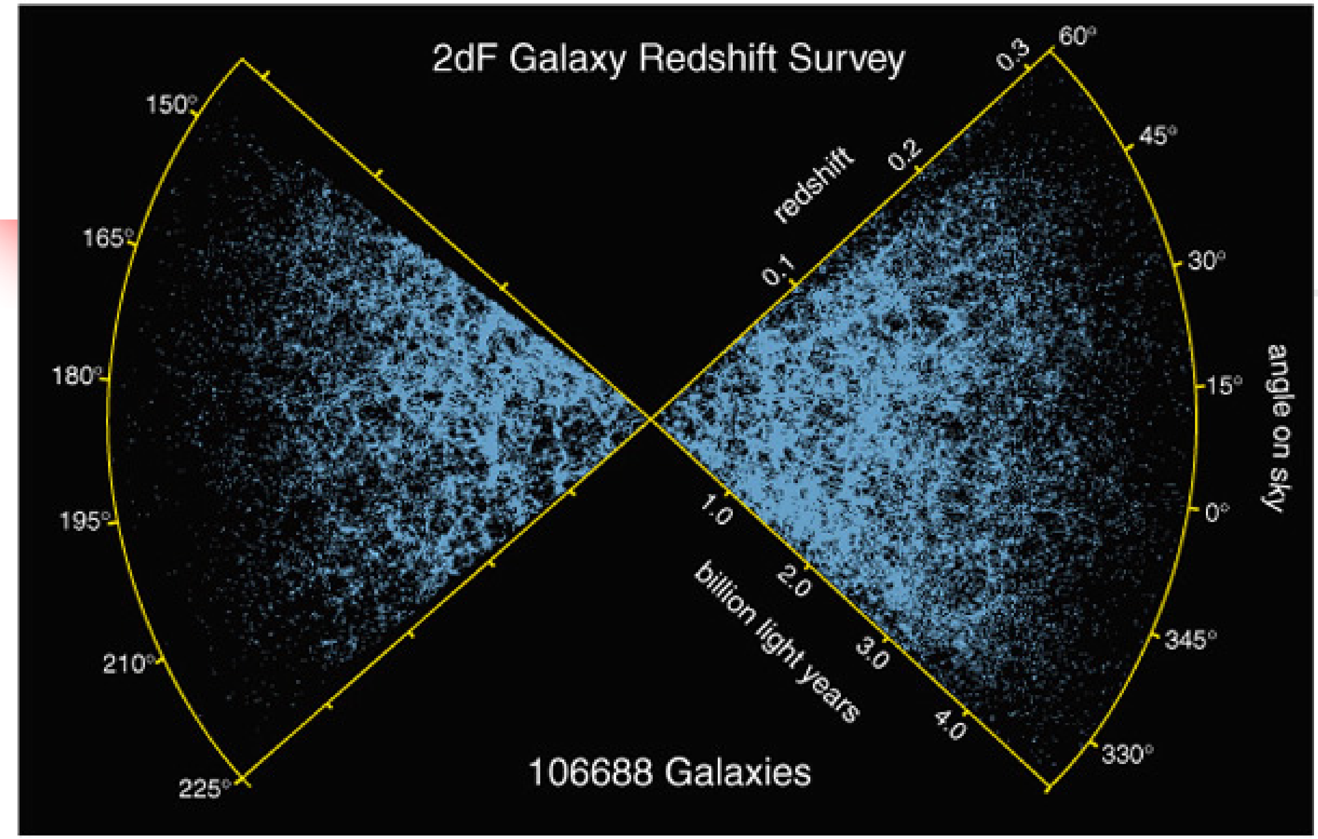


COMA CLUSTER looks different in visible light (*left*) and in x-rays (*right*). In visible light, it appears to be just an assemblage of galaxies. But in x-rays, it is a gargantuan ball of hot gas some five million light-years across.

Clusters and Super-clusters of galaxies

Figure 20.7 Central part of the galaxy cluster Abell 1689. The region pictured is about 2 million light-years across. Almost every object in this photograph is a galaxy belonging to the cluster. Yellowish elliptical galaxies outnumber the whiter spiral galaxies. A few stars from our own galaxy appear in the foreground, looking like white

dots with four spikes in the form of a cross



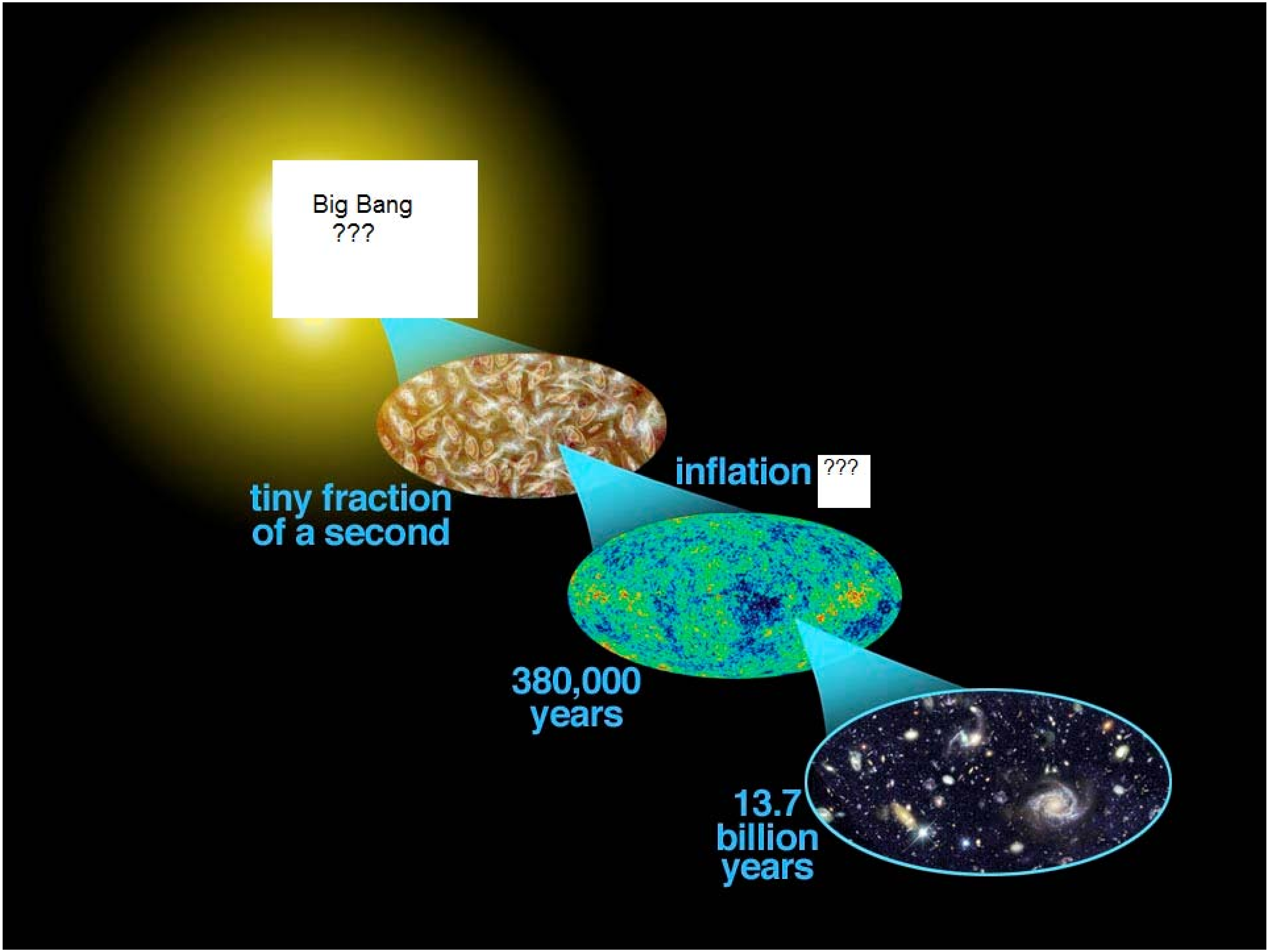
From largest structures to almost homogeneity

