HIGH ENERGY NUCLEAR COLLISIONS

HARD PROBES, HEAVY QUARKS, STRONG GLUON FIELDS

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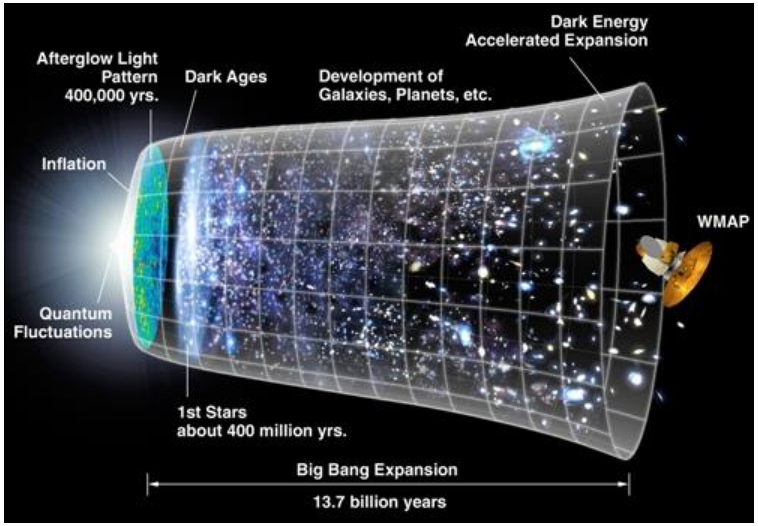
Office of Science
U.S. Department of Energy
JET Collaboration

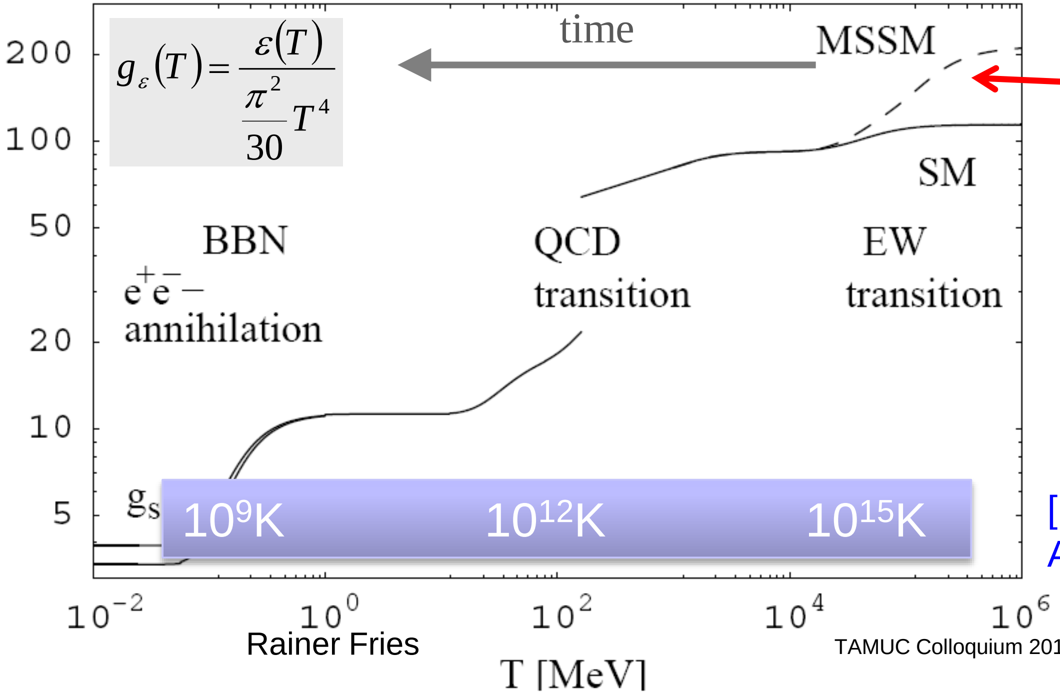
The Early Universe

- Cooling and expansion → succession of phase transitions and freeze-outs.
- Rapid decrease in degrees of freedom.
- We will encounter the same in the Little Bang!



Atomic Physics & Astronomy: Atoms, Galaxies, …

Nuclear Physics: Strong Force/QCD



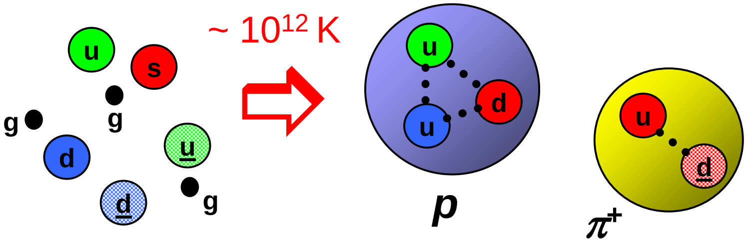
HEP: new particles?

- QCD transition @ ~ few μs @ ~ 10^{12} K
- Quarks and gluons

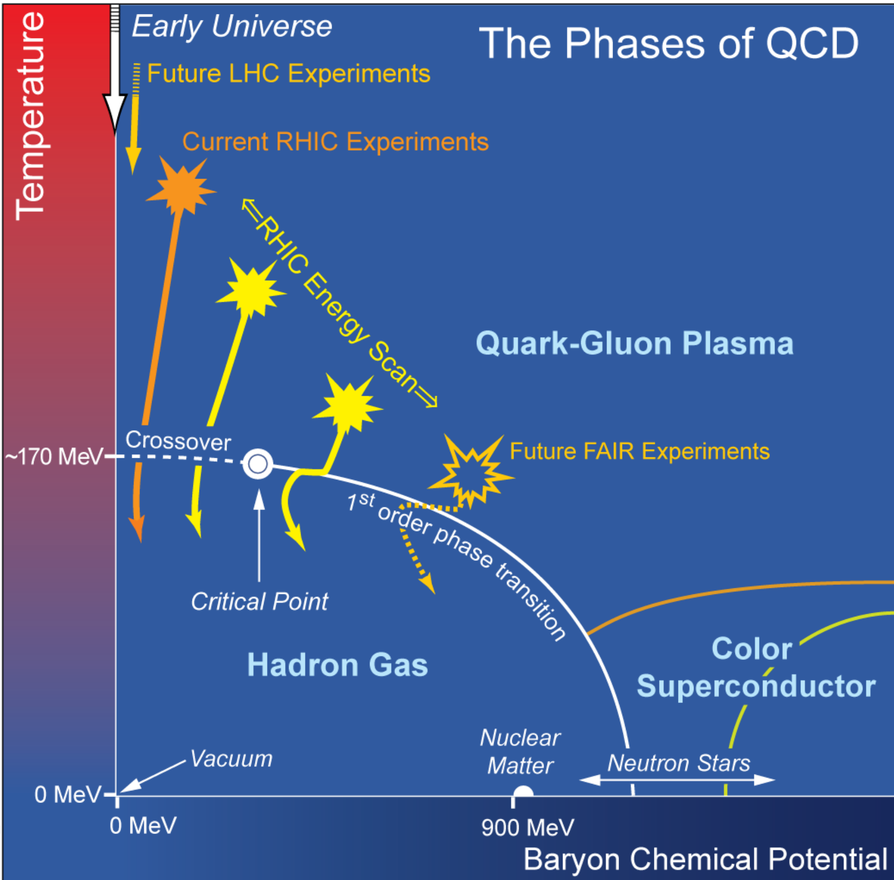
[D.J. Schwarz, Ann. Phys. 12, 220 (2003)]

The QCD Transition in the Cosmos

- Quantum Chromodynamics (QCD) = Theory of the Strong Force
- Fundamental degrees of freedom = quarks and gluons ("partons")
    - Partons are not the ground states of a QCD system
    - Transition to bound states (protons, neutrons, pions, …)

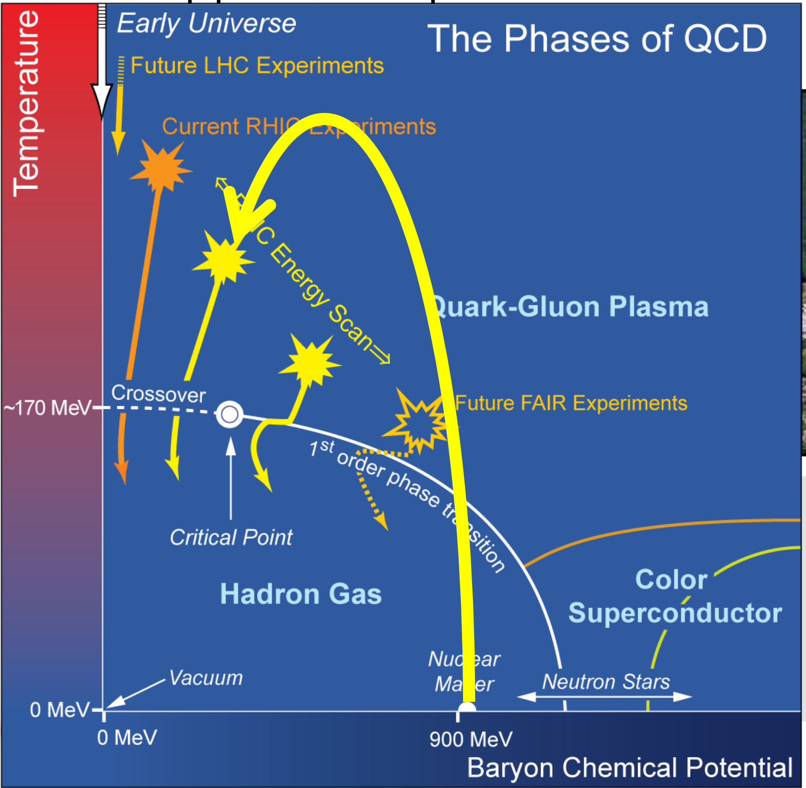


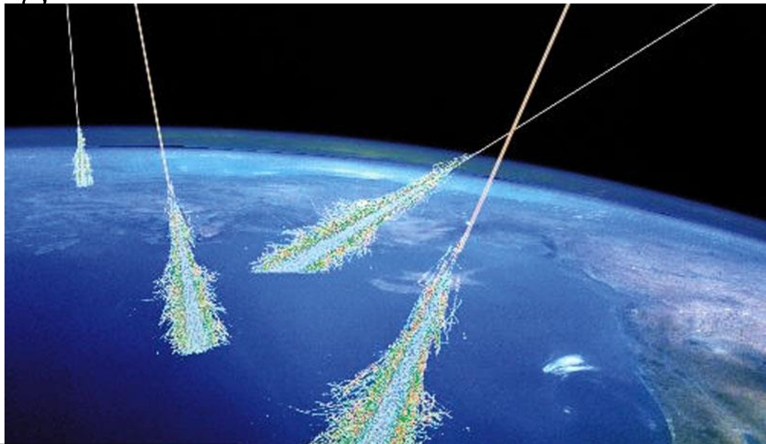
- Thermodynamic and transport properties of QCD matter?
- QCD phase diagram?
- Properties of the QCD phase transition?



The 'Little Bang'

- Luckily we don't have to go back to the Big Bang.
- Take heavy nuclei in present day colliders.
- Create a "fireball" with $T \sim 10^{12}$ K, $p \sim 10^{35}$ Pa with lifetime ~ 10^{-22} s.
- Another scenario: Cosmic rays hitting the upper atmosphere.



