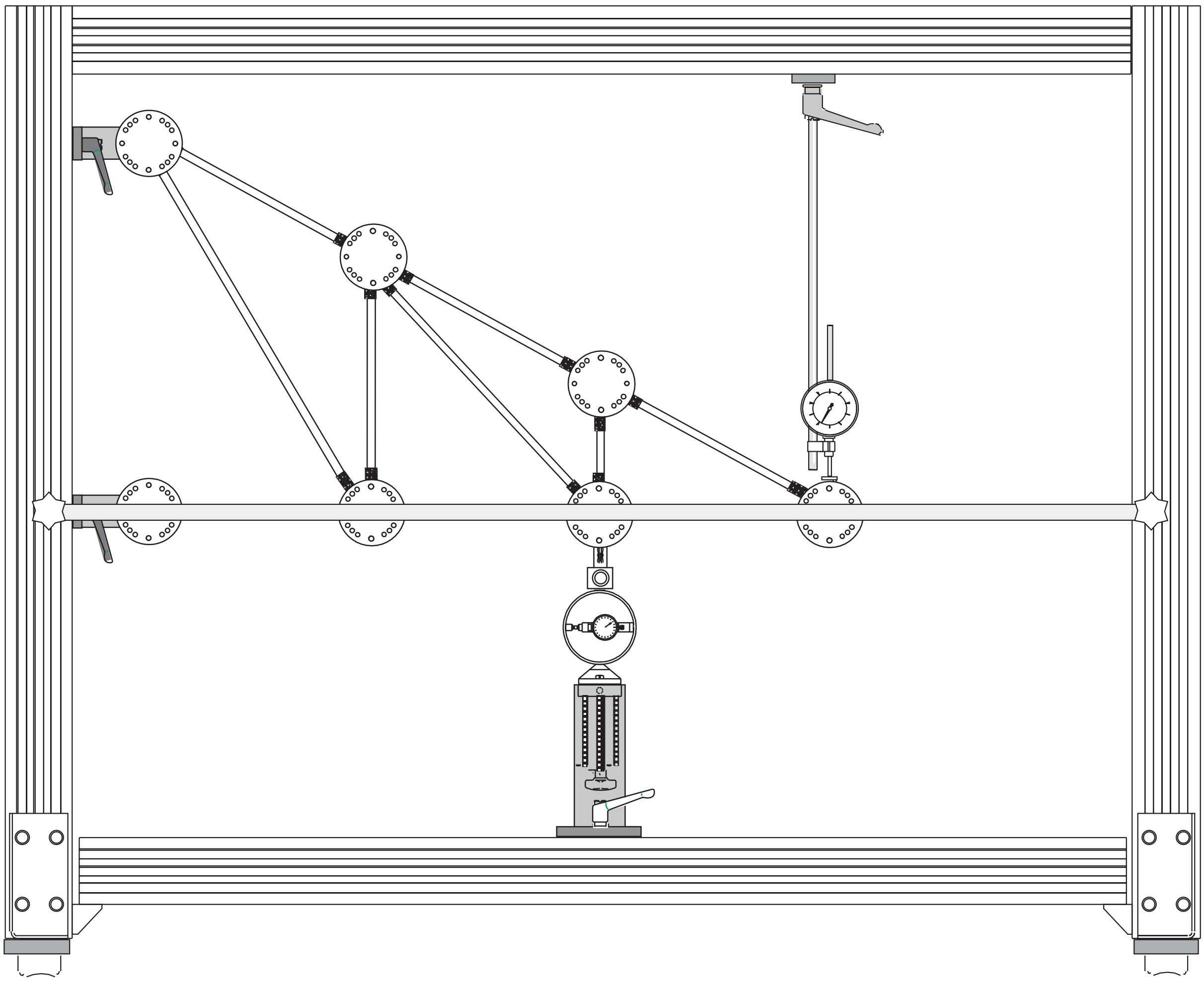
Experiment Instructions

SE 110.44 Deformation of Trusses

SE 110.44 DEFORMATION OF TRUSSES



Experiment Instructions

Please read and follow the safety regulations before the first installation!