Note: The universe become almost homogeneous and isotropic

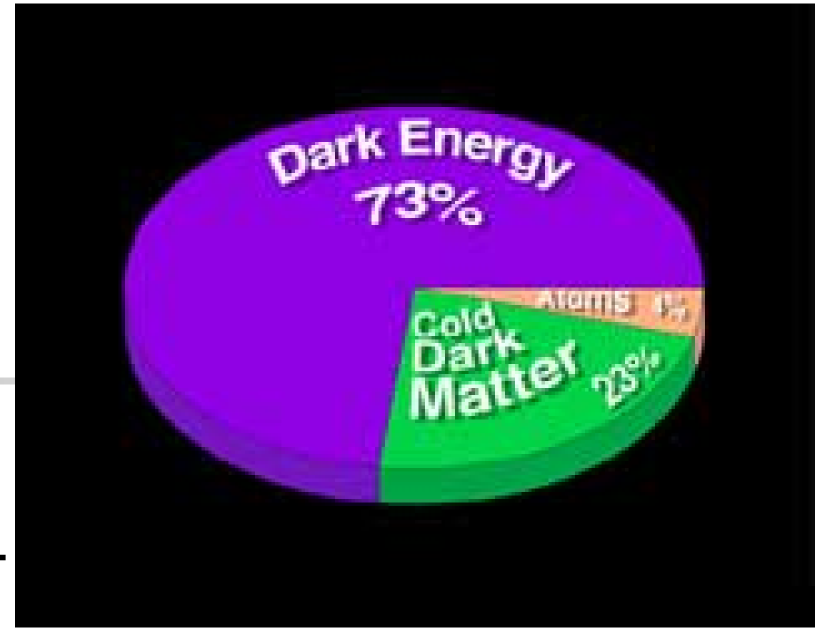
The galaxies in these two slices of the sky extend much deeper into space–up to 4 billion light-years. Many voids, walls, and strings of galaxies hundreds of millions of light-years in size are evident as tiny details in this image. However, the distribution of galaxies on scales larger than a billion light-years is nearly uniform (SDSS > we used 265000 galaxies for intrinsic correlations between galaxies)

What is astrophysics?
What is cosmology?

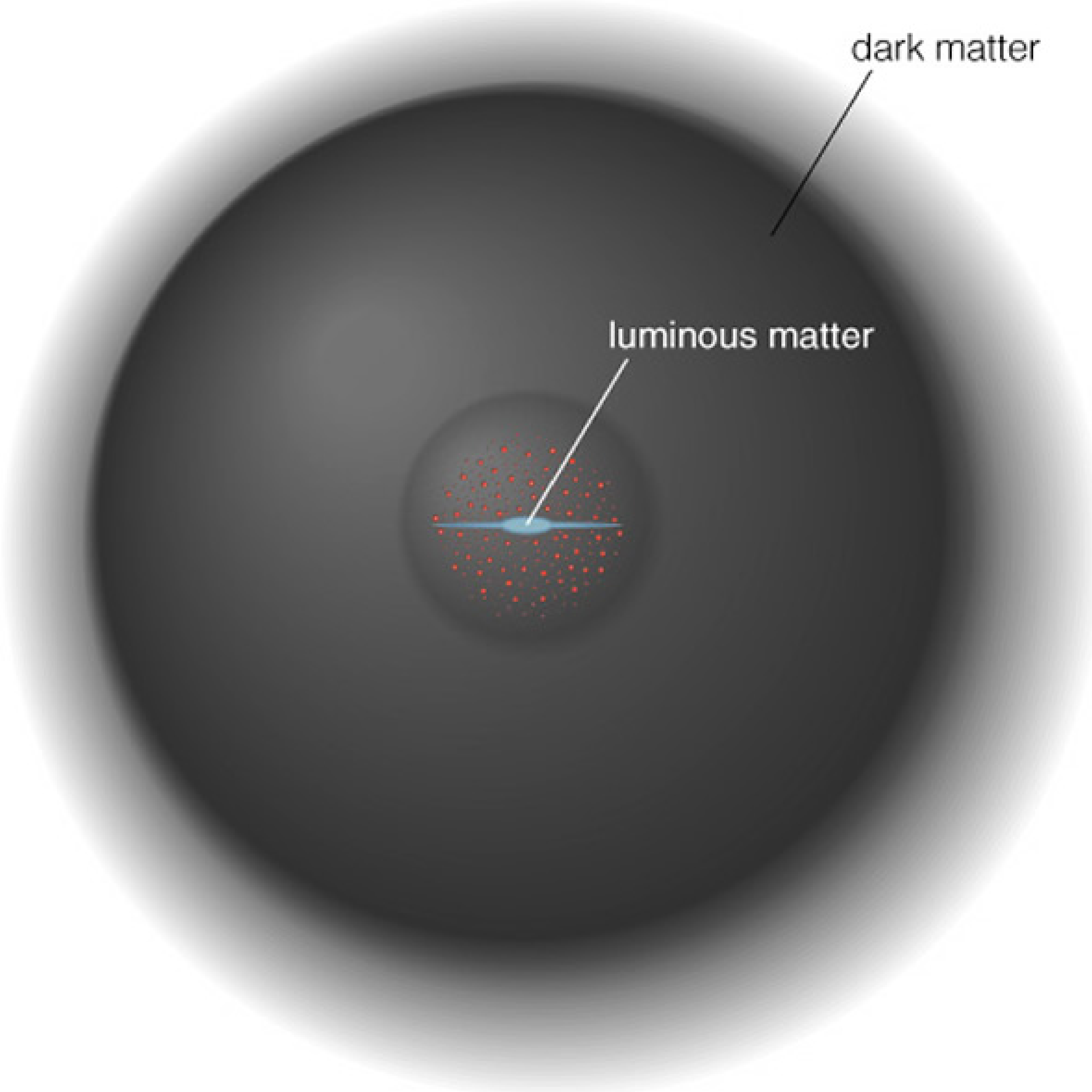
- Our solar system is made of our star the Sun and the planets orbiting around it
- Millions of other starts are regrouped into galaxies (our galaxy is the Milky Way)
- Millions of galaxies are regrouped into clusters of galaxies and super clusters of galaxies
- The Universe is made of millions and millions of clusters and super clusters of galaxies
- **Astrophysics is the science that study the laws of physics that governs the astronomical objects at the different scales of distance (Note: this is different from astronomy which can be exclusively descriptive)**
- **Cosmology is the science that studies the physics and astrophysics of the universe as a whole and also phenomena at very large scales of distance in the Universe**
- **Both fields require a good deal of physics and mathematics**
- Type of work:
  - Observational cosmology (experiments, observations, data reductions, …)
  - Pure mathematical cosmology (mathematical relativity, higher dimensions, string cosmology, …)
  - Theoretical cosmology (fitting cosmological models of the universe to the data, developing new cosmological techniques to analyze the data, developing and testing new theories to explain observations, …)
  - Numerical cosmology (simulations of structures, simulations of effects,)
  - (NOTE: Usually, research projects in cosmology include more than one type of work from the list above)


Big Bang
???
inflation ???
tiny fraction
of a second
380,000
years
13.7
billion
years