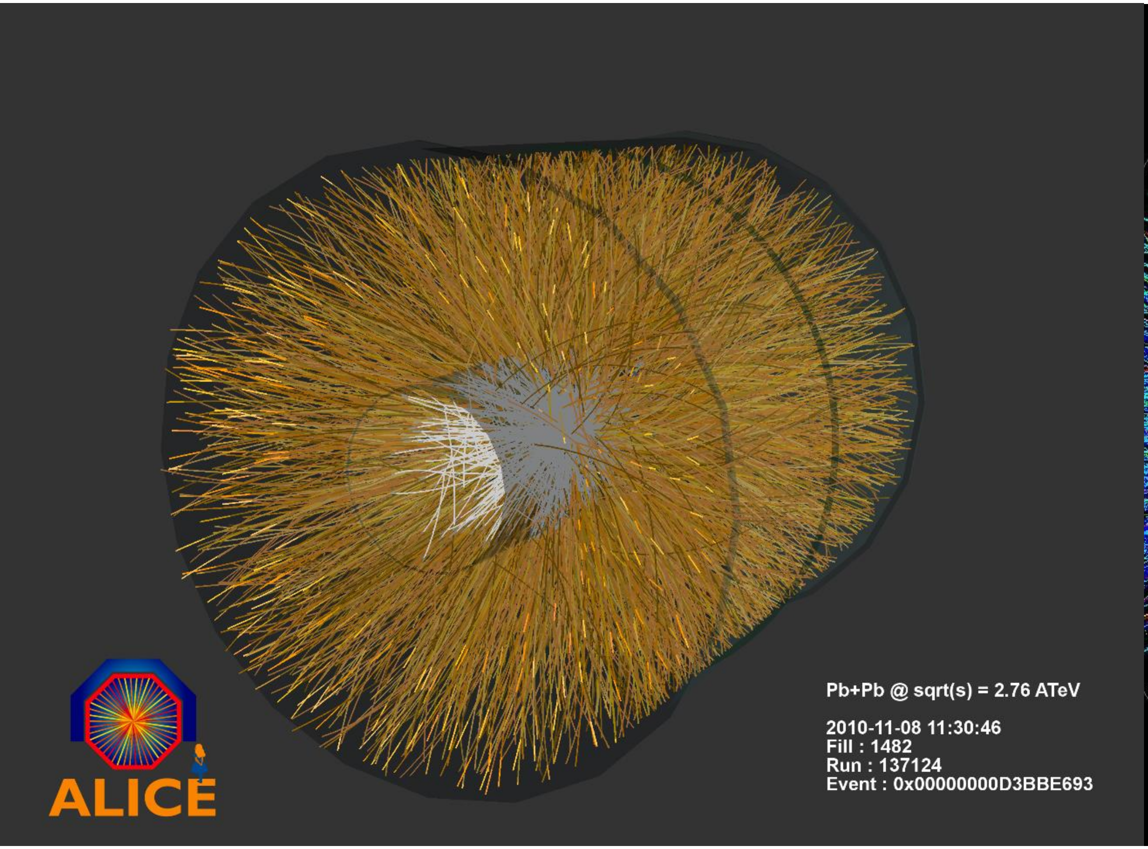


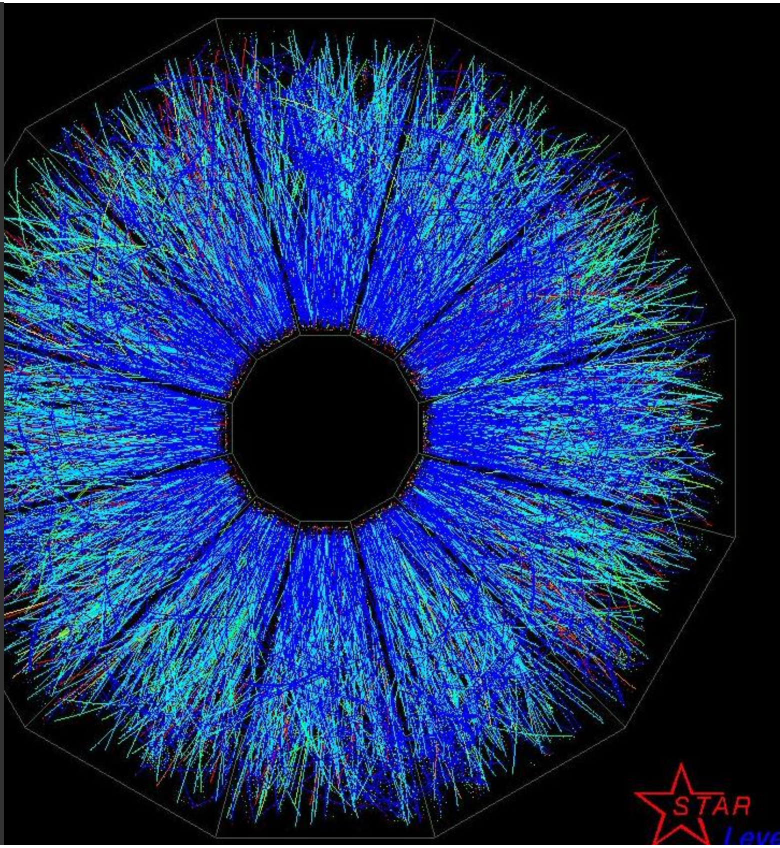
RHIC:
s_{NN} = 500 GeV (p+p)
s_{NN} = 200 GeV (Au+Au)
s_{tot} = 40 TeV (Au+Au)
T_{max} ~ 400 MeV

LHC:
s_{NN} = 14/7 TeV (p+p)
s_{NN} = 5.5/2.76 TeV (Pb+Pb)
s_{tot} = 1.1/0.55 PeV (Pb+Pb)
T_{max} ~ 800 MeV

High Energy Nuclear Collisions

- Thousands of particles created.
- Directed kinetic energy of beams → mass (particle) production + thermal motion + collective motion





QCD in Two Slides

- Electrodynamics: U(1) gauge field
  - ☐ Field equations: Maxwell

$$\partial_\mu F^{\mu\nu} = eJ^\nu$$

  - ☐ Field strength tensor

$$F^{\mu\nu} = \frac{i}{e}\left[D^\mu, D^\nu\right] = \partial^\mu A^\nu - \partial^\nu A^\mu$$

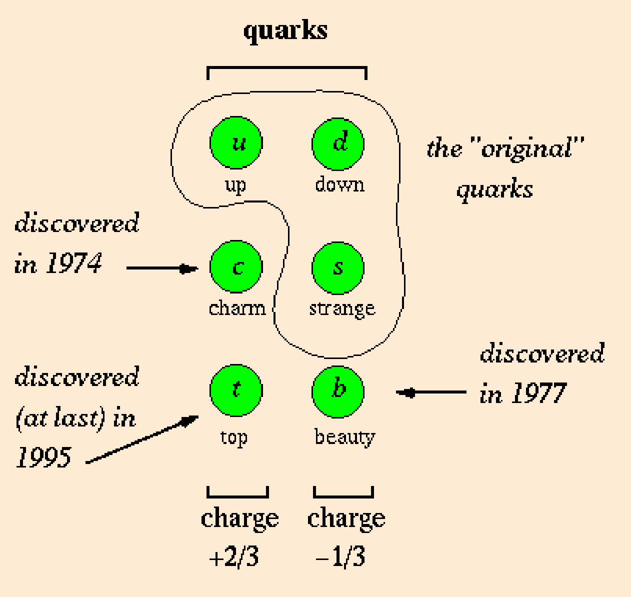
- QCD: SU(3) gauge field
  - ☐ Field equations: Yang-Mills

$$\boxed{D_\mu} F^{\mu\nu} = gJ^\nu$$

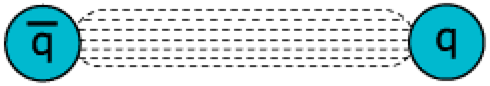
  - ☐ Field strength tensor

$$F^{\mu\nu} = \frac{i}{e}\left[D^\mu, D^\nu\right] = \partial^\mu A^\nu - \partial^\nu A^\mu \boxed{-ig\left[A^\mu, A^\nu\right]}$$

- Quarks



- Notable difference in dynamics:
  - ☐ Non-abelian fields/self-interacting force carriers
  - ☐ Larger coupling (g ›› e)
  - ☐ Long distance forces via QCD strings.

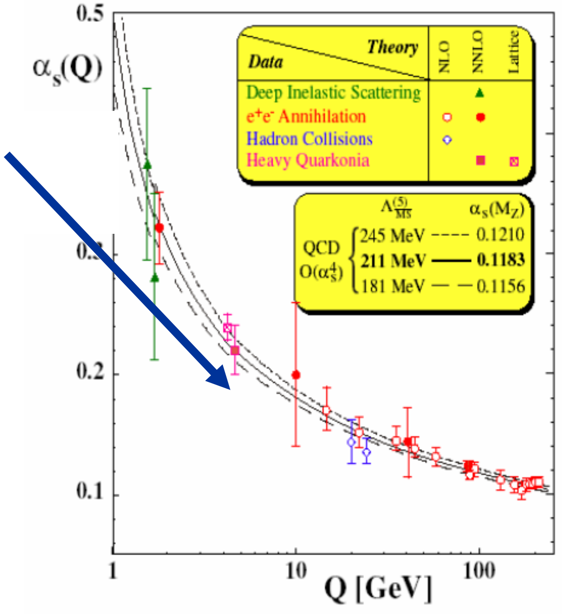


QCD in Two Slides

- Asymptotic freedom at large momentum

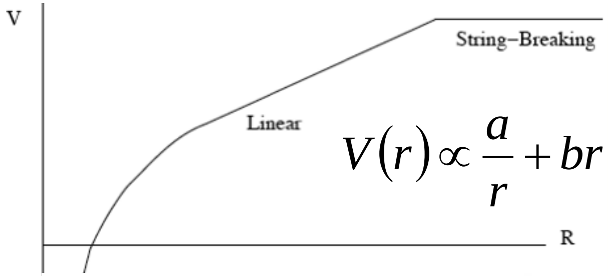
$$\alpha_s = \frac{g^2}{4\pi} = \frac{1}{\beta_0 \ln Q^2 / \Lambda_{QCD}^2}$$

Running of the strong coupling constant

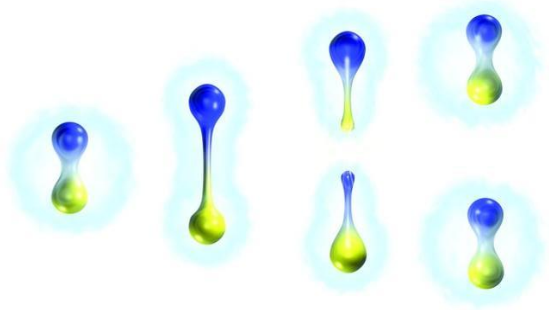


- Chiral Symmetry Breaking in the ground state
  - Chiral condensate
  - QCD mass generation: ~ 5 MeV → ~ 300 MeV

- Confinement



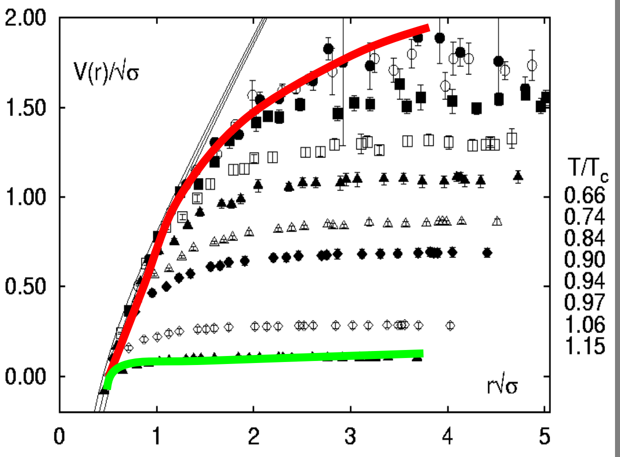
$$V(r) \propto \frac{a}{r} + br$$



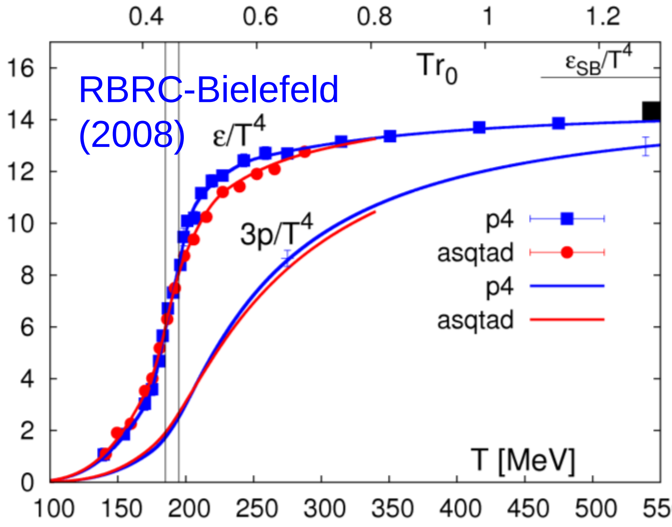
- Can we solve QCD?
  - Perturbation theory (works only in selected cases)
  - Lattice (numerically very expensive, works only in selected cases)
  - Models, effective theories (too many!)

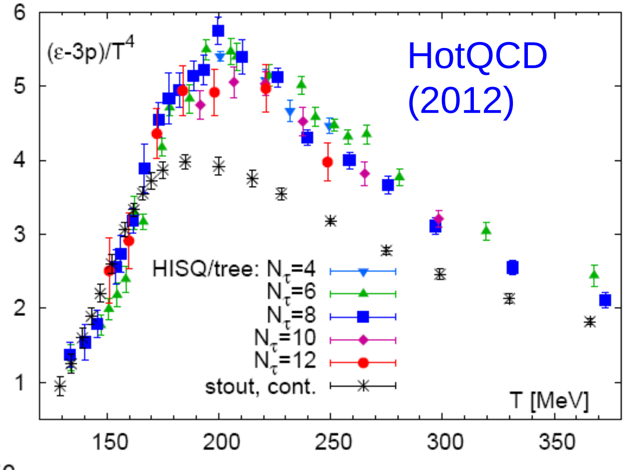
The QCD Transition

- Best knowledge so far: lattice QCD at small μ_B.
  - Deconfinement → vanishing confinement potential
  - Chiral symmetry restoration
- Chiral critical temperature at $\mu_B = 0$.
  - $T_c = 154 \pm 9$ MeV [RBC-Bielefeld]
  - $T_c = 151$ MeV [Wuppertal-Bielefeld]
- Equation of state at $\mu_B = 0$:
  - Cross over for realistic s quark masses, pQCD works above ~$3T_c$.
- Finite μ_B: critical point expected close to $T = T_c$ and $\mu \sim 200$-400 MeV.

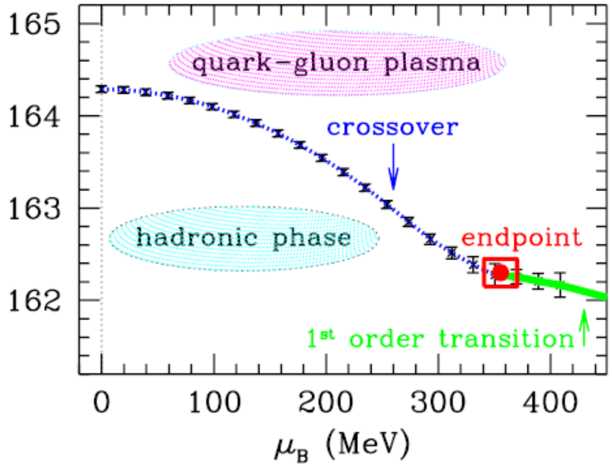


Static quark potential (Karsch et al.)