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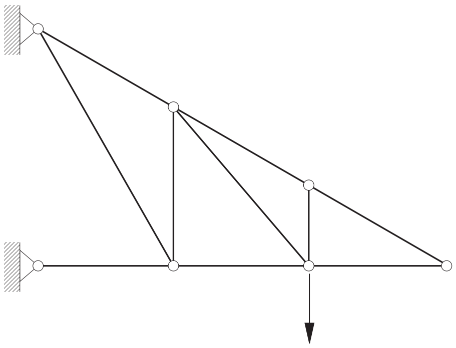
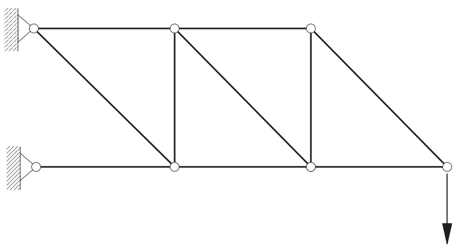
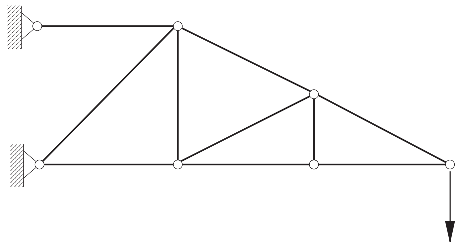
1 Introduction







The exercise set **deflection of trusses SE 110.44** is intended for use with the universal test frame **SE110**.

The exercise set allows the experimental investigation of deflection of trusses under load. The Castigliano theorems can be tested on it.

The exercise set SE110.44 boasts the following properties:

– Assembly of many different trusses with a range of bars in an appropriate gradation of lengths.

– The fixed length means that there is no need for time-consuming adjustment of the bars.

– Easy and quick connection of the rods via special snap connections and node disks.

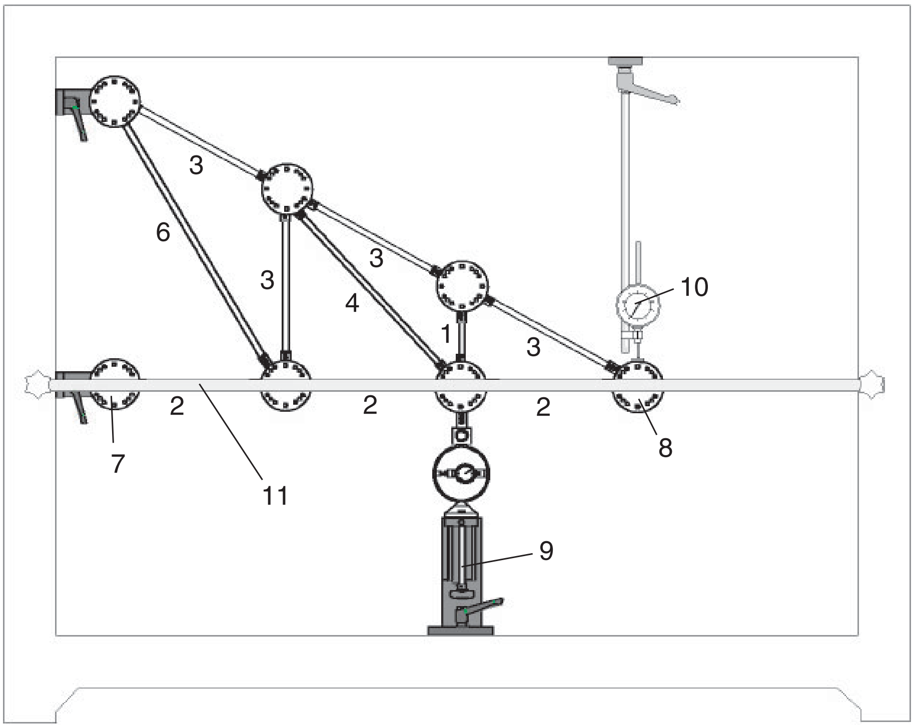
– Up to 12 bars possible in one node.