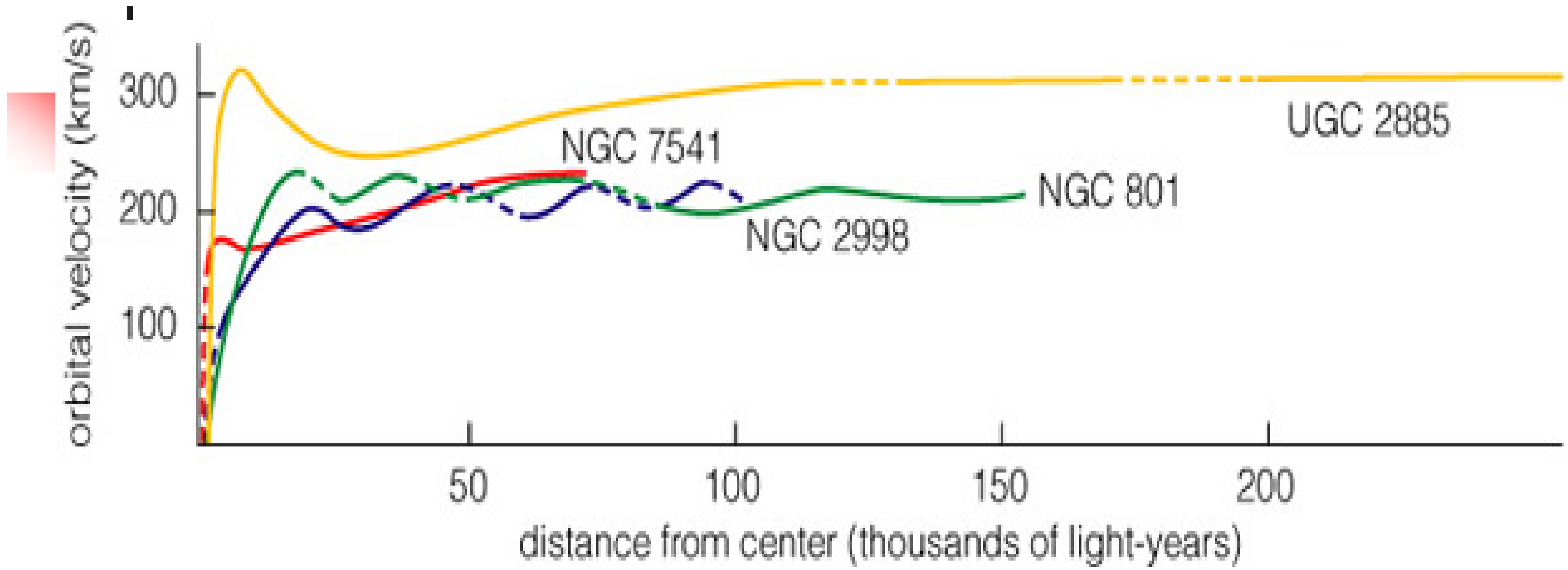
from current observations?



- The age of the universe is ~1
- The total matter-energy density in the universe is very close to some critical density
- Unexpectedly, the expansion of the universe is accelerating instead of decelerating. Why?
- There is Dark Matter in the Universe



The dark matter associated with a spiral galaxy occupies a much larger volume than the galaxy's luminous matter. The radius of this dark-matter halo could be as much as 10 times larger than the galaxy's halo of stars.



Actual rotation curves of four spiral galaxies. They are all nearly flat over a wide range of distances from the center, indicating that dark matter is

common in spiral galaxies

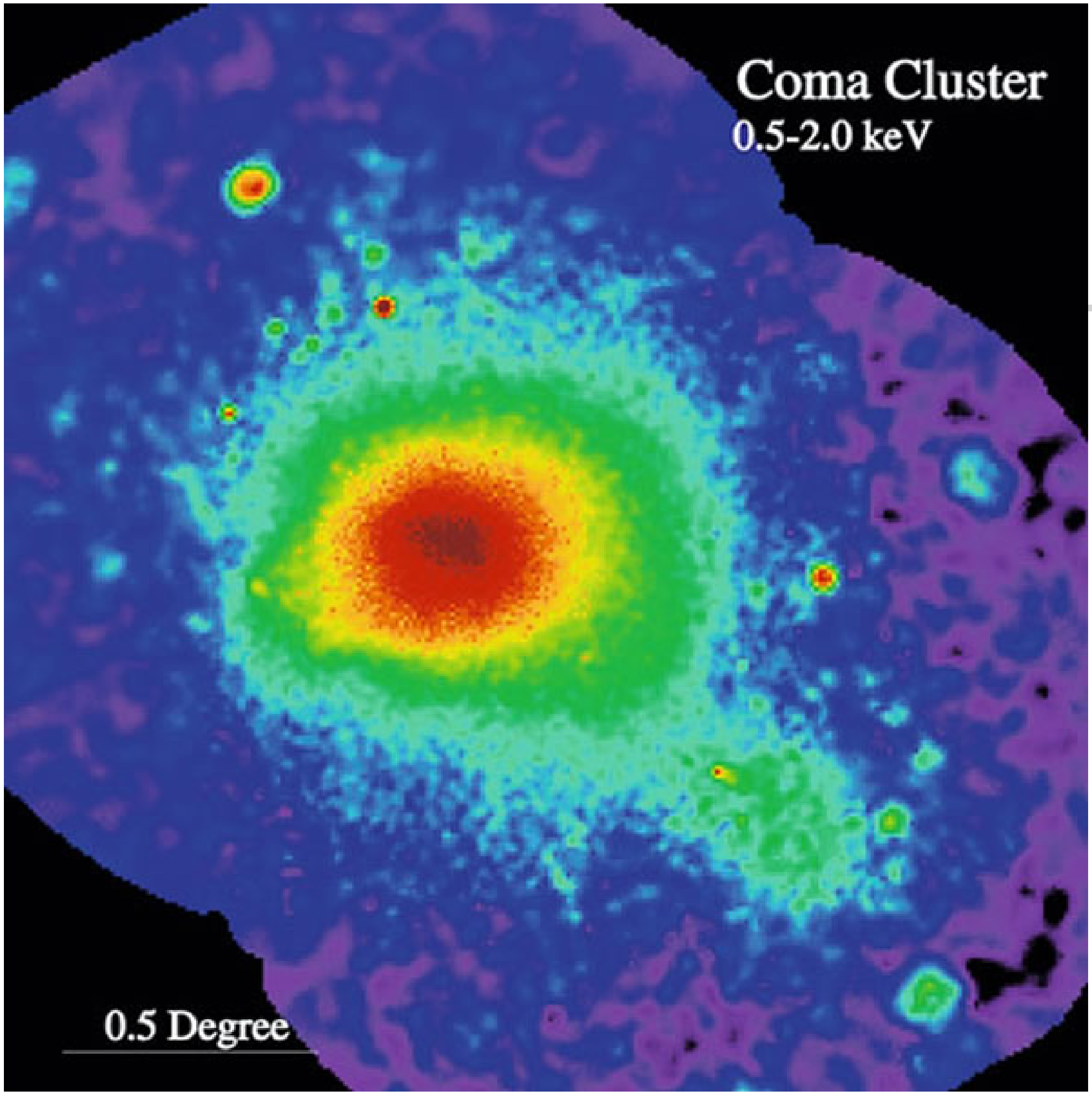


Figure 22.8 The Coma Cluster of galaxies in both visible light and X-ray light. *a* Virtually every object pictured in this visible-light photo is a galaxy in the Coma Cluster. Measuring the motions of these galaxies reveals that the Coma Cluster contains about 1015 solar masses of matter. Because the total luminosity of all the cluster's galaxies is less than 1013LSun, the mass-to-light ratio of the cluster exceeds 100 solar masses per solar luminosity. We therefore conclude that the cluster contains far more dark matter than luminous matter. (The picture shows the central 3 million light-years of the cluster.) *b* This false-color image shows X-ray emission from the extremely hot gas (the intracluster medium) that fills the Coma Cluster. The temperature of this gas–almost 100 million degrees–also indicates that the Coma Cluster contains about 1015 solar masses of matter. Thus, the X-ray observations confirm the amount of dark matter estimated from the visible-light observations of galaxy motions. (The whole X-ray map shows a region about 14 million light-years across. The dark red and orange regions are bright in X-ray light and mark the center of the cluster. The green and blue regions are less bright.)

Skip: Method two: parameterization of the Growth rate of large scale structure

- large scale matter density perturbation, $\delta = \delta\rho_m / \rho_m$, satisfies the ODE:

$$\ddot{\delta} + 2H\dot{\delta} - 4\pi G_{eff}\,\rho_m \delta = 0$$

- The ODE can be written in terms of the logarithmic growth rate $f = d\ln\delta / d\ln a$ as:

$$f' + f^2 + \left(\frac{\dot{H}}{H^2} + 2 \right) f = \frac{3}{2}\frac{G_{eff}}{G}\Omega_m$$

where the underlying gravity theory is expressed via the expression for G_{eff} , *H(z), and* $\Omega_m(z)$.

Skip: A constant growth rate index parameter

- The growth function f *can be* approximated using the ansatz [Peebles, 1980; Fry, 1985; Lightman & Schechter, 1990]

$$f = \Omega_m^{\gamma}$$

where γ is the growth index parameter

- It was found there that

$$f(z) = \Omega_m^{0.6} \qquad\qquad f = \Omega_m^{4/7}$$

were good approximations for matter dominated models.