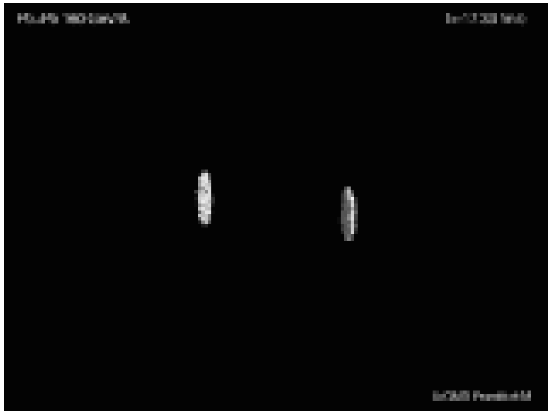


[Fodor, Katz, JHEP 04, 050 (2004)]

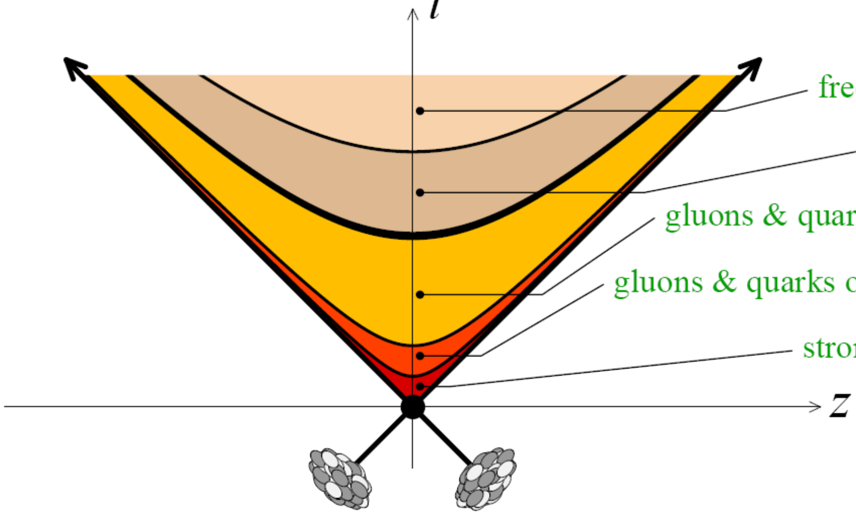


Nuclear Collisions In Two Slides

- Basic geometry:
  - Lorentz contraction of the nuclei $L \sim R/\gamma \to 0$
  - Approximate boost-invariance in beam direction a la Bjorken (expansion with ~c).
  - Delayed transverse expansion.
  - For arbitrary impact parameter *b*: elliptic deformation in transverse direction.
- Time evolution:



[Henning Weber: Pb+Pb @ SPS]



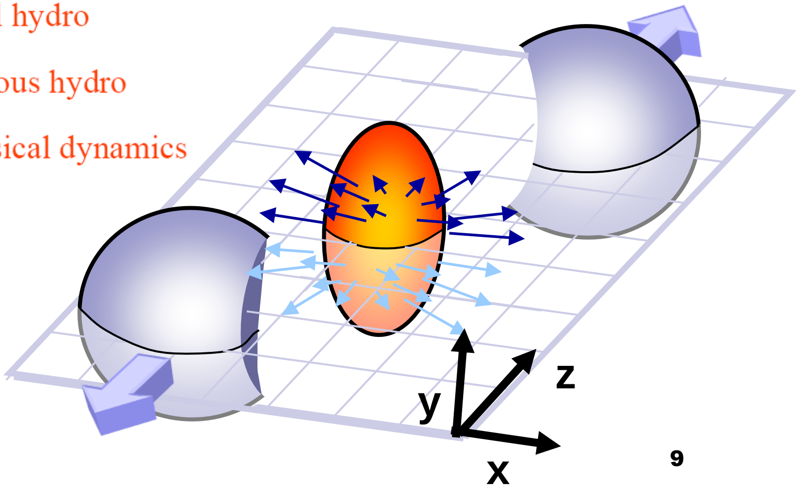
freeze out

hadrons ⟶ kinetic theory

gluons & quarks in eq. ⟶ ideal hydro

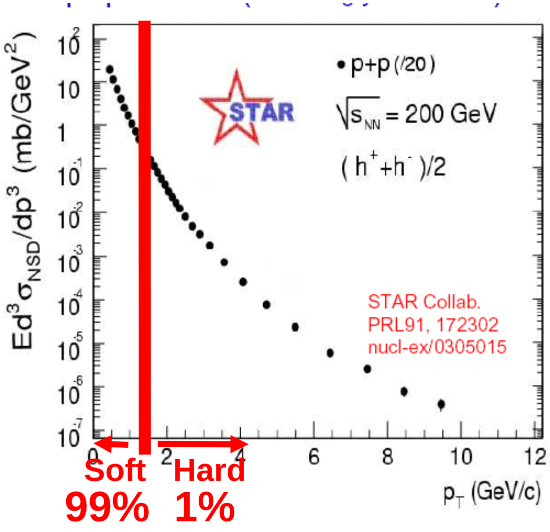
gluons & quarks out of eq. ⟶ viscous hydro

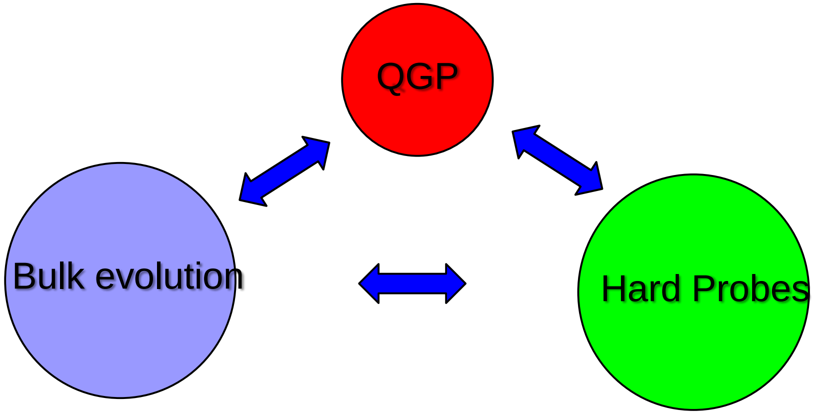
strong fields ⟶ classical dynamics



Nuclear Collisions in Two Slides

- Hierarchy in Momentum Space:
- Soft particles ($P_T <$ 1-2 GeV): >99%, “bulk” fireball
  - Thermalization → Quark Gluon Plasma → hydrodynamic expansion and cooling
- Hard particles ($P_T >$ 1 -2GeV): <1%, rare “hard” probes
  - QCD jets with FSI (but no thermalization)
  - Probes for the QGP
  - Also includes particles not participating in the strong force (photons, leptons): EM probes

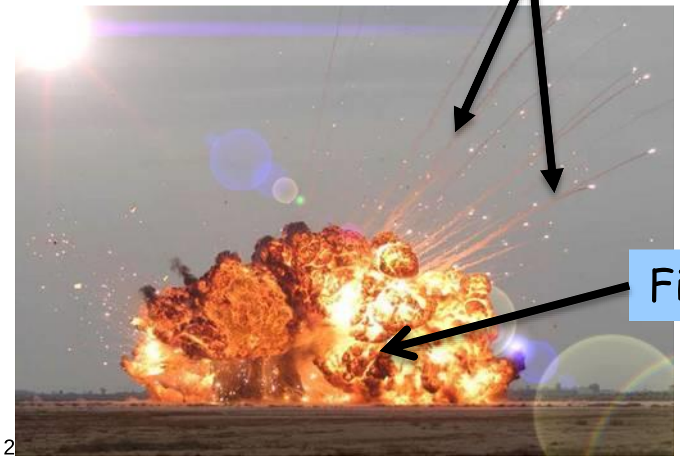



QGP

Bulk evolution

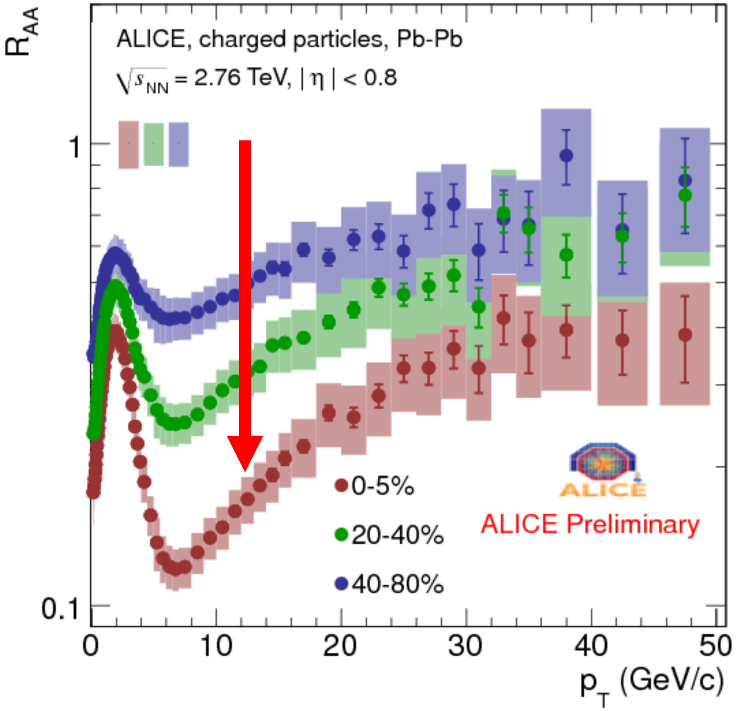
Hard Probes

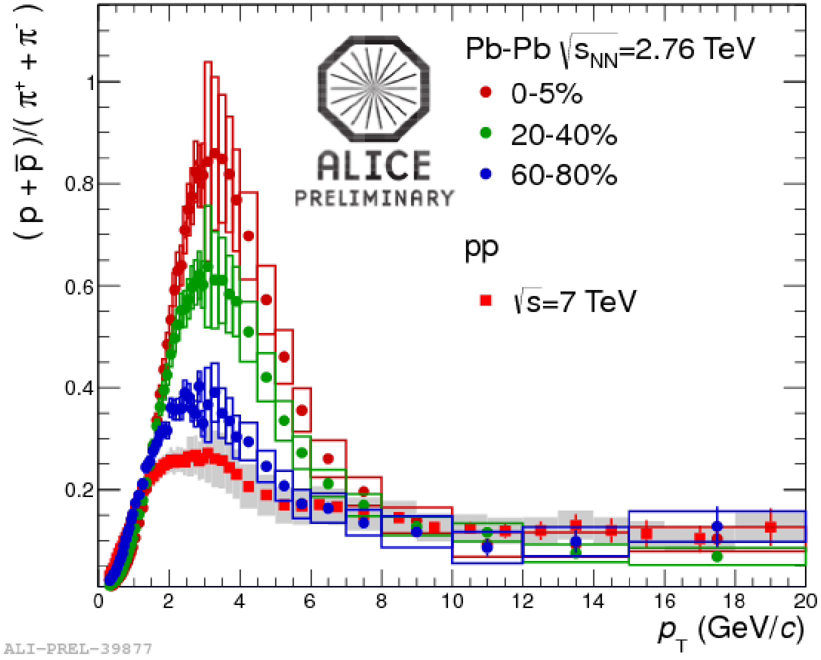
Some particles accelerated much more: fast probes.


Fireball

Spectra and R_{AA}

- Nuclear modification factor $R_{AA} = \frac{dN^{AA}/dp_T}{N_{coll}\, dN^{pp}/dp_T}$
- Proton/pion ratio.



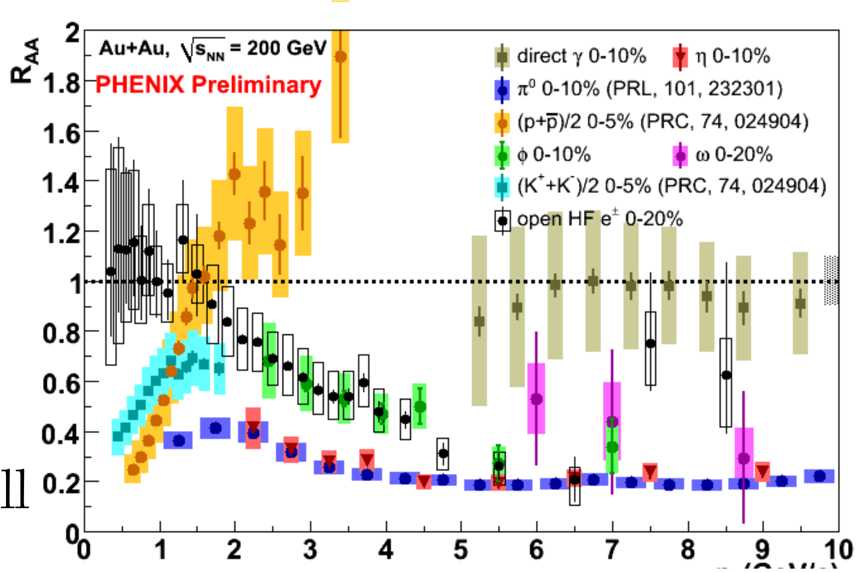


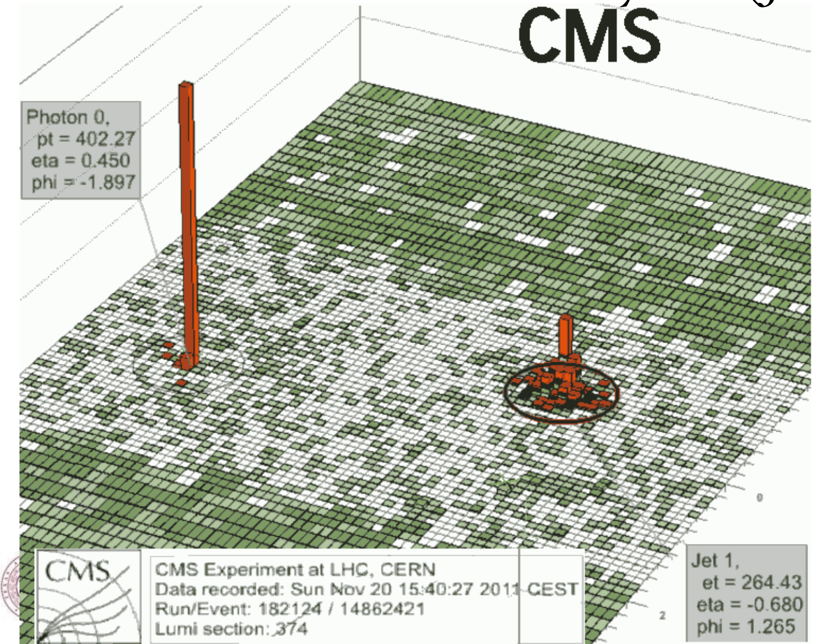
- 3 distinct regions (soft/intermediate/hard) clearly visible.
- High momentum suppression ($R_{AA} < 1$): jet quenching

Hard Probes

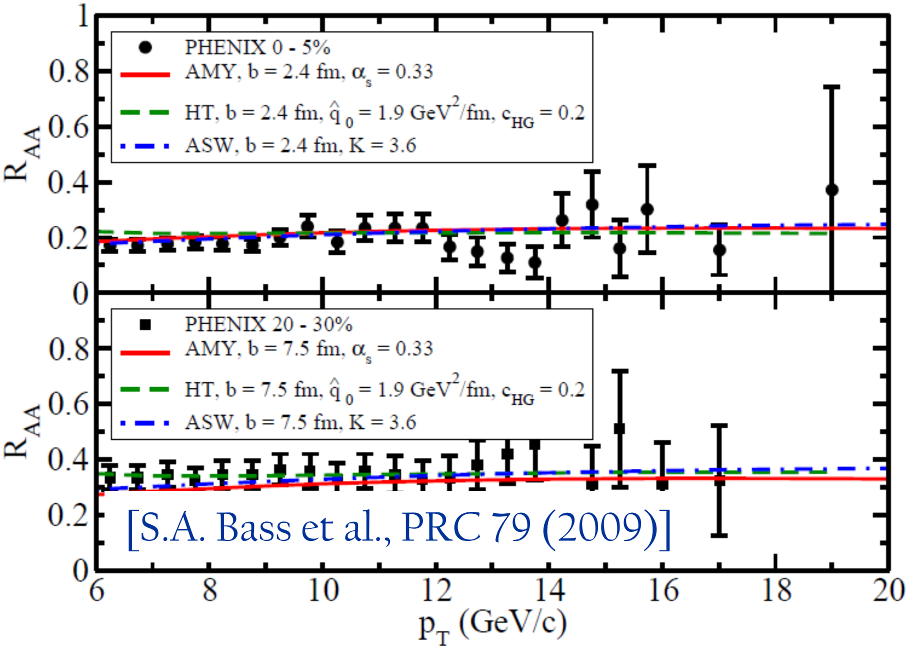
- High energy quarks/gluons ➩ QCD jets
- RHIC era: hadrons
- LHC era: jet reconstruction
- Consensus estimate for parton energy loss $\hat{q} = 1.5 - 4.5$ GeV^2/fm at T ~ 350 MeV.