– Loading unit with spindle drive and annular force meter.

– Bars compatible with the truss FL110.

2 Unit description

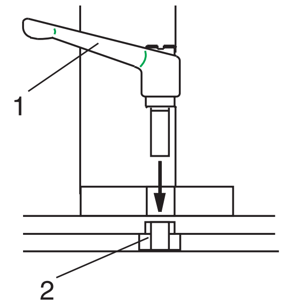
2.1 Unit assembly

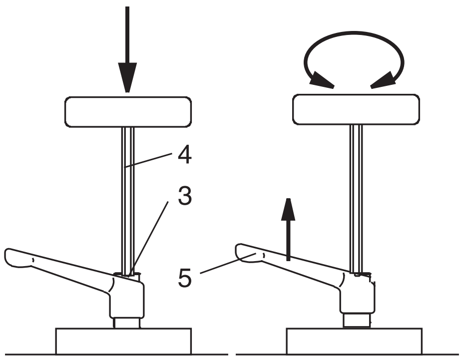


1 Bar 1, 150 mm, L/2
2 Bar 2, 259 mm, L√3/2
3 Bar 3, 300 mm, L
4 Bar 4, 397 mm, L√7/2
5 Bar 5, 424 mm, L√2 (not illustrated)
6 Bar 6, 520 mm, L√3
7 Support with node disk
8 Node disk
9 Loading unit with annular force meter and holder
10 Dial gauge
11 Cross member for the side stability of the truss

Fig. 2.1 Exercise set SE 110.44 “Deflection of trusses” shown with the frame SE 110

2.2 Assembly instructions for the frame SE 110





The elements are placed on the base frame and secured with clamping levers. Centring is performed through the slots of the profile.

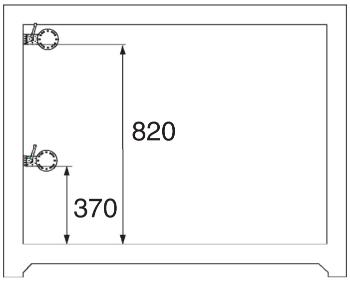
– Screw clamping lever (1) through the base plate from above into the slot nut (2) located in the slot .

If the clamping lever cannot be screwed far enough, proceed as follows:

– Press the axle of the clamping lever (3) firmly downwards with a screwdriver (4).
– Release the connection between the hand lever and axle by pulling the hand lever (5) upwards.
– The axle of the clamping lever can now be turned with the screwdriver.
– After the hand lever is released, it once again engages securely in the axle.

If the hand lever (4) comes up against a stop when tightening or releasing, it can be brought into a different position by pulling it up and turning it.

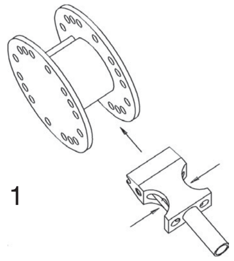
2.3 Truss assembly



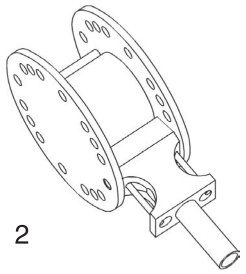
Mount both supports on the vertical part of the frame.

Upper support at a height of 820 mm,

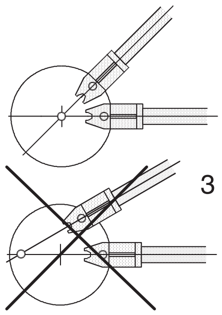
lower support at a height of 370 mm.



– To do this, press together the connecting bolts of the snap closure with your fingers and push the snap closure between the perforated plates of the node (1).