Growth rate index parameter and [General Relativity + Dark Energy] models

- [L. Wang and Steinhardt, 1998] considered Dark Energy models with slowly varying *w* and derived

$$\gamma = \frac{3(1-w)}{5-6w} + \frac{3}{125}\frac{(1-w)(1-3w/2)}{(1-6w/5)^2(1-12w/5)}(1-\Omega_m) \qquad f = \Omega_m^\gamma$$

- with the asymptotic early value $\gamma_\infty = \frac{3(1-w)}{(5-6w)}$
- [see also for example Linder and Cahn, 2007; Mortonson, Hu, Huterer, 2009; Zhang et al. 2007; Gong, 2008; Polarski and Ganouji, 2008, Gong, Ishak, A. Wang 2009 …]
- The approximation provides a fit of about 1% to the growth function f as numerically integrated from the ODE

The growth index parameter as a discriminator for Gravity Theories

- The asymptotic constant growth index parameter takes distinctive value for distinct gravity theories
- Thus, can be used to probe the underlying gravity theory and the cause of cosmic acceleration
- $\gamma=6/11=0.545$ for the Lambda-Cold-Dark-Matter model. (i.e. for $w=-1$)
- $\gamma=11/16=0.687$ for the flat DGP model [e.g. Linder and Cahn, 2007; Gong 2008].

Redshift parameterization for the growth rate index

[Polarski and Gannouji, PLB, 2008; Ishak and Dossett, PRD 2009; Gong, Ishak, Wang, PRD 2009]

- A parameterization that interpolates between a small/intermediate redshift expression and an asymptotic constant value at high redshifts:

$$\gamma(a) = \tilde{\gamma}(a)\ \frac{1}{1+(a_{ttc}/a)} + \gamma_{early}\ \frac{1}{1+(a/a_{ttc})}$$

or

$$\gamma(z) = \tilde{\gamma}(z)\ \frac{1}{1+\frac{1+z}{1+z_{ttc}}} + \gamma_{early}\ \frac{1}{1+\frac{1+z_{ttc}}{1+z}}$$

where z_{ttc} is a transition resdhift from an early-time, almost constant value, to the following redshift dependent form

$$\gamma(a)_{late} = \tilde{\gamma}(a) = \gamma_0 + (1-a)\gamma_a$$ or $$\gamma(z)_{late} = \tilde{\gamma}(z) = \gamma_0 + \left(\frac{z}{1+z}\right)\gamma_a$$

The effect of spatial curvature: degeneracy

(Huterer, Linder, Hu, PRD, 2008??; Gong, Ishak, Wang, PRD 2009)

For the curved dark energy model with constant equation of state w, we have

$$\frac{\dot{H}}{H^2} = \frac{1}{2}\Omega_k - \frac{3}{2}[1 + w(1 - \Omega_m - \Omega_k)]. \tag{3}$$

The energy conservation equation tells us that

$$\Omega_m' = 3w\Omega_m(1 - \Omega_m - \Omega_k) - \Omega_m\Omega_k. \tag{4}$$

Substituting Eqs. (3) and (4) into Eq. (2), we get

$$[3w\Omega_m(1 - \Omega_m - \Omega_k) - \Omega_m\Omega_k]\frac{df}{d\Omega_m} + f^2 + \left[\frac{1}{2} + \frac{1}{2}\Omega_k - \frac{3}{2}w(1 - \Omega_m - \Omega_k)\right] f = \frac{3}{2}\Omega_m. \tag{5}$$

Plugging $f = \Omega_m^\gamma$ into Eq. (5), we get

$$[3w(1-\Omega_m-\Omega_k)-\Omega_k]\Omega_m \ln\Omega_m \frac{d\gamma}{d\Omega_m} + \left(\gamma - \frac{1}{2}\right)[3w(1-\Omega_m-\Omega_k)-\Omega_k] + \Omega_m^\gamma - \frac{3}{2}\Omega_m^{1-\gamma} + \frac{1}{2} = 0. \tag{6}$$

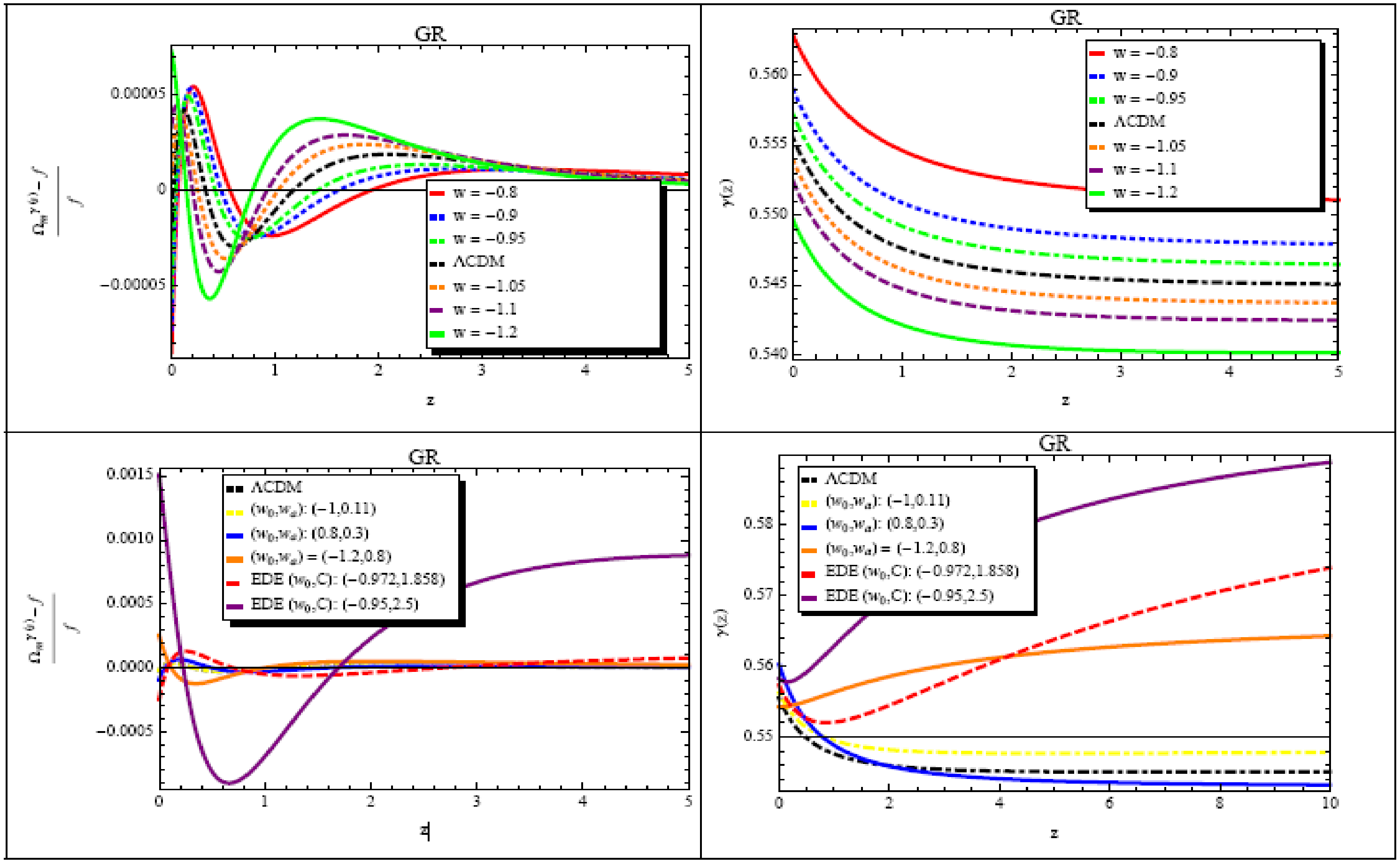
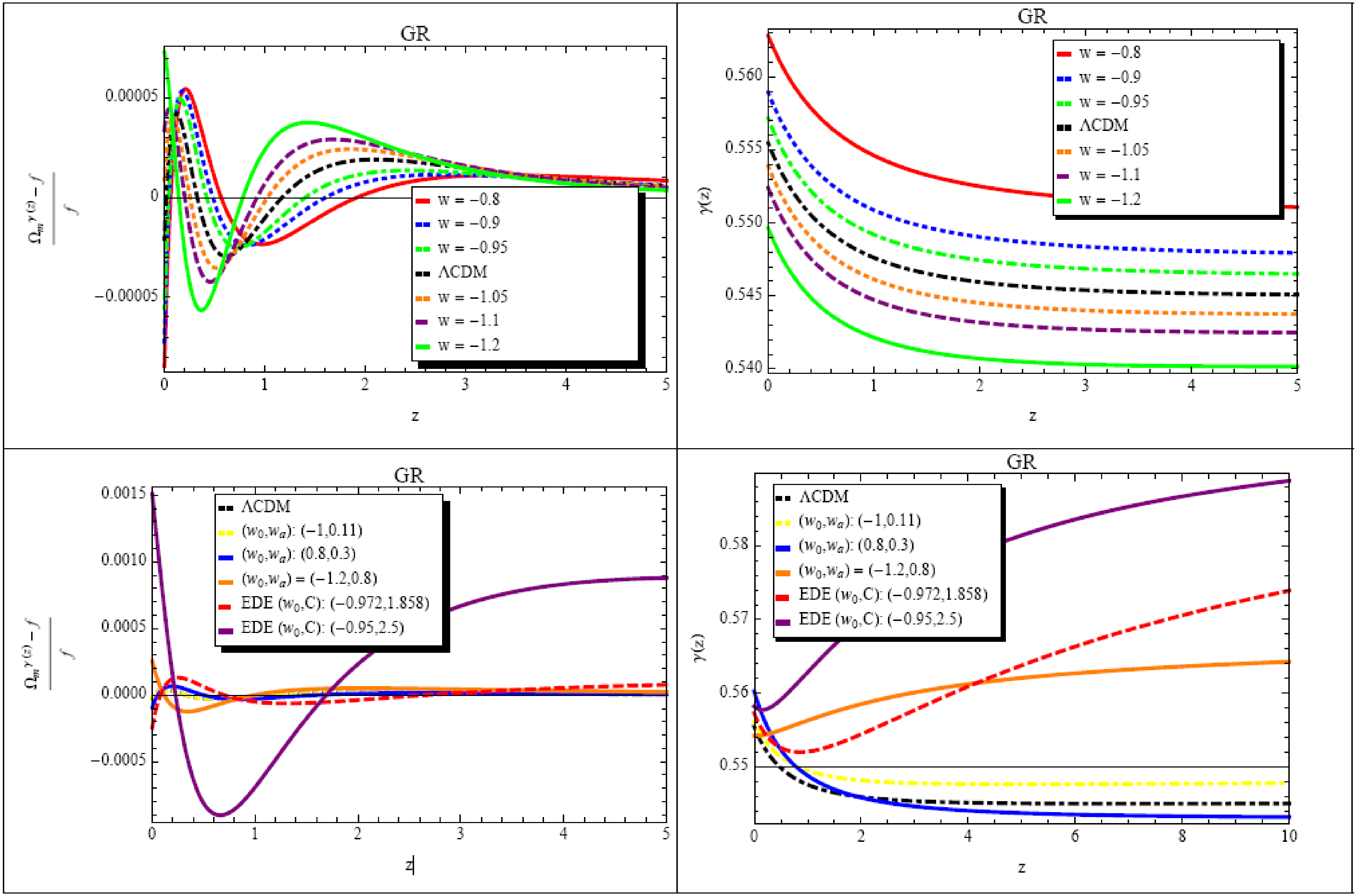