- Jets require in-medium jet shower MC: will be done in the next 2 years (JET)





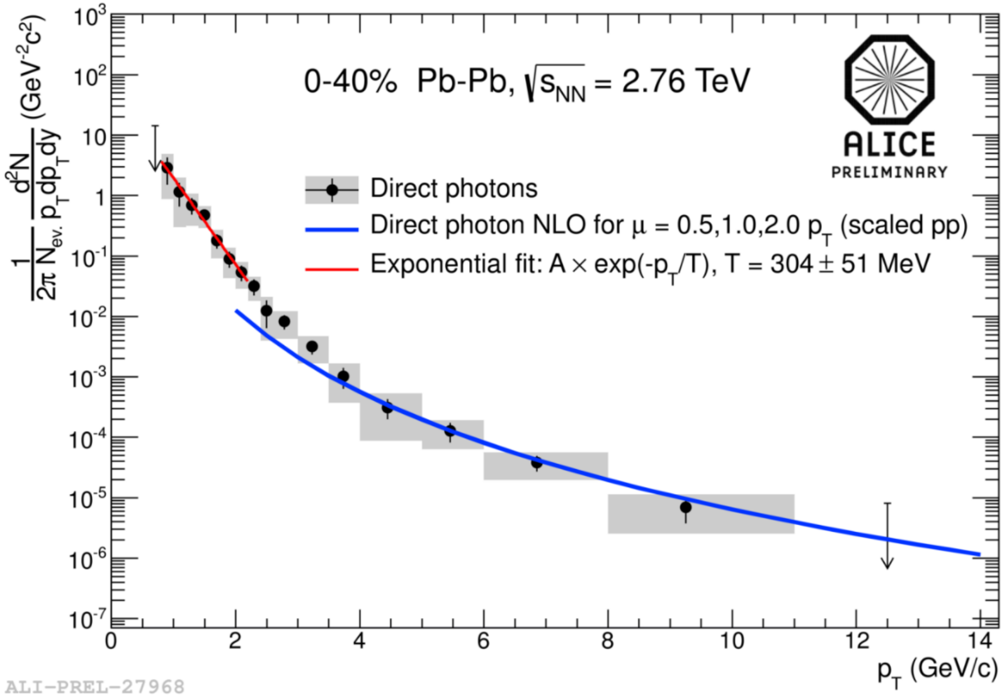
$$\hat{q} = \frac{\langle k_T^2 \rangle}{\lambda}$$

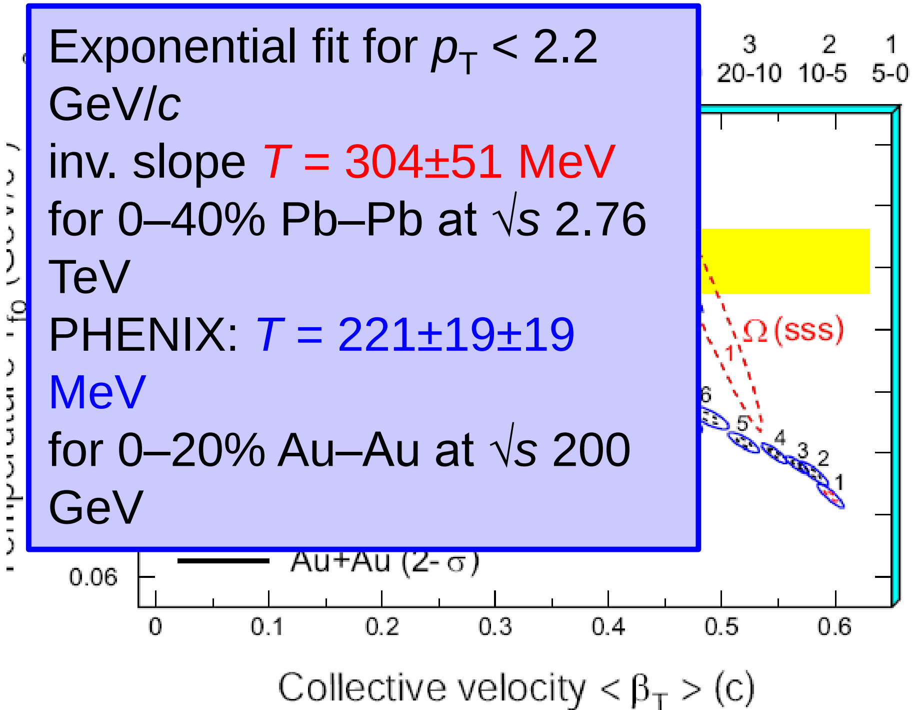


[S.A. Bass et al., PRC 79 (2009)]

Equilibrium and Flow

- Hadrons found in chemical and kinetic equilibrium with kinetic freeze-out temperature T_{kin} ~ 100 MeV.
- Spectra for hadrons below 2 GeV exhibit blast wave shape = thermal distribution + collective flow.
- Local thermal equilibrium + pressure gradients evolving over time
- Hydrodynamic behavior
- Average temperature from photon spectrum

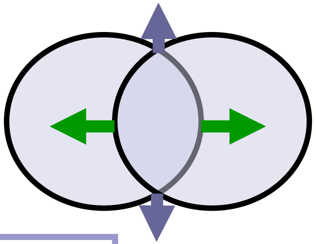


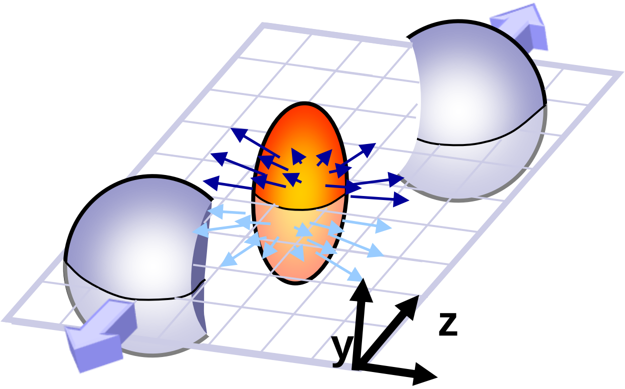


Exponential fit for $p_T < 2.2$ GeV/*c*
inv. slope *T* = 304±51 MeV
for 0–40% Pb–Pb at √*s* 2.76 TeV
PHENIX: *T* = 221±19±19 MeV
for 0–20% Au–Au at √*s* 200 GeV

Elliptic Flow

- Initial spatial eccentricity in transverse plane at finite b → final momentum space anisotropy of produced particles.
- Analysis of final particle anisotropy in terms of harmonics:
  - ☐ v_2 = elliptic flow

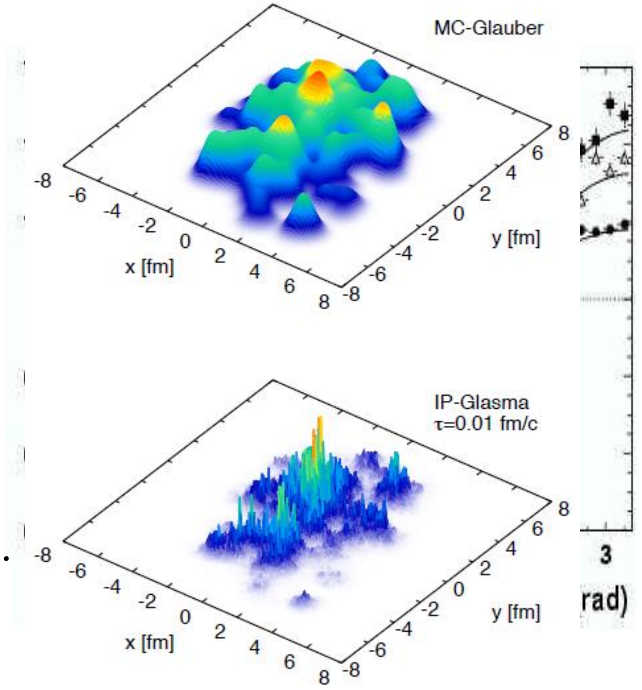




$$\boxed{\frac{dN}{P_T dP_T d\varphi} = \frac{dN}{2\pi P_T dP_T}\left[1+2\sum_n v_n(P_T)\cos(n\varphi+\delta_n)\right]}$$

- Fluctuations are important (odd coefficients!)
- Excellent test for hydro and transport models.

- Workhorse: relativistic, dissipative (2nd order) hydrodynamics, often coupled with hadronic transport (“afterburner”)
- v_n data require short equilibration times ~ 0.2–1 fm/c.

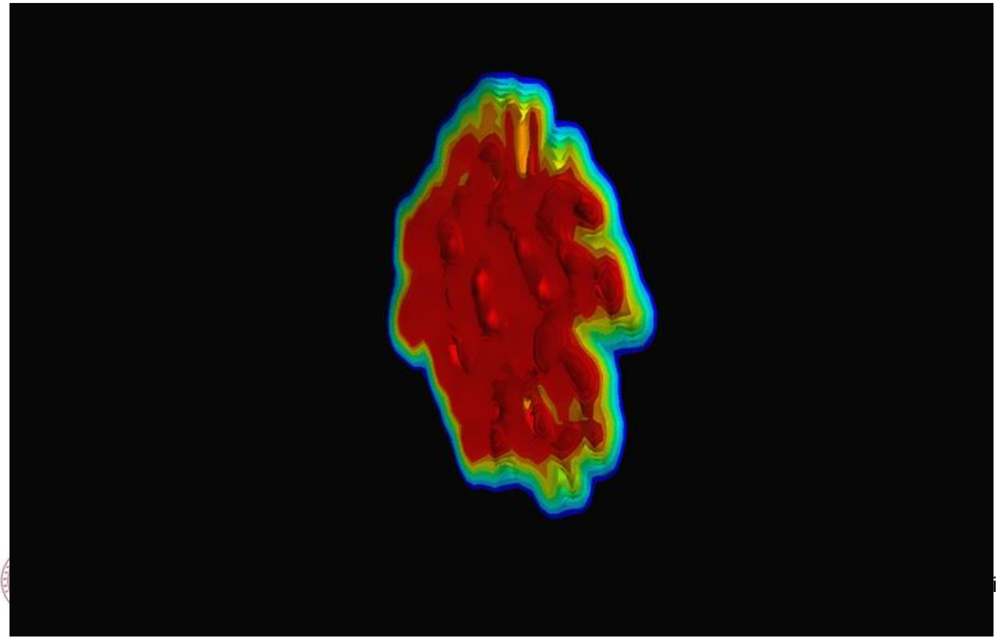


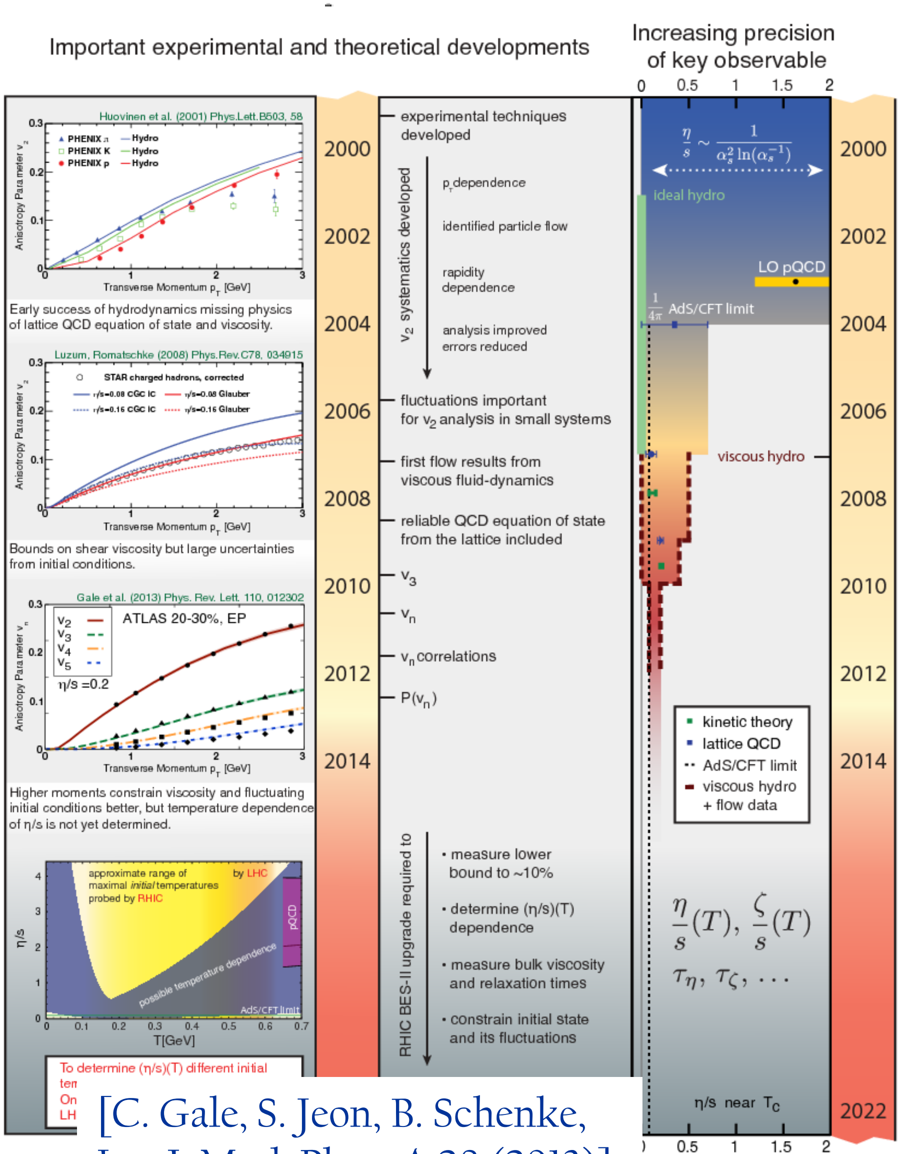
[B. Schenke, P. Tribedy, R. Venugopalan, Phys. Rev. Lett. 108 (2012)]

Hydrodynamic Simulations

- Very good description of spectra and flow variables.
- Estimates for QCD shear viscosity.
- Lattice equation of state.
- Fluctuations provide constraints on initial conditions.