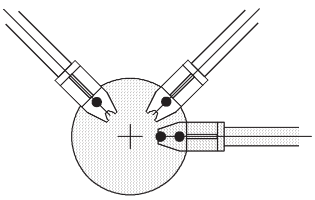


– Then allow the connecting bolts to engage in accordance with the bar angle (2).

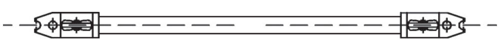


– Longer bar axles must pass through the mid-point of the node disk (3).

ATTENTION: In order to assemble a stable truss, a bar of a node must in each case engage in the locking pin of the node disk. This securely connects the node disk with a bar.



2.4 Bars



The bars are made of tubular PVC with an outside diameter of 16 mm and a wall thickness of 1.8 mm.

They are graduated in steps of:

150 mm

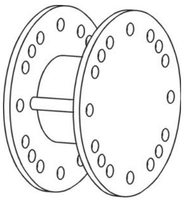
259 mm

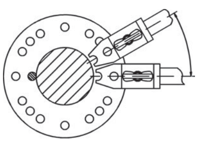
300 mm and

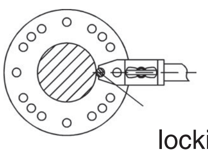
424 mm

At the ends of the bars there are special snap closures that engage in the node disks.

2.5 Node disks







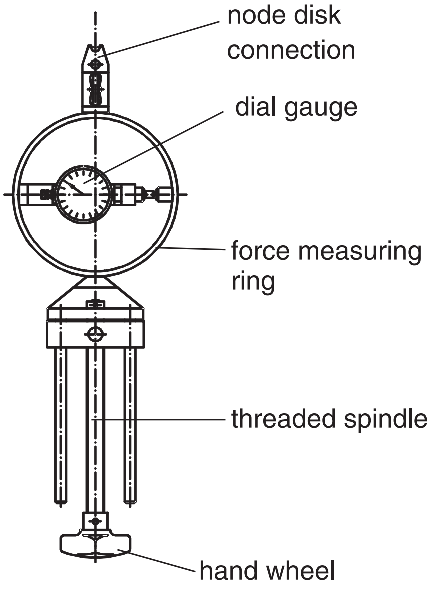
The nodes consist perforated circular disks. In each disk there are 16 holes so that scale divisions of 30° and 45° are possible.

The smallest angle spacing between two bars is 30°. This enables up to 12 bars to engage on one node.

The bars are connected freely articulated in the truss plane. The deflection is rigid and perpendicular to the truss plane, with the result that the level truss has sufficient lateral stability.

One bar of a node in each case must be rigidly connected to the node disk via a second locking pin. This is necessary in order to obtain a rigid, statically certain truss. Otherwise, the node disk would itself become an additional bar of the truss as a result of the spatially dispersed linkage points.

2.6 Force measuring unit

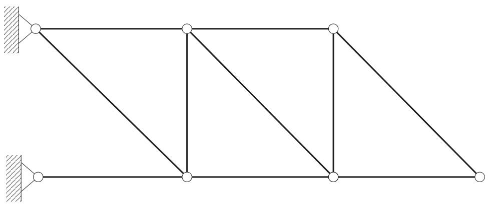


The external stresses are applied to the truss via the force measuring unit, and the support reactions are measured. The core is a force measuring ring. This deforms elastically under the influence of external force. This deflection is measured with a position measuring gauge and is a direct measurement of the force.

In order to be able to generate a force, the respective test object (in other words the truss) must first be pre-stressed. Play in the nodes is eliminated in order to achieve this. Pre-stressing is performed via a fine threaded spindle and a hand wheel.

The force measuring unit can be clamped to the frame at any point so that it can be moved to different angles. It enables tensile and pressure forces of up to 200 N to be applied and measured.