FIG. 1: GR - Dark Energy Models. TOP LEFT: We consider the QCDM models with a constant equation of state and plot the relative error $\frac{\Omega_m^{\gamma(z)}-f}{f}$ in order to compare the fit of the proposed parameterization to that of the growth rate, f, that is numerically integrated from the growth ODE. For the ΛCDM, we find the best fit parameters $\gamma_0 = 0.5655$ and $\gamma_a = -0.02710$ when $\gamma_\infty^{\Lambda CDM} = 6/11$. The fit approximate the growth function f to better than 0.004% while the best fit constant $\gamma_{const}^{\Lambda CDM} = 0.5509$ approximates the growth to 0.6%. Using our redshift dependent parameterizations of growth index provides an improvement to the fit of the growth of about a factor 150. TOP RIGHT: We plot $\gamma(z) = \tilde{\gamma}(z)\,\frac{1}{1+\frac{1+z}{1+z_{ttc}}} + \gamma_\infty\,\frac{1}{1+\frac{1+z_{ttc}}{1+z}}$ for various values of the constant equation of state w showing very little dispersion of the order of 0.015 at any given redshift. BOTTOM LEFT:We consider the QCDM models with a variable equation of state, as well as some Early Dark Energy models and plot the relative error $\frac{\Omega_m^{\gamma(z)}-f}{f}$ in order to compare the fit of the proposed parameterization to that of the growth rate, f, that is numerically integrated from the growth ODE. We find using our redshift dependent parameterizations of the growth index are able to approximate the growth to within 0.15%. BOTTOM RIGHT: We plot $\gamma(z) = \tilde{\gamma}(z)\,\frac{1}{1+\frac{1+z}{1+z_{ttc}}} + \gamma_\infty\,\frac{1}{1+\frac{1+z_{ttc}}{1+z}}$ for various dark dnergy models with a varying equation of state $w(a)$ including some early dark energy models.

Growth index parameter for GR + Dark Energy models. LEFT: Very precise parameterization. RIGHT: Very little dispersion around the γ=6/11=0.545