[Lumpy MUSIC (Schenke et al.)]



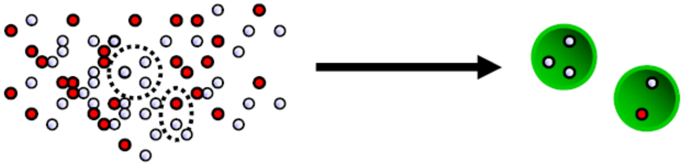


[C. Gale, S. Jeon, B. Schenke, Int. J. Mod. Phys. A 28 (2013)]

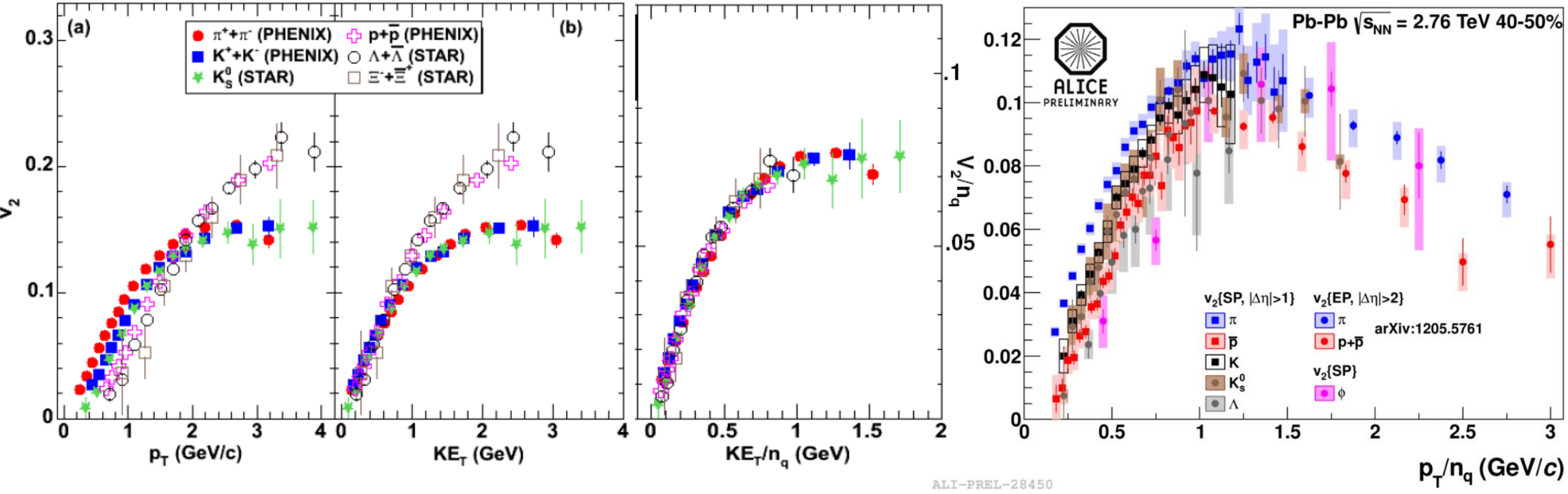
Quark Recombination: Parton Flow

- Quark recombination models of hadronization predict a scaling of elliptic flow with valence quark number:

$$v_2^M(p_t) = 2v_2^p\left(\frac{p_t}{2}\right) \quad \text{and} \quad v_2^B(p_t) = 3v_2^p\left(\frac{p_t}{3}\right)$$



- Low P_T: scaling with kinetic energy (hydro + freeze-out hierarchy; not a recombination effect) [He, Fries and Rapp, PRC 82 (2010)]



- Flow has partonic origin!

Heavy Quark Probes

The following is based on work with M. He and R. Rapp at Texas A&M.

[M. He, RJF and R. Rapp, Phys. Rev. C 86, 014903 (2012)]
[M. He, RJF and R. Rapp, Phys. Lett. B 701, 445 (2011)]
[M. He, RJF and R. Rapp, Phys. Rev. C 85, 044911 (2012)]
[M. He, RJF and R. Rapp, Phys. Rev. Lett. 110, 112301 (2013)]
[M. He et al., Phys. Rev. E in press]

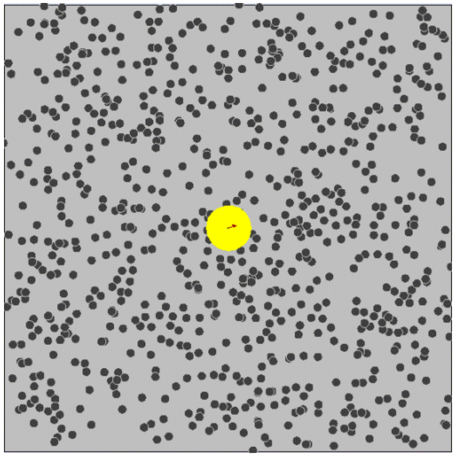
Heavy Quark: Dynamics

- Heavy quarks Q (charm, bottom) and heavy hadrons:
  - Kinetic equilibration rates parametrically suppressed by T/m_Q
  - Equilibration times ~ lifetime of the medium
- Degree of thermalization and collective motion (flow) = measure for HQ-medium interactions.
- Fokker-Planck dynamics, stochastically realized by Langevin equations

$$d\mathbf{x} = \frac{\mathbf{p}}{E}dt,$$

$$d\mathbf{p} = \boxed{-\Gamma(p)\mathbf{p}dt} + \boxed{\sqrt{2D(\mathbf{p}+d\mathbf{p})\,dt}\rho}$$

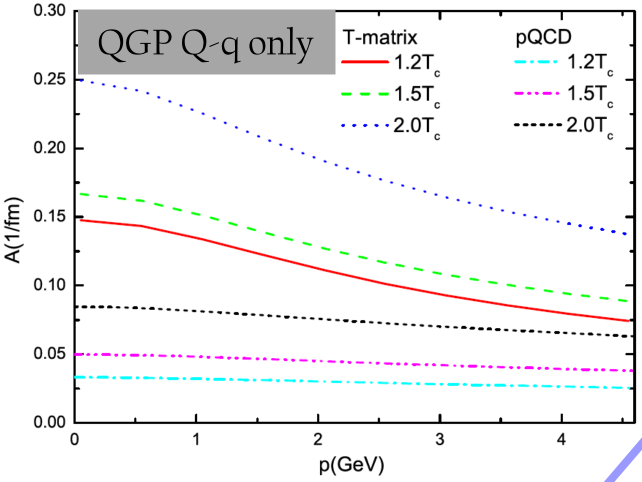
[B. Svetitsky, Phys. Rev. C 37, 2484 (1987)]
[H. van Hees and R. Rapp, Phys. Rev. C 71, 034907 (2005)]
[T. Koide, Kodama, Phys. Rev. E 83 (2011)]



[Paco and Hwang: Brownian Motion Gas Model Applet]

- Physical picture: many uncorrelated momentum kicks needed to change heavy flavor momentum → drag force and Brownian motion.

Heavy Quarks: Simulation Setup



- Heavy quark relaxation rates in QGP from elastic scattering.
- Potential constrained by Q-Q results for lattice QCD.
- Resonant correlations up to 1.5 T_c
- Physical picture: "*D*-meson"-like states survive above T_c.

[F. Riek and R. Rapp, PRC 82, (2010)]

Initial charm quark spectrum.

Langevin simulation of HQ in QGP

Simulation of bulk medium through hydrodynamics

Hadronization of HQs

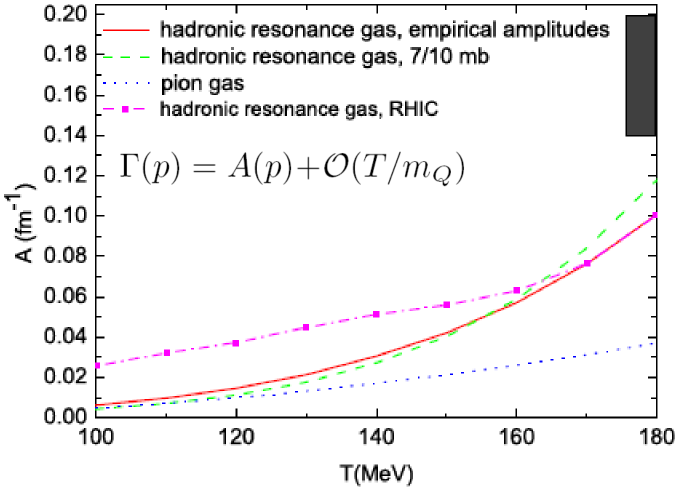
Transport coefficients from heavy flavor scattering rates in the medium

Langevin simulation of heavy hadrons in hadronic medium

Phenomenology, semi-leptonic decays, etc

- *D*-meson relaxation rates in hot hadron gas.
- Rough estimate for *B* mesons: scale *A* with *T*/*m*.

[M. He, RJF and R. Rapp, PLB 701, 445 (2011)]



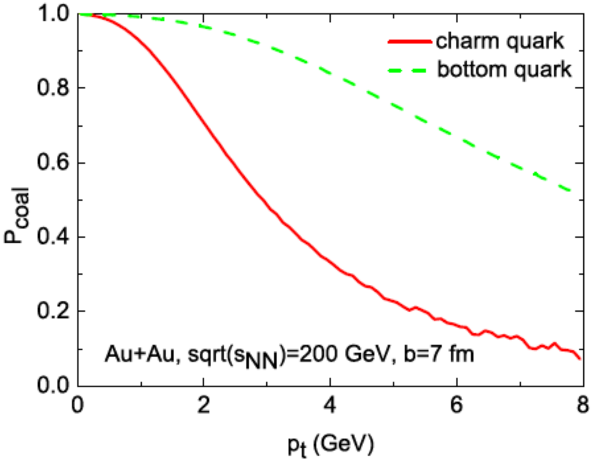
Heavy Quarks: Hadronization

- Resonance recombination based on a Boltzmann equation (respects kinetic equilibrium) + fragmentation
- How to decide recombination vs fragmentation rate?

Q-q recombination rate ~ Q-q in medium scattering rate!

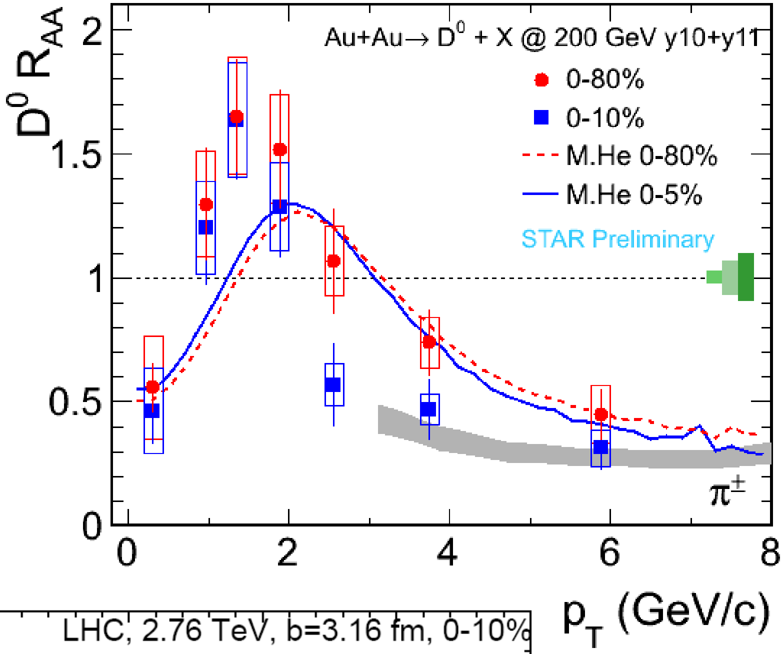
$$P_{\mathrm{coal}}(p) = \Delta\tau_{\mathrm{res}}\ \Gamma_{Q}^{\mathrm{res}}(p)$$

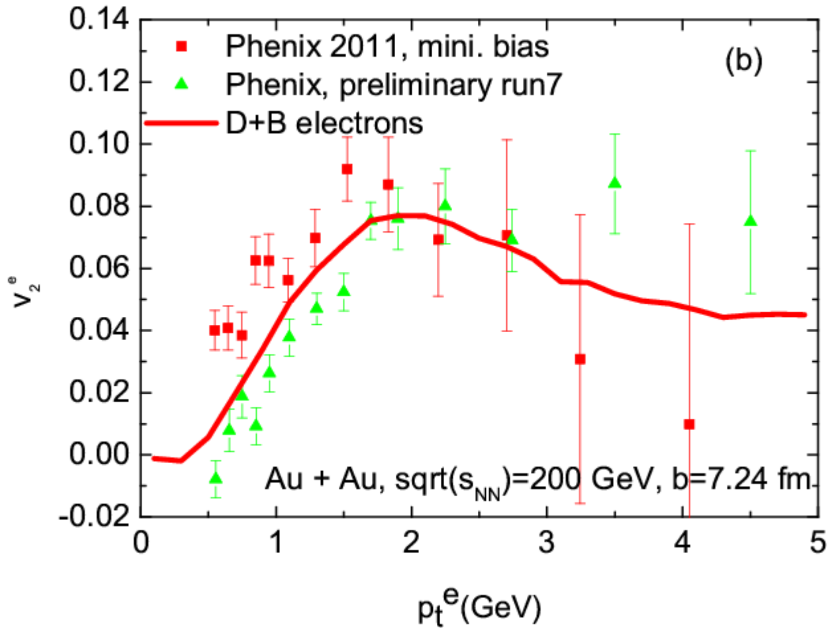
- Consistent with in-medium dynamics.
  - Low momenta = recombination dominated (co-moving thermal partons!)
  - High momenta = fragmentation dominated (no co-moving thermal partons)
- Total recombination probability averaged over fluid cells in lab frame:



Heavy Quarks: Conclusions

- D meson R_{AA} exhibits characteristic “flow bump”.
- Clearly seen in data: charm flows at RHIC+LHC.
- Elliptic flow a bit on the low side but consistent.
- Consistent with relaxation rates $A \sim 0.1 - 0.15$ 1/fm around T_c.