2.7 Examples of other trusses

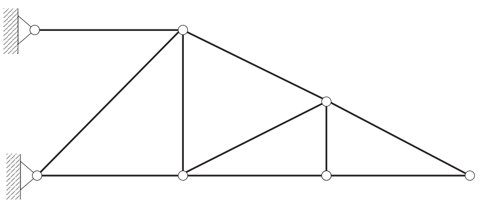


Parts required:

7 x bar 3 (300 mm)

3 x bar 5 (424 mm)

5 x node disk



Parts required:

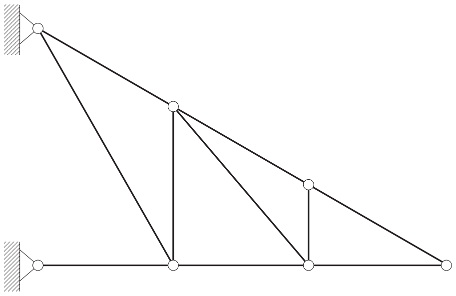
1 x bar 1 (150 mm)

2 x bar 2 (259 mm)

6 x bar 3 (300 mm)

1 x bar 5 (424 mm)

5 x node disk



Parts required:

1 x bar 1 (150 mm)

3 x bar 2 (259 mm)

4 x bar 3 (300 mm)

1 x bar 4 (397 mm)

1 x bar 6 (520 mm)

5 x node disk

3 Safety

3.1 Risks to the unit and function

Attention: Do not overload the truss.

Max. permissible load ± 200 *N*

4 Theory

4.1 Shape changing work in rod supports and springs

When elastic systems are stressed, they deform proportional to the load increase. Here, the external forces and moments do work along the displacement or rotation of their application points. This work of the external forces and moments is stored in elastic systems and is designated the shape changing work. This is not lost. Rather, it is released again when the force is relieved (e.g. steel springs). The work of the external loads arising from temperature expansion is not stored in mechanical systems and is not released when the force is relieved. It is not shape changing work.

Shape changing work is the energy stored in the elastic system. It corresponds to the work of the external forces and moments in the load-related deflection of the elastic system.

The **work of a force** is the product of the displacement travel with the force components in the direction of displacement.

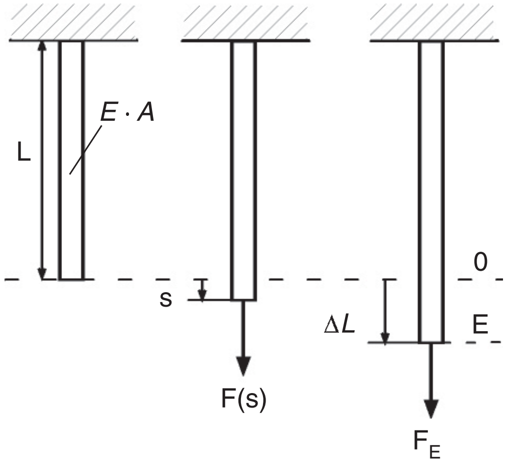
$$W = \int_{(s)} F(s)ds$$

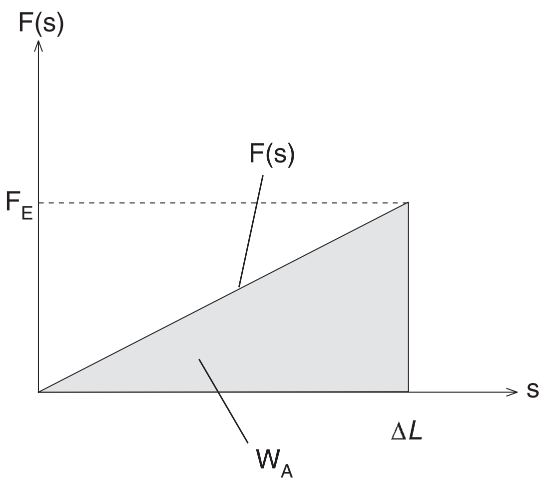
$F(s)$: Force components in the direction of displacement

The **work of a moment** is the product of the rotation angle in the radian measure with the moment components in the rotation direction

$$W = \int_{(\varphi)} M(\varphi)d\varphi$$

$M(\varphi)$: Moment components in the rotation direction





Example: Tension bar

The force should not be applied suddenly. Instead, it should start at zero and increase at a rate that the bar can follow.