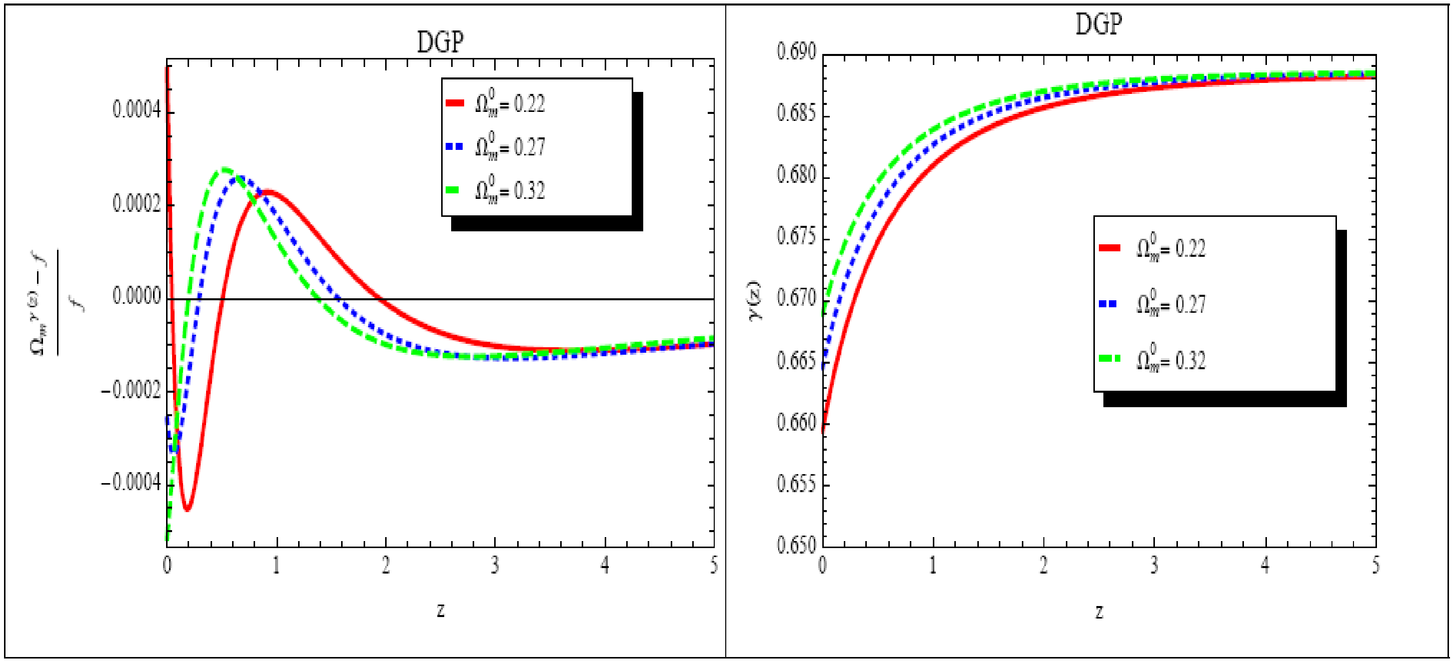
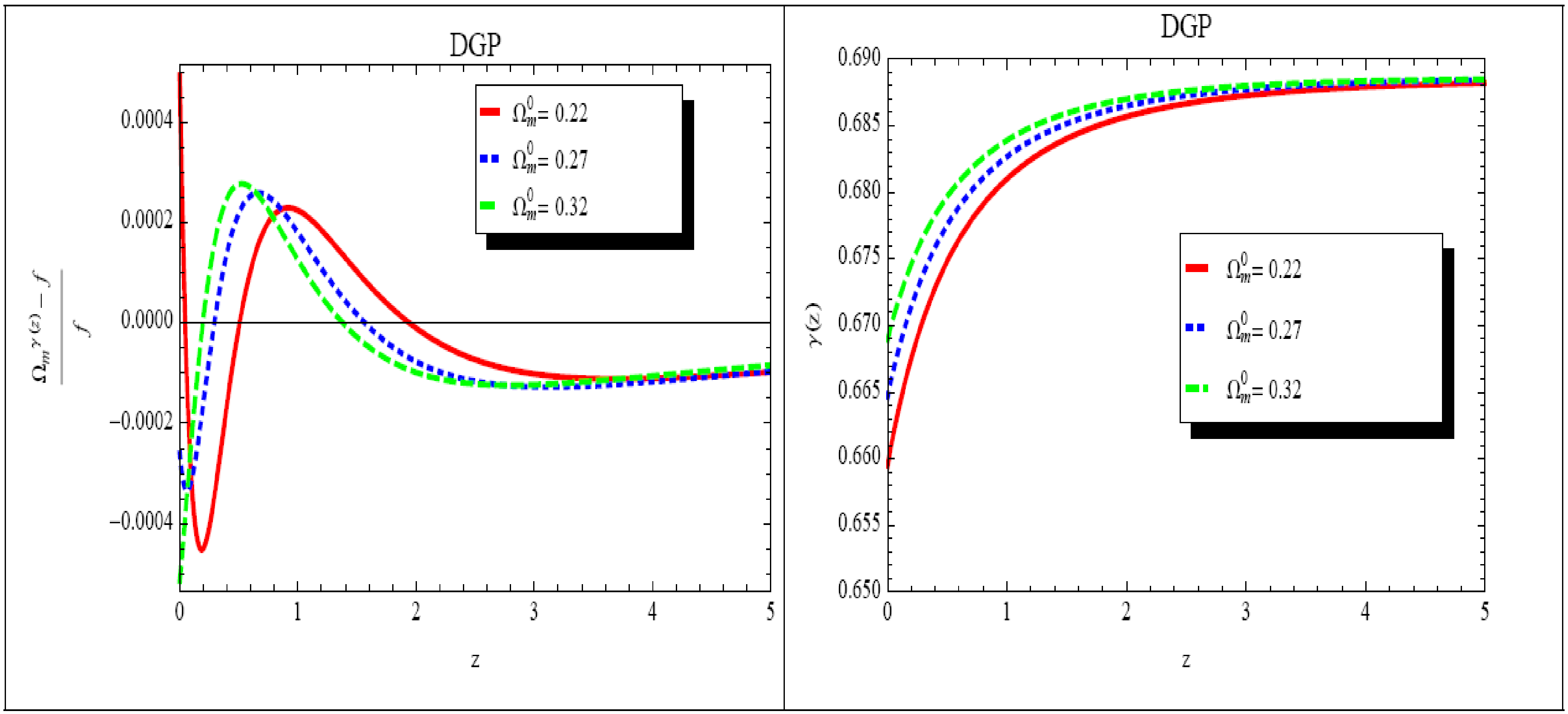


FIG. 2: DGP models. LEFT: We consider the DGP model and plot the relative error $\frac{\Omega_m^{\gamma(z)}-f}{f}$ in order to compare the fit of the proposed parameterization to that of the growth rate f_{num} that is numerically integrated from the growth ODE. We find the best fit parameters $\gamma_0 = 0.6418$ and $\gamma_a = 0.06261$ for $\Omega_m^0 = 0.27$ when $\gamma_\infty^{DGP} = 11/16$. The fit approximates the growth function f to better than 0.04% while the best fit constant $\gamma_{const}^{DGP} = 0.6795$ approximates the growth to 1.95%. So using our redshift dependent parameterization of the growth index provides an improvement to the fit of about a factor 50 for the DGP model. RIGHT: We plot $\gamma(z) = \tilde{\gamma}(z)\,\frac{1}{1+\frac{1+z}{1+z_{ttc}}} + \gamma_\infty^{DGP}\,\frac{1}{1+\frac{1+z_{ttc}}{1+z}}$ for various values of Ω_m^0 showing very little dispersion of the order 0.01 or less at any redshift.

Growth index parameter for DGP models.
LEFT: Very precise parameterization.
RIGHT: Very little dispersion around the γ=11/16=0.687



Parameters for various QCDM models.		
(w_0, w_a)	γ_0	γ_a
$(-0.8, 0)$	0.5690	-0.02131
$(-0.9, 0)$	0.5683	-0.022525
$(-0.95, 0)$	0.5676	-0.02699
$(-1, 0)$	0.5655	-0.02718
$(-1.05, 0)$	0.5635	-0.02735
$(-1.1, 0)$	0.5617	-0.02749
$(-1.2, 0)$	0.5583	-0.02771
$(-1, 0.11)$	0.5641	-0.02464
$(-0.8, -0.3)$	0.5720	-0.03074
$(-1.2, 0.8)$	0.5409	-0.01417
Parameters for some EDE models.		
(w_0, C)	γ_0	γ_a
$(-0.972, 1.858)$	0.5498	-0.02915
$(-0.95, 2.5)$	0.5165	-0.05578
Parameters for various DGP models.		
Ω_m^0	γ_0	γ_a
0.22	0.6314	0.07324
0.27	0.6418	0.06261
0.32	0.6504	0.05279

TABLE I: We list the parameter values for in our interpolation parameterization for various QCDM, EDE, and DGP models. These values were found by fitting our parameterization to the numerically integrated solution of ODE for the growth function, f (e.g. we use for $\gamma(z)$, Eqs.(18) with(9) for dark energy models, and Eqs. (25) with (9) for DGP models). We see that the QCDM and EDE models have a negative values for the parameter γ_a, while the DGP models have a positive value for γ_a, thus providing parameter that observational data can constrain to distinguish between the two gravity theories, additionally γ_0 takes on distinct values for each theory.

Simulated future growth data

- Using discussions from current papers of galaxy redshift distortions and Lyman alpha, we extrapolated two scenarios for uncertainties on future data.
- A **pessimistic scenario** where we assume that we will have more data but the uncertainties will get only slightly better then the ones of the current data: that is 20% for the range $0<z<= 2.0$ and 30% for $2<z<= 4.0$
- A **moderate scenario** with: 10% uncertainty (an improvement of a factor of 2) for the range $0<z= 2.0$ and 20% (an improvement of a factor of 1.5) for $2<z<= 4.0$.
- We generated 80 points (or bins) for the growth rate that are almost equally spaced by $\Delta z=0.05$ between redshifts 1 and 4