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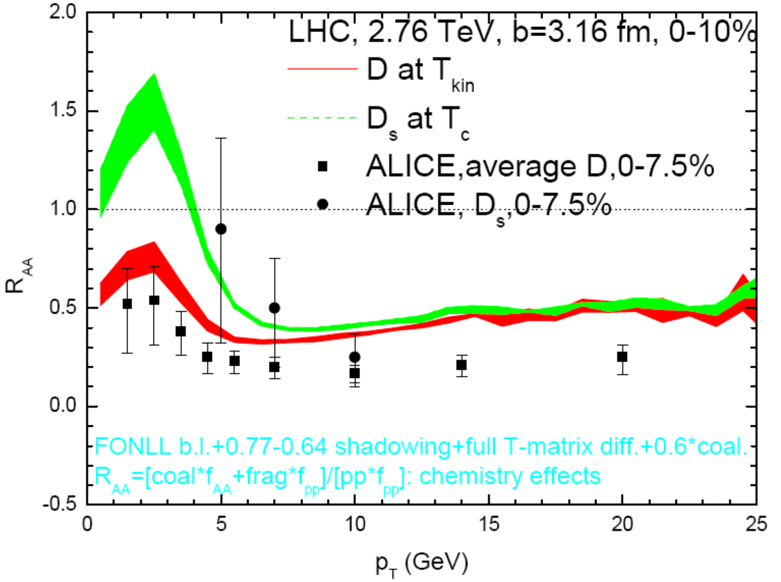


The D_s as a Signature

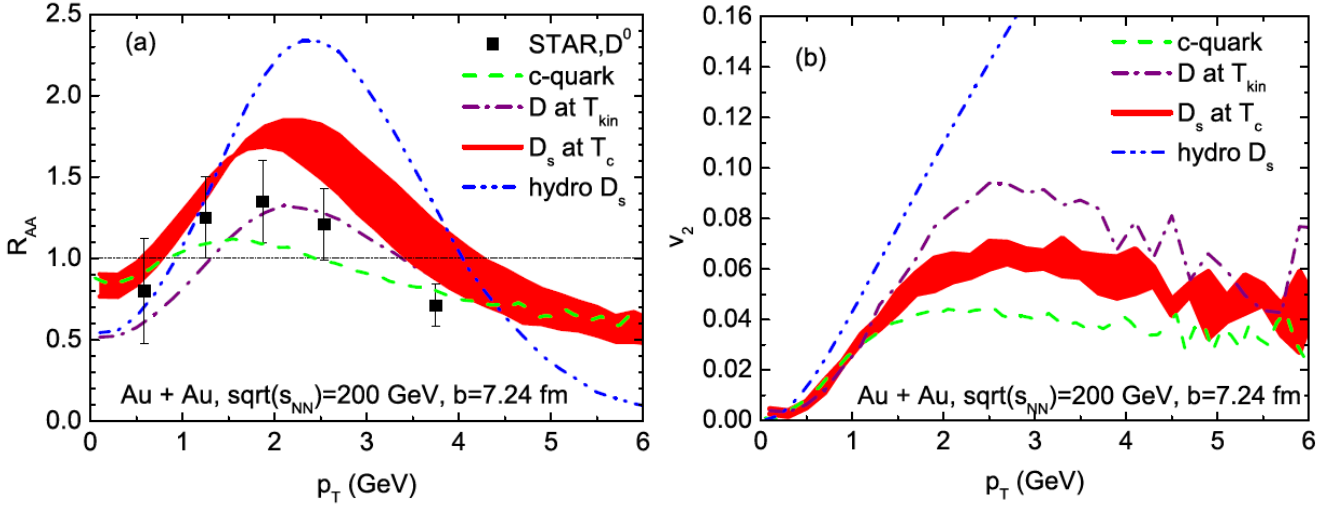
- D_s = charm-strange bound state.

- Signature 1: D_s vs D R_{AA} is a measure for strength of recombination vs fragmentation.
  - Charm in D_s and D suffer from same drag and diffusion up to T_c.
  - If charm fragments: D_s/D as in p+p.
  - If charm recombines: D_s picks up enhanced strangeness → D_s enhanced.
  - Numerical check: hadronic phase does not destroy this signal.

- Signature 2: D_s vs D v_2 can measure the relative strength of D_s vs D interactions in the hadronic phase.
  - D_s is an analogue to multi-strange hadrons in the light sector.
  - If there is early freeze-out it can be read of from the D_s vs D v_2.

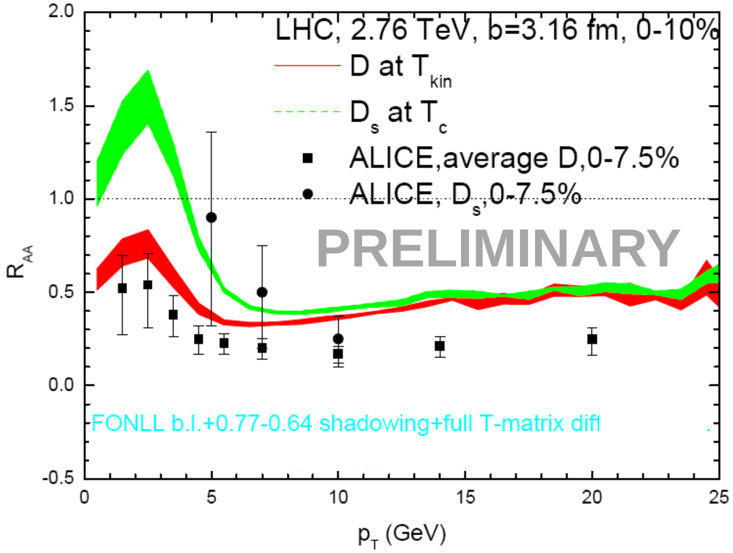
D_s Predictions

- RHIC:



- LHC: first data shown by ALICE at QM 2012
  - D_s enhancement seen.

- Importance of recombination for heavy quarks confirmed.



Global Event Dynamics from Gluon Fields

The following is based on work with G. Chen, and J. Kapusta.

[RJF, J. Kapusta, Y. Li, Nucl. Phys. A774, 861 (2006)]
[G. Chen, RJF, Phys. Lett. B 723, 417 (2013)]

Global Event Dynamics from Gluon Fields

- The initial stage of the collision before local thermalization is poorly understood.

- Color Glass Condensate (CGC) is an attractive candidate for a "$\tau = 0$" effective theory.

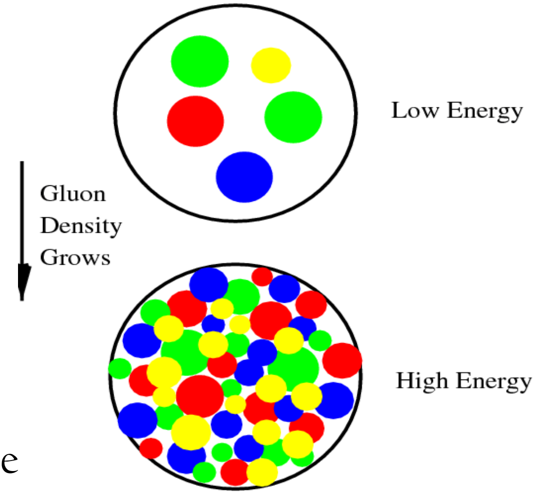
- Initial energy density including fluctuations: very successful

  [B. Schenke, P. Tribedy, R. Venugopalan, Phys. Rev. Lett. 108 (2012)]

- Initial flow: ??

Color Glass

- Nuclei/hadrons at asymptotically high energy:
  - ☐ Saturated gluon density ~ $Q_s^{-2} \rightarrow$ scale $Q_s \gg \Lambda_{QCD}$
  - ☐ Probes interact with many quarks + gluons coherently.
  - ☐ Large occupation numbers → quasi-classical fields.
  - ☐ Large nuclei are better: $Q_s \sim A^{1/3}$



- Effective Theory a la McLerran & Venugopalan
  - ☐ For intersecting light cone currents J_1, J_2 (given by SU(3) charge distributions ρ_1, ρ_2) solve Yang-Mills equations for gluon field $A^\mu(\rho_1, \rho_2)$.

$$\left[D_\mu, F^{\mu\nu}\right] = J_1^\nu + J_2^\nu$$

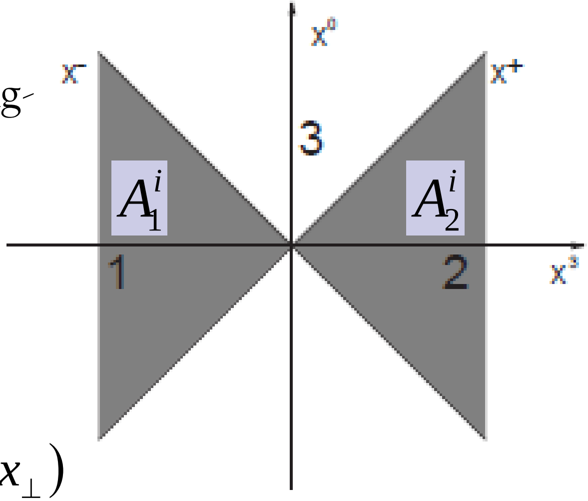
  - ☐ Calculate any observable $O(\rho_1, \rho_2)$ from the gluon field.
  - ☐ Compute expectation value of O by averaging over ρ_1, ρ_2. since those are arbitrary frozen fluctuations of a color-neutral object.

$$\langle O \rangle_\rho = \int [d\rho_1][d\rho_2] O(\rho_1, \rho_2) W(\rho_1, \rho_2)$$

[L. McLerran, R. Venugopalan]
[A. Kovner, L. McLerran, H. Weigert]
…

Colliding Nuclei

- Yang-Mills equations: two sources ρ_1, ρ_2
  - Intersecting light cone currents J_1, J_2 (given by ρ_1, ρ_2) solve Yang-Mills equations for gluon field $A^\mu(\rho_1, \rho_2)$.
- Forward light cone (3): free Yang-Mills equations for fields A, $A^i_\perp$



$$\frac{1}{\tau^3}\partial_\tau \tau^3 \partial_\tau A - \left[D^i,\left[D^i,A\right]\right] = 0$$

$$\frac{1}{\tau}\left[D^i,\partial_\tau A_\perp^i\right] - ig\tau\left[A,\partial_\tau A\right] = 0$$

$$\frac{1}{\tau}\partial_\tau \tau \partial_\tau A_\perp^i - ig\tau^2\left[A,\left[D^i,A\right]\right] - \left[D^j,F^{ji}\right] = 0$$

$$A^\pm = \pm x^\pm A(\tau, x_\perp)$$

$$A^i = A_\perp^i(\tau, x_\perp)$$

[A. Kovner, L. McLerran, H. Weigert]

- Boundary conditions on the forward light cone:

$$A_\perp^i(\tau = 0, x_\perp) = A_1^i(x_\perp) + A_2^i(x_\perp)$$

$$A(\tau = 0, x_\perp) = -\frac{ig}{2}\left[A_1^i(x_\perp), A_2^i(x_\perp)\right]$$

- MV setup is boost-invariant, but not symmetric between + and − direction.

Gluon Fields in The Forward Lightcone

- Goals:
  - Calculate fields and energy momentum tensor of *early time* gluon field as a function of space-time coordinates.
  - Analyze energy density and flow field.
  - Derive constraints for further hydrodynamic evolution of equilibrating QGP.
- Small-time expansion

$$A(\tau, x_\perp) = \sum_{n=0}^{\infty} \tau^n A_{(n)}(x_\perp)$$

$$A_\perp^i(\tau, x_\perp) = \sum_{n=0}^{\infty} \tau^n A_{\perp(n)}^i(x_\perp)$$

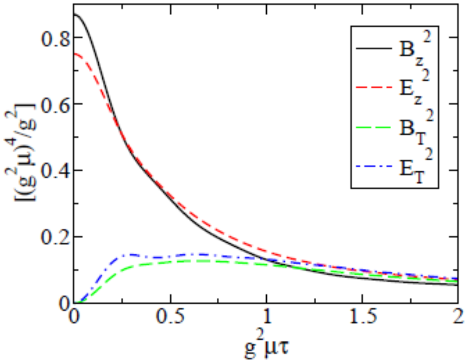
- Results: recursive solution for gluon field:

$$A_{(n)} = \frac{1}{n(n+2)} \sum_{k+l+m=n-2} \left[D_{(k)}^i, \left[D_{(l)}^i, A_{(m)}\right]\right]$$

$$A_{\perp(n)}^i = \frac{1}{n^2}\left(\sum_{k+l=n-2} \left[D_{(k)}^j, F_{(l)}^{ji}\right] + ig \sum_{k+l+m=n-4} \left[A_{(k)}, \left[D_{(l)}^i, A_{(m)}\right]\right]\right)$$

Numerical solution

[T. Lappi]



$$A_{\perp(0)}^i(x_\perp) = A_1^i(x_\perp) + A_2^i(x_\perp)$$

$$A_{(0)}(x_\perp) = -\frac{ig}{2}\left[A_1^i(x_\perp), A_2^i(x_\perp)\right]$$

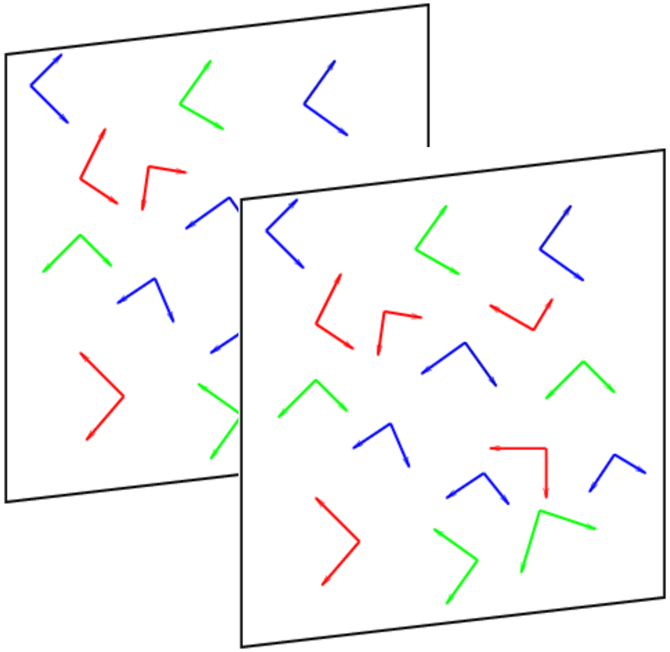
[RJF, J. Kapusta, Y. Li, 2006]

[Fujii, Fukushima, Hidaka, 2009]

Result: Fields

- Before the collision: color glass = pulse of strictly transverse (color) electric and magnetic fields, mutually orthogonal, with random color orientations, in each nucleus.