The shaded area in the F-s diagram corresponds to the external work W_A that is done by the force F(s) up to a limit value F_E. This work is stored in the tension bar and should be expressed below by the section size F_L of the state of equilibrium.

The following applies:

$$s = \frac{F(s) \cdot L}{E \cdot A} \qquad F(s) = \frac{E \cdot A}{L} \cdot s$$

$$\Delta L = \frac{F_E \cdot L}{E \cdot A}$$

whereby E = module of elasticity (E-module)

A = cross section effective for shape change

$$W_{0\to E} = \int_{s=0}^{s=\Delta L} F(s) \cdot ds = \int_{s=0}^{s=\Delta L} \frac{E \cdot A}{L} \cdot s \cdot ds$$

$$W_{0\to E} = \frac{E \cdot A}{L} \cdot \frac{1}{2} \cdot s^2 \Big/_{0}^{\Delta L}$$

$$W_{0\to E} = \frac{1}{2} \cdot \frac{E \cdot A}{L} \cdot \Delta L^2 = \frac{1}{2} \cdot \frac{F_E^2 \cdot L}{E \cdot A}$$

In the case of fully applied force in the state of equilibrium, $F_L = F_E$ applies and therefore

$$W = W_{0\to E} = \frac{1}{2} \cdot \frac{F_L^2 \cdot L}{E \cdot A}$$

4.2 The Castigliano theorem

(Italian master builder 1847 – 1884)

Requirements:

- Linear-elastic material behaviour (Hook's Law must apply)
- No expansions arising from temperature changes

The Castigliano theorem:

- The partial derivation of the entire shape changing work of a system after the external force gives the displacement components of the force application point in the force direction as a result of all external loads that were taken into account for the formulation of the shape changing work.
- The partial derivation based on an external moment gives the rotation angle component of the moment application point in the radian measure.
- The partial derivation based on a statically uncertain variable is always zero.

$$w_i = \frac{\partial U}{\partial F_i}; \quad \varphi_i = \frac{\partial U}{\partial M_i}; \quad 0 = \frac{\partial U}{\partial X_i}$$

U: Shape change work of the entire system

w_i: Displacement component of the application point of F_i in the direction of F_i

φ_i: Rotation angle (in radian measure) of the moment application point of M_i in the direction of M_i

F_i: External force

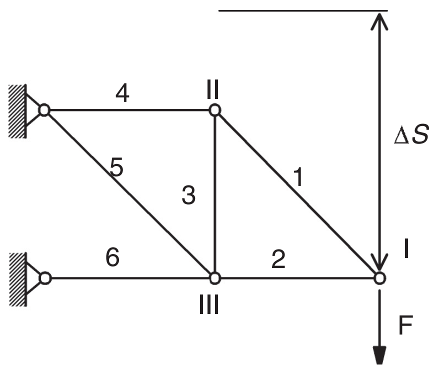
M_i: External moment

X_i: Statically uncertain variable (force or moment)

5 Experiments

Determining the vertical displacement (in the direction of the load) of the truss at the location where a load of 200 N acts

5.1 The structure of the truss



The deflection in the y direction of the node I is to be determined in the truss shown via a load F. To do this, assemble the truss in line with the information in Sections 2.2 to 2.6.

Mount the load application unit directly under node I. (Attention: The exact alignment has a positive influence on the results).

Fit and align the dial gauge over node I to determine the displacement.