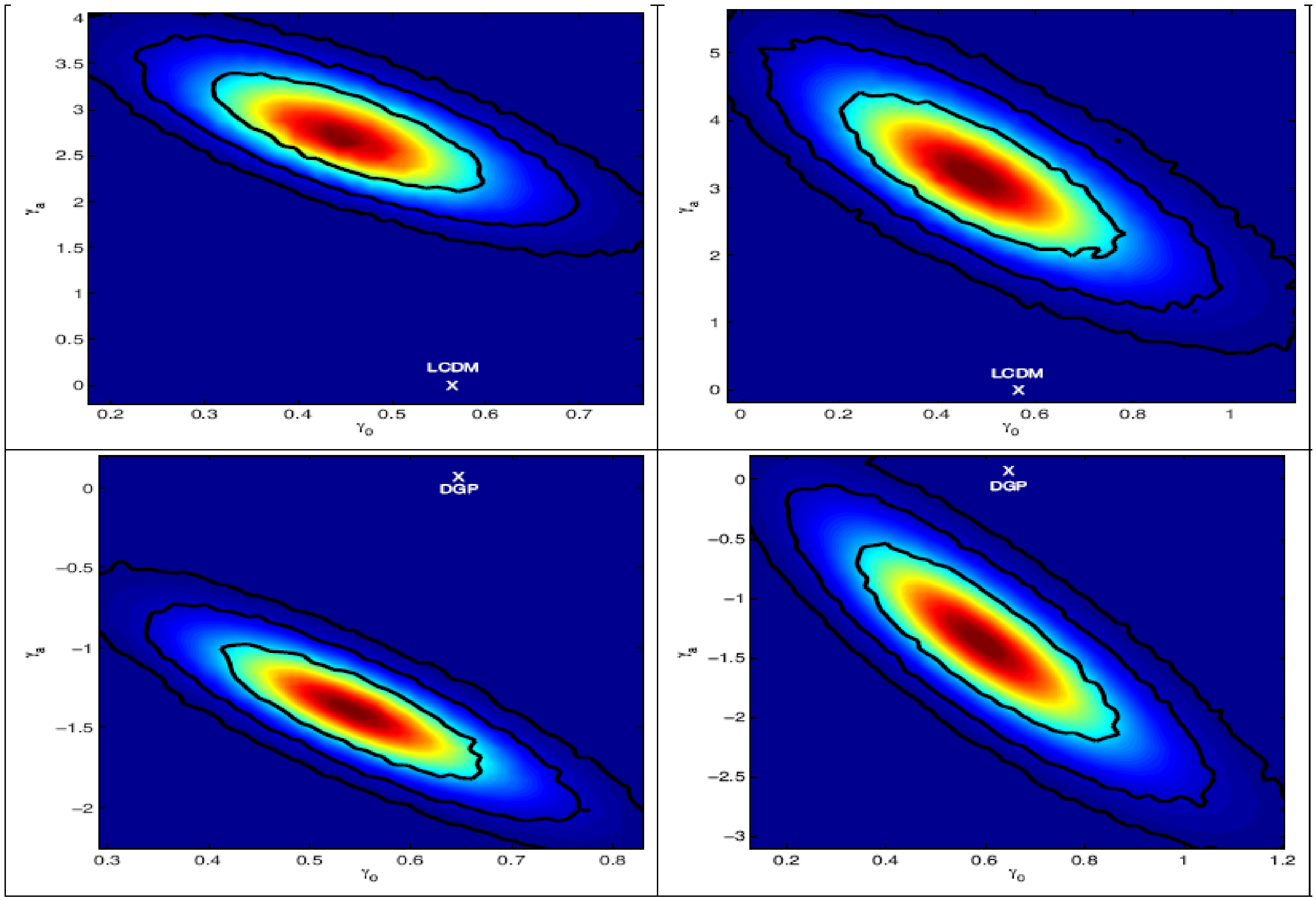
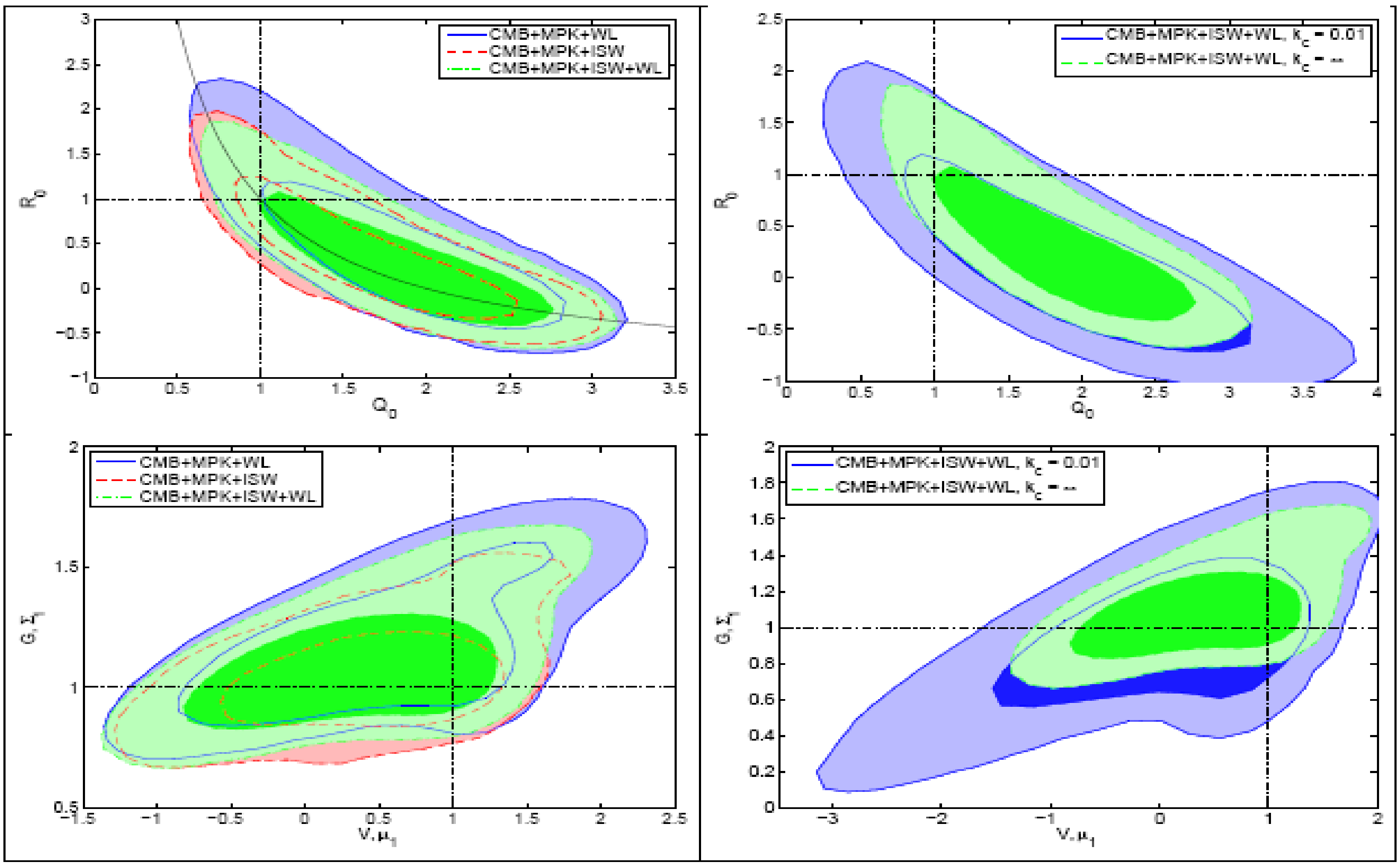


FIG. 3: Interpolated parameterization. TOP LEFT: Moderate scenario fitting fiducial DGP data on an assumed LCDM background. TOP RIGHT: Pessimistic scenario fitting fiducial DGP data on an assumed LCDM background. BOTTOM LEFT: Moderate scenario fitting fiducial LCDM data on an assumed DGP background. BOTTOM RIGHT: Pessimistic scenario fitting fiducial LCDM data on an assumed DGP background. As shown on the figures, in each case the incorrect assumed background model is ruled out to 99.7%.

USING REAL DATA: Figures of merit and constraints from testing General Relativity using the latest cosmological data sets including refined COSMOS 3D weak lensing (Jason Dossett, Jacob Moldenhauer, Mustapha Ishak)

Phys.Rev.D84:023012,2011 (The University of Texas at Dallas)



The standard cosmological model and its cosmological parameters

- Physically meaningful cosmological parameters are used to characterize FLRW sub-models

- parameter tells us something about the properties of the universe, for example:
  - H_0 is the Hubble expansion rate parameter today
  - $\Omega_M \equiv \rho_M / \rho_c$ is the fraction of the matter energy density in the critical density
- A note about the number of cosmological parameters

Growth rate index parameter and [General Relativity + Dark Energy] models

- [L. Wang and Steinhardt, 1998] considered Dark Energy models with slowly varying *w* and derived

$$\gamma = \frac{3(1-w)}{5-6w} + \frac{3}{125}\frac{(1-w)(1-3w/2)}{(1-6w/5)^2(1-12w/5)}(1-\Omega_m) \qquad f = \Omega_m^\gamma$$

- with the asymptotic early value $\gamma_\infty = \frac{3(1-w)}{(5-6w)}$
- [see also for example Linder and Cahn, 2007; Mortonson, Hu, Huterer, 2009; Zhang et al. 2007; Gong, 2008; Polarski and Ganouji, 2008, Gong, Ishak, A. Wang 2009 …]
- The approximation provides a fit of about 1% to the growth function f as numerically integrated from the ODE