$$F_1^{i+} = \delta\left(x^-\right)A_1^i$$



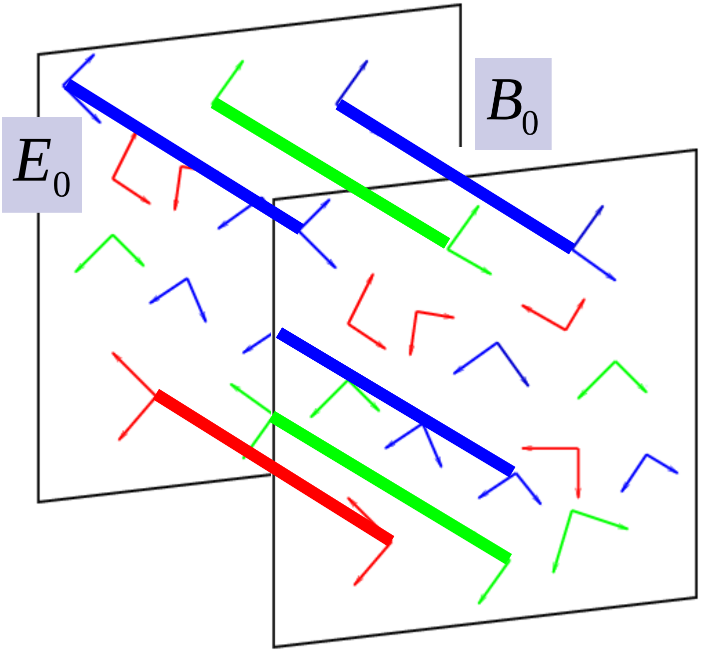
$$F_2^{i-} = \delta\left(x^+\right)A_2^i$$

Result: Fields

- Before the collision: color glass = pulse of strictly transverse (color) electric and magnetic fields, mutually orthogonal, with random color orientations, in each nucleus.
- Immediately after overlap (forward light cone, $\tau \to 0$): strong *longitudinal* electric & magnetic fields. Non-abelian effect!

$$F^{+-}_{(0)} = ig\left[A_1^i, A_2^i\right] \longleftarrow E_0$$

$$F^{21}_{(0)} = ig\varepsilon^{ij}\left[A_1^i, A_2^j\right] \longleftarrow B_0$$

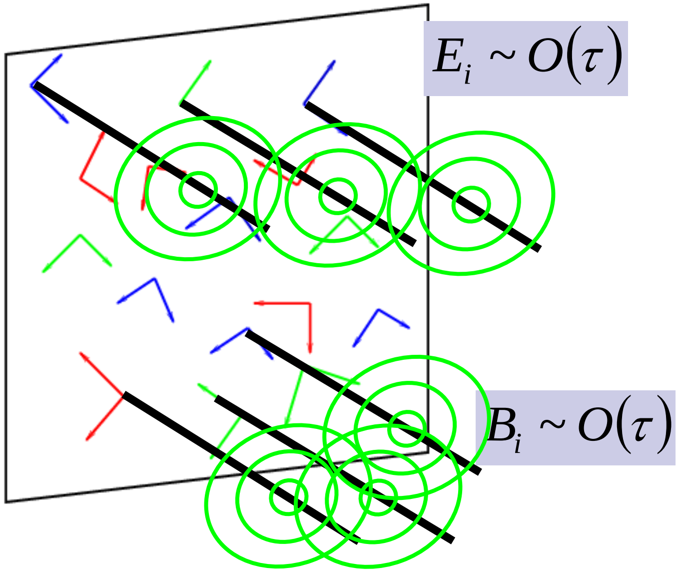


[L. McLerran, T. Lappi, 2006]
[RJF, J.I. Kapusta, Y. Li, 2006]

Result: Fields

- Before the collision: color glass = pulse of strictly transverse (color) electric and magnetic fields, mutually orthogonal, with random color orientations, in each nucleus.
- Immediately after overlap (forward light cone, $\tau \rightarrow 0$): strong *longitudinal* electric & magnetic fields. Non-abelian effect!
- *Transverse* E, B fields start linearly in time τ

$$F^{i\pm}_{(1)} = -\frac{e^{\pm\eta}}{2\sqrt{2}}\left(\varepsilon^{ij}\left[D^{j}_{(0)}, B_0\right] \pm \left[D^{i}_{(0)}, E_0\right]\right)$$



[RJF, J.I. Kapusta, Y. Li, 2006]
[G. Chen, RJF, 2013]

Energy-Momentum Tensor

- Initial ($\tau = 0$) structure of the energy-momentum tensor:

Gauge fields:

$$T_{\mathrm{f}}^{\mu\nu} = 2\,\mathrm{Tr}\left[F^{\mu\lambda}F_{\lambda}^{\nu}\right] + \frac{1}{2}g^{\mu\nu}\,\mathrm{Tr}\left[F^{\kappa\lambda}F_{\kappa\lambda}\right]$$

Particle distributions *f*:

$$T_{\mathrm{p}}^{\mu\nu} = \int \frac{d^3p}{p^0} p^\mu p^\nu f(\vec{p})$$

$$T_{\mathrm{f}(0)}^{\mu\nu} = \begin{pmatrix} \varepsilon_0 & & & \\ & \varepsilon_0 & & \\ & & \varepsilon_0 & \\ & & & -\varepsilon_0 \end{pmatrix} \rightarrow T_{\mathrm{equ}}^{\mu\nu} = \begin{pmatrix} e & & & \\ & p & & \\ & & p & \\ & & & p \end{pmatrix}$$

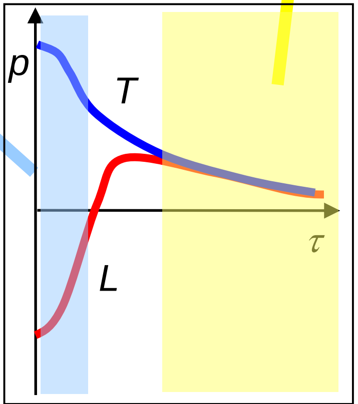
Transverse pressure
$P_T = \varepsilon_0$

Longitudinal pressure
$P_L = -\varepsilon_0$

Later: Ideal plasma
(local rest frame)

$$\varepsilon_0 = \frac{1}{2}\left(E_0^2 + B_0^2\right)$$

- Toward equilibrium: pressure isotropization
(in local rest frame)



Energy Momentum Tensor

- General structure up to order τ^2:

[RJF, J.I. Kapusta, Y. Li, (2006)]
[G. Chen, RJF (2013)]

$$\boxed{\frac{1}{2}\left(E^2+B^2\right)} \qquad \boxed{\vec{S}=\vec{E}\times\vec{B}}$$

$$T_{\mathrm{f}}^{\mu\nu}=\begin{pmatrix} \boxed{\varepsilon_0+O\left(\tau^2\right)} & \alpha^1\cosh\eta+\beta^1\sinh\eta & \alpha^2\cosh\eta+\beta^2\sinh\eta & O\left(\tau^2\right) \\ \alpha^1\cosh\eta+\beta^1\sinh\eta & \varepsilon_0+O\left(\tau^2\right) & O\left(\tau^2\right) & \alpha^1\sinh\eta+\beta^1\cosh\eta \\ \alpha^2\cosh\eta+\beta^2\sinh\eta & O\left(\tau^2\right) & \varepsilon_0+O\left(\tau^2\right) & \alpha^2\sinh\eta+\beta^2\cosh\eta \\ O\left(\tau^2\right) & \alpha^1\sinh\eta+\beta^1\cosh\eta & \alpha^2\sinh\eta+\beta^2\cosh\eta & -\varepsilon_0+O\left(\tau^2\right) \end{pmatrix}$$

- Transverse Poynting vector gives transverse flow.

$$\alpha^i=-\frac{\tau}{2}\nabla^i\varepsilon_0$$

Like hydrodynamic flow, determined by gradient of transverse pressure $P_T=\varepsilon_0$; even in rapidity.

$$\beta^i=\frac{\tau}{2}\varepsilon^{ij}\left(\left[D^j,B_0\right]E_0-\left[D^j,E_0\right]B_0\right)$$

Non-hydro like; odd in rapidity ??

- Example for second order:

Depletion/increase of energy density due to transverse flow

$$T^{00}=\varepsilon_0-\frac{\tau^2}{8}\left[2\nabla^i\alpha^i+\sinh 2\eta\,\nabla^i\beta^i+\left(2-\cosh 2\eta\right)\delta\right]$$

Due to longitudinal flow

Averaging

- Take expectation values.
- Energy density ~ product of nuclear gluon distributions ~ product of color source densities

$$\varepsilon_0 = \frac{g^6 N_c\left(N_c^2-1\right)}{8\pi}\mu_1\mu_2\ln^2\frac{Q^2}{m^2}$$

[T. Lappi, 2006]
[RJF, Kapusta, Li, 2006]
[Fujii, Fukushima, Hidaka, 2009]

- "Hydro" flow:

$$\alpha^i = -\tau\frac{g^6 N_c\left(N_c^2-1\right)}{64\pi^2}\nabla^i\left(\mu_1\mu_2\right)\ln^2\frac{Q^2}{m^2}$$

[G. Chen, RJF, 2013]
[G. Chen et al., in preparation]

- Odd flow term:

$$\beta^i = -\tau\frac{g^6 N_c\left(N_c^2-1\right)}{64\pi^2}\left(\mu_2\nabla^i\mu_1-\mu_1\nabla^i\mu_2\right)\ln^2\frac{Q^2}{m^2}$$

Transverse Field: Abelian Arguments

- Once the (non-abelian) longitudinal fields E_0, B_0 are seeded, the *averaged* transverse flow field is an abelian effect.
- Can be understood in terms of Ampere's, Faraday's and Gauss' Law.
  - ☐ Longitudinal fields E_0, B_0 decrease in both *z* and *t* away from the light cone
- Gauss at fixed time *t*:
  - ☐ Long. flux imbalance compensated by transverse flux
  - ☐ Gauss: **rapidity-odd radial** field
- Ampere/Faraday as function of *t*:
  - ☐ Decreasing long. flux induces transverse field
  - ☐ Ampere/Faraday: **rapidity-even curling** field
- Full classical QCD:

$$E^i = -\frac{\tau}{2}\left(\sinh\eta\left[D^i, E_0\right] + \cosh\eta\,\varepsilon^{ij}\left[D^j, B_0\right]\right)$$

$$B^i = \frac{\tau}{2}\left(\cosh\eta\,\varepsilon^{ij}\left[D^j, E_0\right] - \sinh\eta\left[D^i, B_0\right]\right)$$

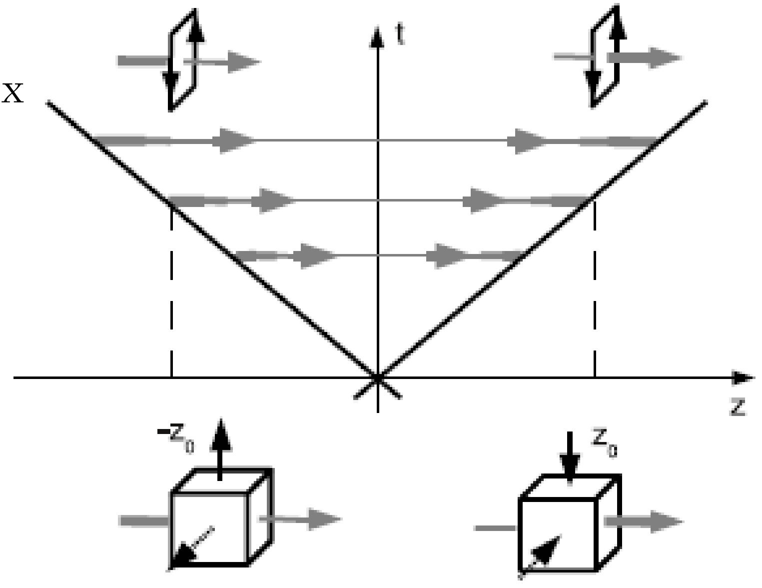
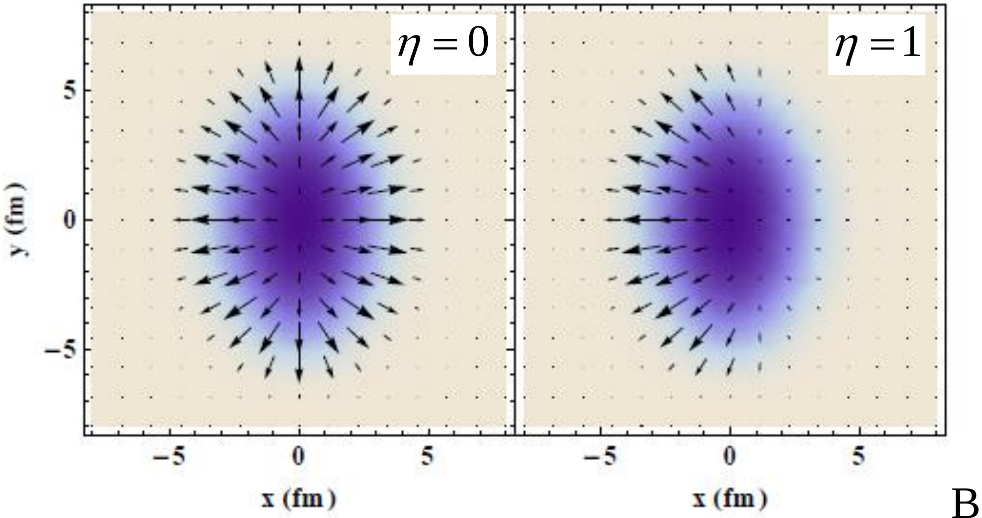


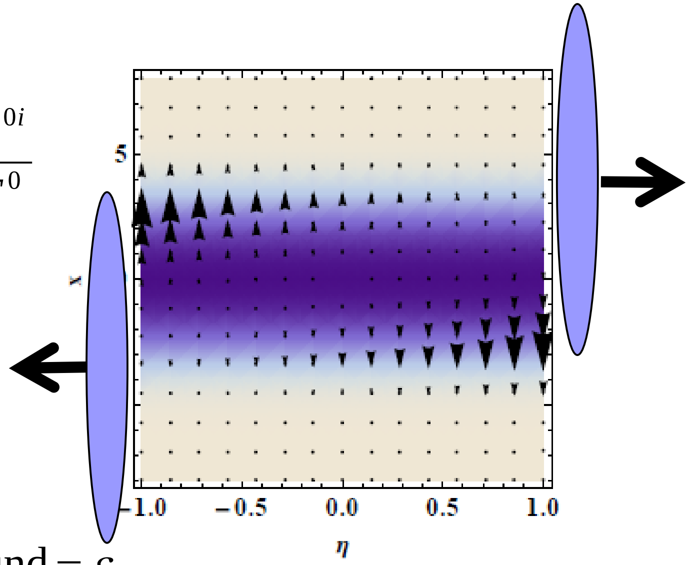
Figure 1: Two observers at $z = z_0$ and $z = -z_0$ test Ampère's and Faraday's Laws with areas a^2 in the transverse plane and Gauss' Law with a cube of volume a^3. The transverse fields from Ampère's and Faraday's Laws (black solid arrows) are the same in both cases, while the transverse fields from Gauss' Law (black dashed arrows) are observed with opposite signs. Initial longitudinal fields are indicated by solid grey arrows, thickness reflects field strength.