5.2 Carrying out the experiment

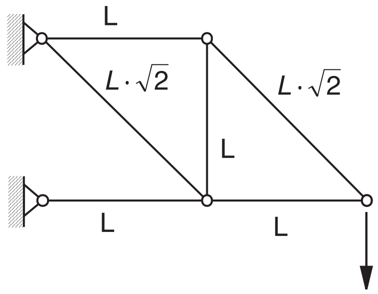
- With the hand wheel of the threaded spindle, increase the pre-stressing until the dial gauge of the force measuring unit just starts to respond.
- Align the scale zero point of the force measuring unit to the pointer.
- Increase the load in steps of 20 N up to 200 N.
- Note deflections and forces.

5.3 Measurement

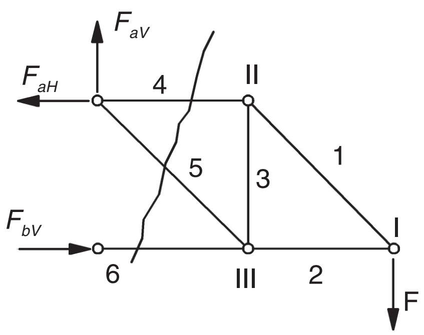
Force encoder		Displacement
F		Δw
Force [N]	Travel [0.01 mm]	Travel [0.01 mm]
0	0	0
20	2	
40	4	
60	6	
80	8	
100	10	209
120	12	
140	14	
160	16	
180	18	
200	20	350

At the start of load application, it must be ensured that the truss bars are not connected without play. The laws of strain only apply when the play has been set.

5.4 Calculating the bar forces



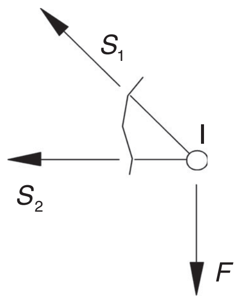
In this example, the bar forces of the truss shown opposite are to be calculated and compared with the experiment results.



A position plan is first compiled with the bar and rod numbers.

The load F is applied to node I in a vertical direction.

The bar forces are calculated via the node equilibrium and Ritter dissection.



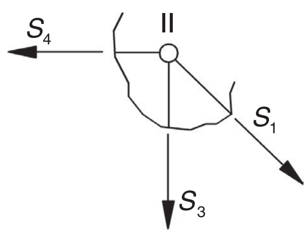
Node I:

$$\sum F_V = 0 = -F + S_1 \cdot \sin 45^\circ$$

$$S_1 = \sqrt{2} \cdot F = 1.41 \cdot F$$

$$\sum F_H = 0 = S_2 + S_1 \cdot \cos 45^\circ$$

$$S_2 = -F$$



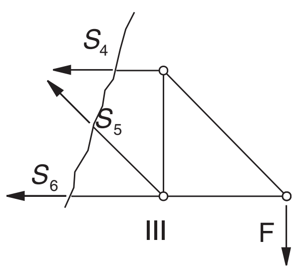
Node II:

$$\sum F_H = 0 = S_4 - S_1 \cdot \cos 45^\circ$$

$$S_4 = F$$

$$\sum F_V = 0 = -S_3 - S_1 \cdot \sin 45^\circ$$

$$S_3 = -F$$



Ritter dissection 1

$$\sum F_V = 0 = -F + S_5 \cdot \cos 45^\circ$$

$$S_5 = \sqrt{2} \cdot F = 1.41 \cdot F$$

$$\sum F_H = 0 = S_4 + S_5 \cdot \sin 45^\circ + S_6$$

$$S_6 = -2 \cdot F$$

Summary of the bar forces						
bar no.:	1	2	3	4	5	6
outside force	1.41F	-F	-F	F	1.41F	-2F
when F=200 N	282 N	-200	-200 N	200 N	282 N	-400 N

5.5 Calculating the displacement using the Castigliano method:

$$w_i = \frac{\partial U_F}{\partial F_i}$$

whereby

w_i: Displacement by F_i

U_F: Shape change energy of any linear elastic system

F_i: Generalised individual force

For articulated bar systems (no moments, only normal forces), the shape change energy U_{F_i} is a function of the section sizes.