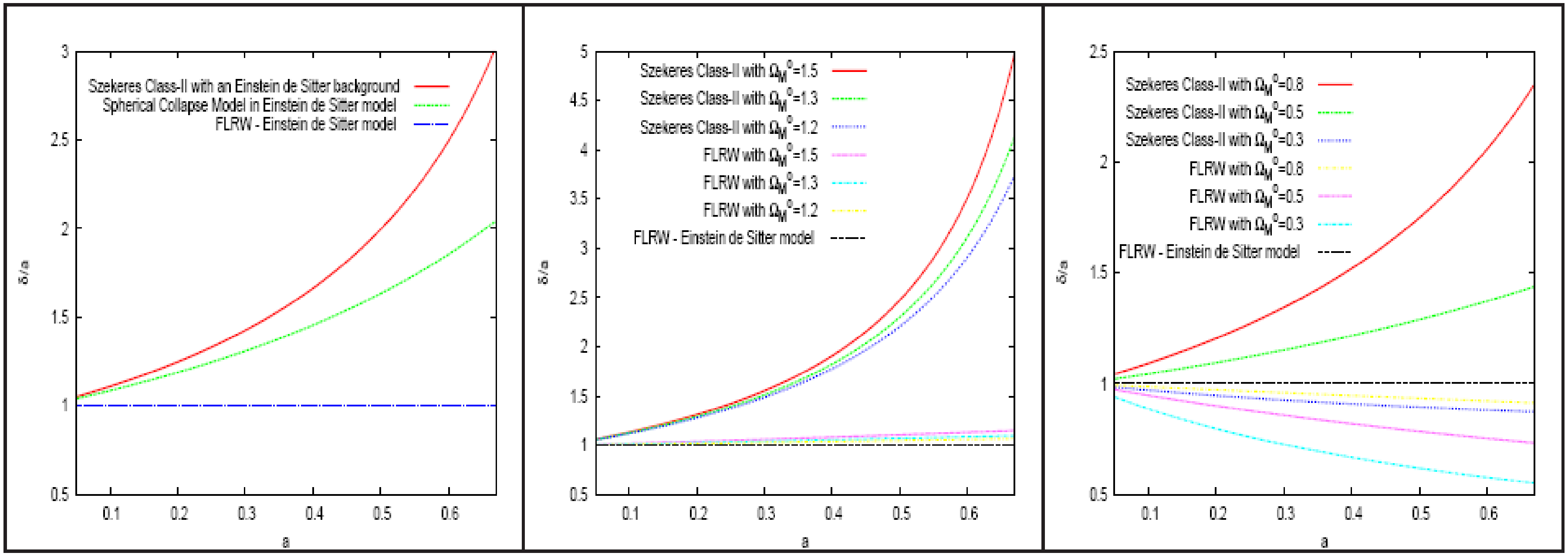


FIG. 1: LEFT: Growth rate of structure in flat Szekeres Class-II models (or Class-I models with a fixed value of z; see section IV) (solid-red curve), the usual spherical collapse model (green-dashed), and the perturbed Einstein-de Sitter (EdS) model (blue-dotted). The Szekeres growth rate is stronger than that of the perturbed EdS by up to a factor of 3. The Szekeres growth rate is also stronger than that of the spherical collapse model. CENTER: Growth rate of structure in positively curved Szekeres Class-II models for various values of Ω_M^0 (or Class-I models with a fixed value of z and various values of $\Omega_M^0(z)$). The growth rate in linearly perturbed FLRW models with the same values of Ω_M^0 are plotted for comparison. RIGHT: Growth rate of structure in negatively curved Szekeres Class-II models for various values of Ω_M^0 (or Class-I models with a fixed value of z and various values of $\Omega_M^0(z)$). The growth rate in linearly perturbed FLRW models with the same values of Ω_M^0 are plotted for comparison as well. In both cases, the Szekeres growth rates are stronger than those of the corresponding perturbed FLRW models by up to 5 times.

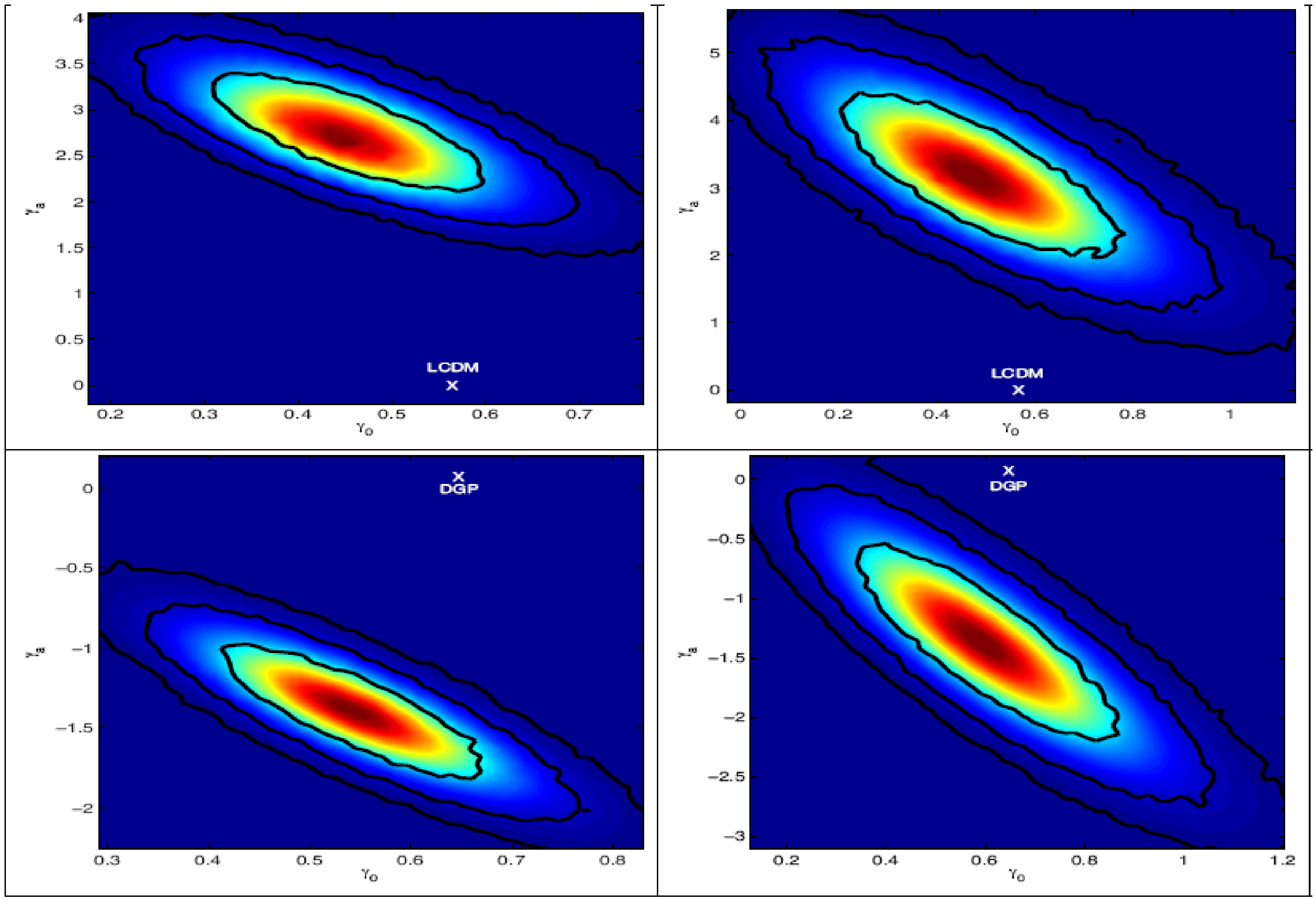


FIG. 3: Interpolated parameterization. TOP LEFT: Moderate scenario fitting fiducial DGP data on an assumed LCDM background. TOP RIGHT: Pessimistic scenario fitting fiducial DGP data on an assumed LCDM background. BOTTOM LEFT: Moderate scenario fitting fiducial LCDM data on an assumed DGP background. BOTTOM RIGHT: Pessimistic scenario fitting fiducial LCDM data on an assumed DGP background. As shown on the figures, in each case the incorrect assumed background model is ruled out to 99.7%.

Superclusters of galaxies. 300 Mpc and 400 MPC superstructures. A tenth of Hubble scales.