$$U_F = \frac{1}{2}\sum_i \int_0^L \left[\frac{F^2(x)}{EA(x)}\right] dx$$

after the partial derivation

$$U_F = \sum_{i=1}^{n} \frac{1}{E_i \cdot A_i} \cdot F_i \cdot L_i$$

with the designations of the truss bars

$$U_F = \left[\frac{1}{A \cdot E_{PVC}} \cdot L_1 \cdot F_1^2 + L_2 \cdot F_2^2 + L_3 \cdot F_3^2 + L_4 \cdot F_4^2 + L_5 \cdot F_5^2 + L_6 \cdot F_6^2\right]$$

The displacement by F_i in the direction of F_i is:

$$w_i = \frac{1}{F_i} \cdot \left[\frac{1}{A \cdot E_{PVC}} \cdot \left(L_1 \cdot F_1^2 + L_2 \cdot F_2^2 + L_3 \cdot F_3^2 + L_4 \cdot F_4^2 + L_5 \cdot F_5^2 + L_6 \cdot F_6^2 \right) \right]$$

with the values

L: pure PVC bar length in m

F: bar forces in N

A: bar cross section in m² effective for the shape change

E: PVC E-module in N/m²

$$w_i = \frac{0.136\ m \cdot (200\ N)^2 + 0.136\ m \cdot (-200\ N)^2 + 0.136\ m \cdot (-400\ N)^2 + 0.26\ m \cdot (282\ N)^2 + 0.26\ m \cdot (282\ N)^2 + 0.136\ m \cdot (-200\ N)^2}{200\ N \cdot 8.03 \cdot 10^{-5}\ m^2 \cdot 1.54 \cdot 10^9\ \frac{N}{m^2}}$$

$$w_i = 0.00321\, m$$

Displacement calculated using the Castigliano method in

the direction of the active force: 3.21 mm

measured displacement: 3.50 mm.

Deflection calculations for points at which no external load is applied can be calculated, for example, using the auxiliary forces method.

6 Appendix

6.1 Worksheets

Experiment:		Date:
		Name:
Load application unit (annular force meter)		Dial gauge
Location:		Location:
Force F	Travel	Displacement Δw
in Newton	in 0.01 mm	in 0.01 mm
0		
20		
40		
60		
80		
100		
120		
140		
160		
180		
200		

6.2 Technical data

Bar – bar lengths

Bar No.:	Length nominal L_N	Unit length	Length betw. bolts L_B	Length PVC tube L_{PVC}	Quantity
1	150	L/2	90	6	2
2	259	$L\cdot\sqrt{3}/2$	199	131	5
3	300	L	240	136	7
4	397	$L\cdot\sqrt{7}/2$	337	233	1
5	424	$L\cdot\sqrt{2}$	364	260	3
6	520	$L\cdot\sqrt{3}$	460	356	1

PVC tube 16 x 1.8 mm
Material: PVC-U
Cross section area: 8.03×10^{-5} m²
Measured PVC E-module: 1540 N/mm²
(referred to the actual PVC tube length)

Tensile strength: $\sigma_z = 50 - 75 \dfrac{N}{mm^2}$

Pressure strength: $\sigma_p = 80 \dfrac{N}{mm^2}$

Node disks

Quantity: 5
Angle gradations: 30°,45°, 60°, 90°

Load application unit

Principle: Annular force meter

Force range:	max. 500 N
Resolution:	10 N/Sct.
Read-off range of the dial gauge:	0-3 mm
Scaling:	1000 N/mm
Adjustment travel:	90 mm

6.3 Formula symbols

A: Cross section area

E: E-module

F: Force, load

L: Bar length

U: Shape-changing energy, work

s: Distance, travel

M: Moment

w: Displacement

6.4 References

Hütte, Die Grundlagen der Ingenieurwissenschaften, 29th Edition, Springer-Verlag

6.5 Scope of delivery

1 x experimental set SE 110.44 Deflection of trusses

1 x experiment instructions SE 110.44