Phenomenology: $b \neq 0$

- Odd flow needs an asymmetry: e.g. finite impact parameter
- Flow field for Au+Au collision, b = 4 fm.

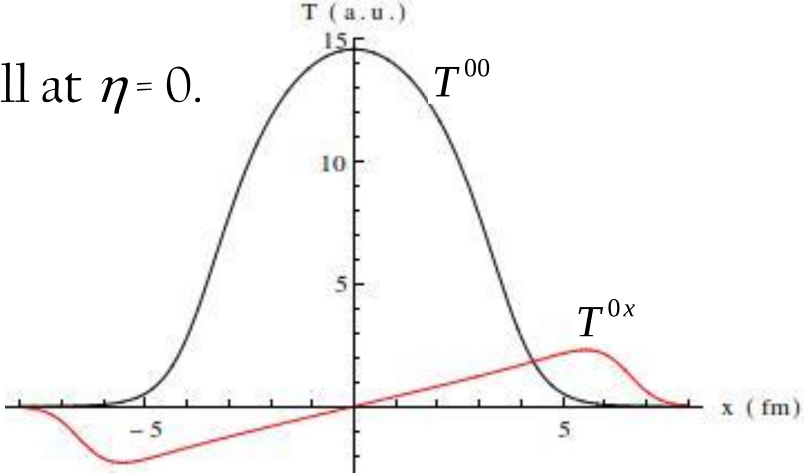


$$V^i = \frac{T^{0i}}{T^0}$$



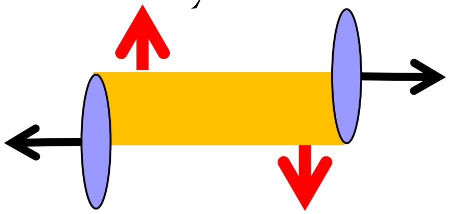
Background = ε_0

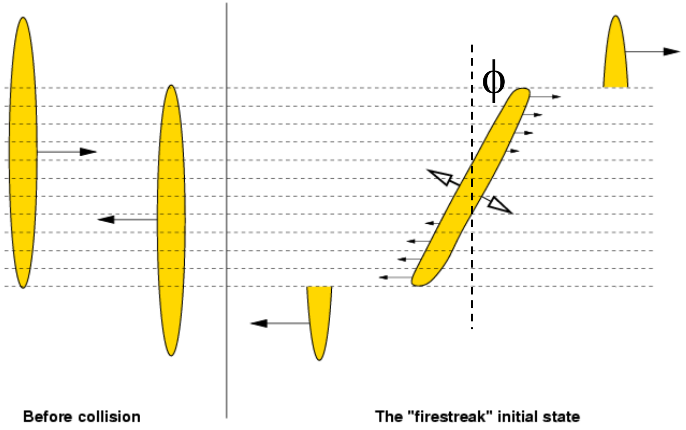
- Radial flow following gradients in the fireball at η = 0.
- Clearly: directed flow away from η = 0.
- Fireball tilted, angular momentum.
- Careful: time τ ~ 0.1-0.2 fm/c



Phenomenology: $b \neq 0$

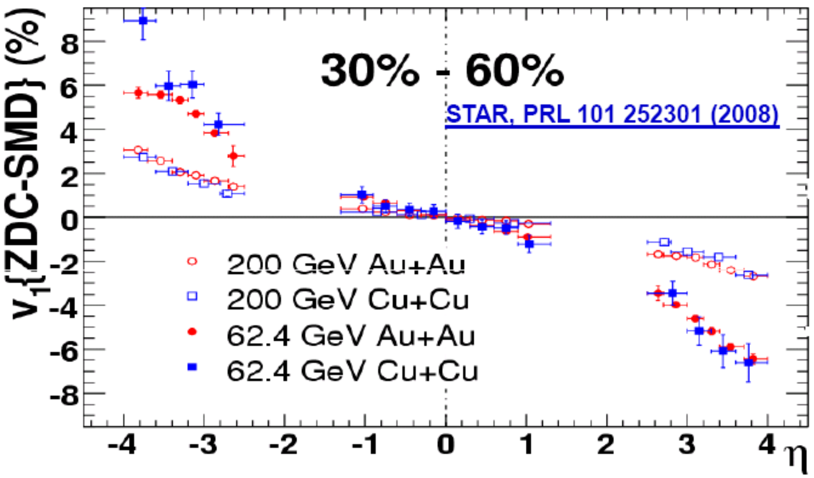
- Angular momentum is natural: some old models have it, most modern hydro calculations don't.
  - Do we underestimate flow by factors of cos ϕ?



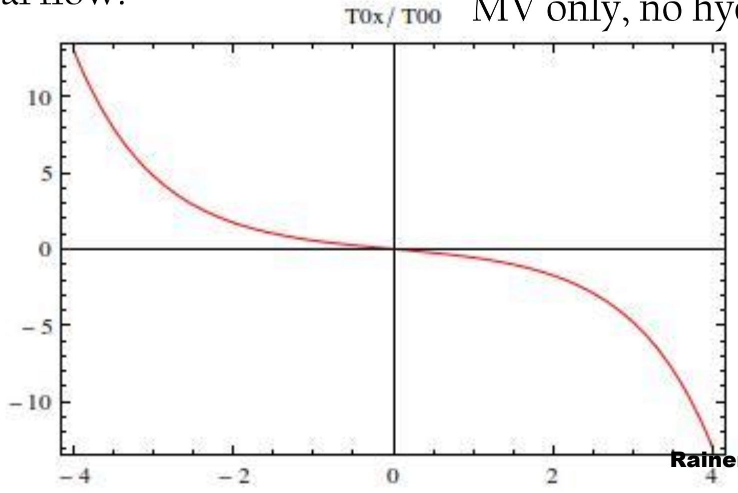


[Gosset, Kapusta, Westfall (1978)]

- Note that boost-invariance is not broken.
- Directed flow v_1:
  - Hydro needs tilted initial conditions or initial flow.



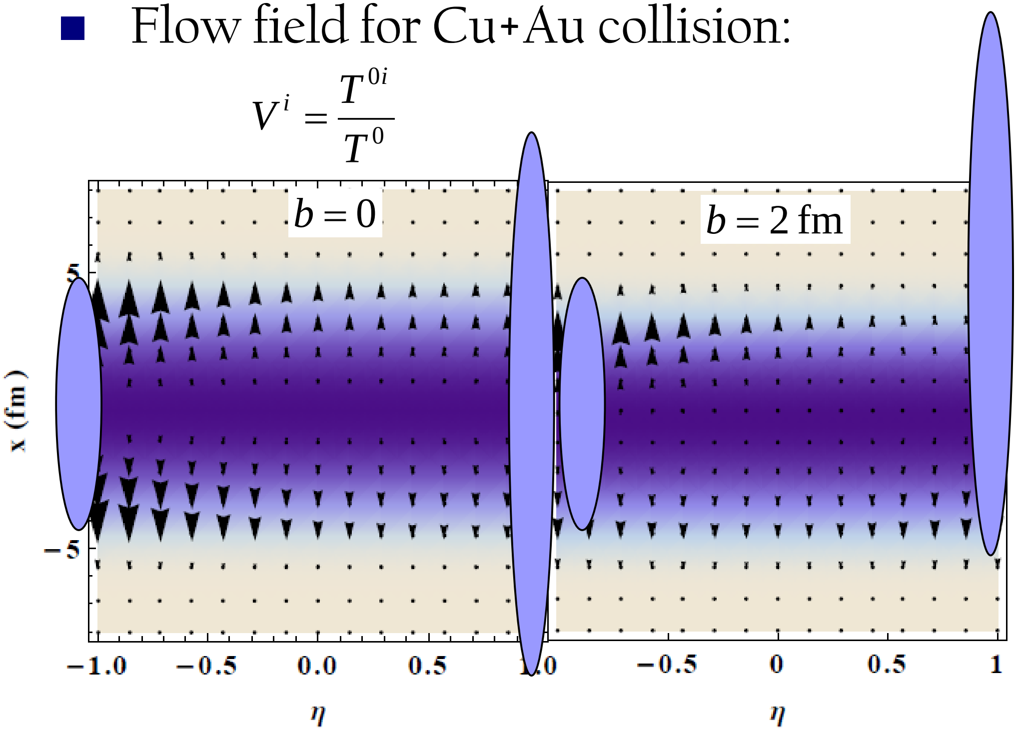
MV only, no hydro



Phenomenology: A ≠ B

- Odd flow needs an asymmetry: e.g. asymmetric system
- Flow field for Cu+Au collision:

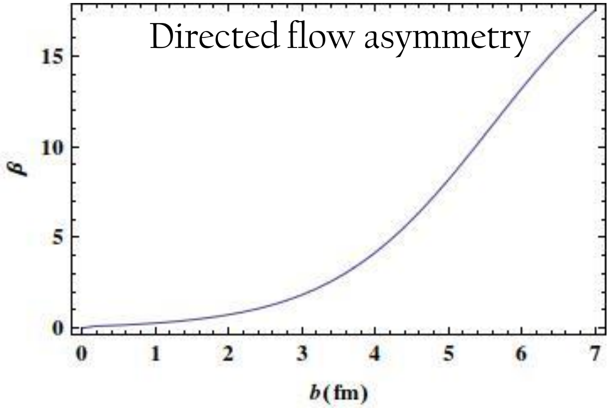
$$V^i = \frac{T^{0i}}{T^0}$$

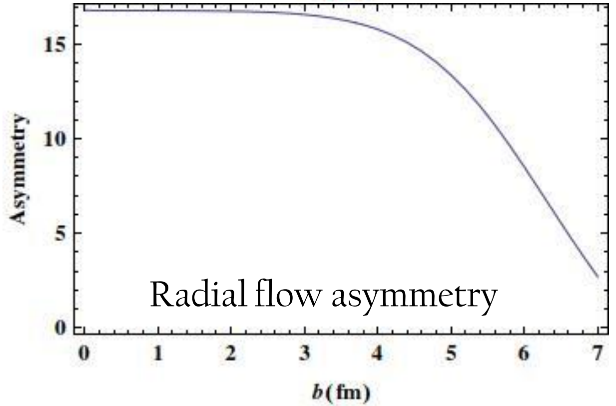


- Odd flow increases expansion in the wake of the larger nucleus, suppresses flow on the other side.
- Should lead to very characteristic flow patterns in asymmetric systems.

Example: Forward-backward asymmetries. Here: p+Pb

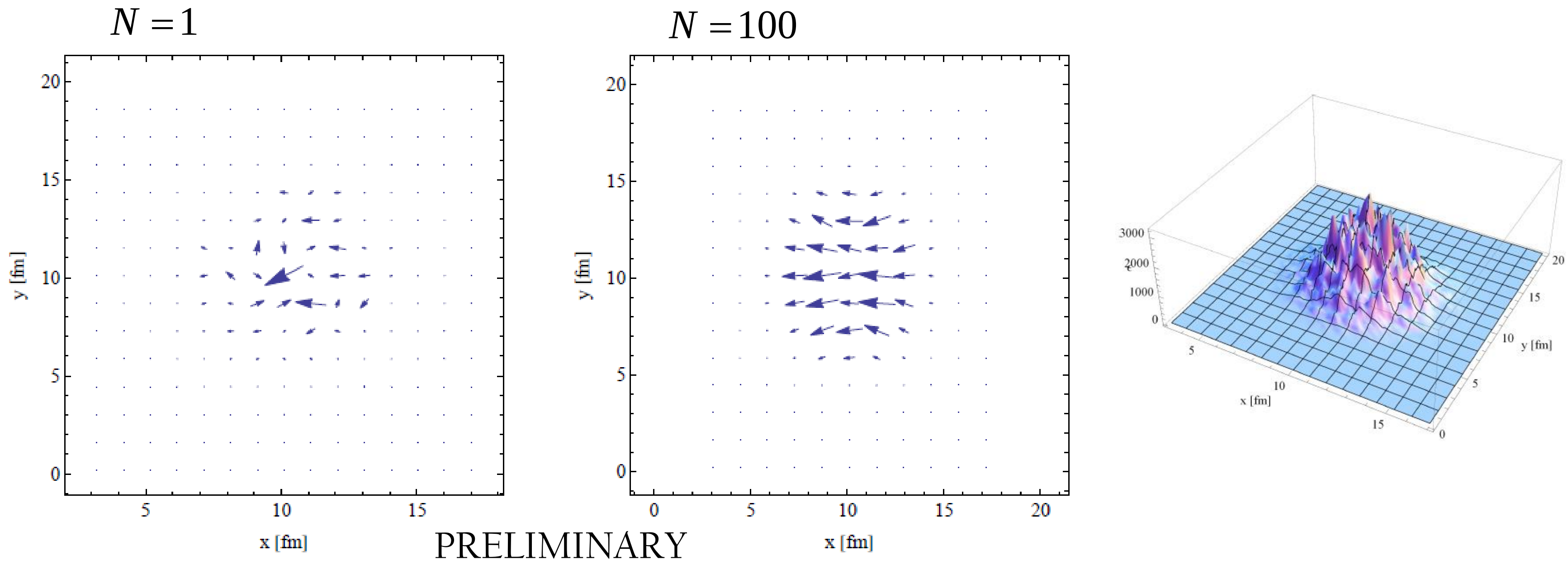
$$\frac{\langle T^{0x}\rangle}{\langle T^{00}\rangle}(\eta = -2) - \frac{\langle T^{0x}\rangle}{\langle T^{00}\rangle}(\eta = +2)$$





Event-By-Event Picture

- Numerical simulation of β in Au+Au, sampling charge distributions in the nuclei.



$N = 1$

$N = 100$

PRELIMINARY

- Individual events dominated by fluctuations.
- Averaging N › 100 events: recover directed flow.

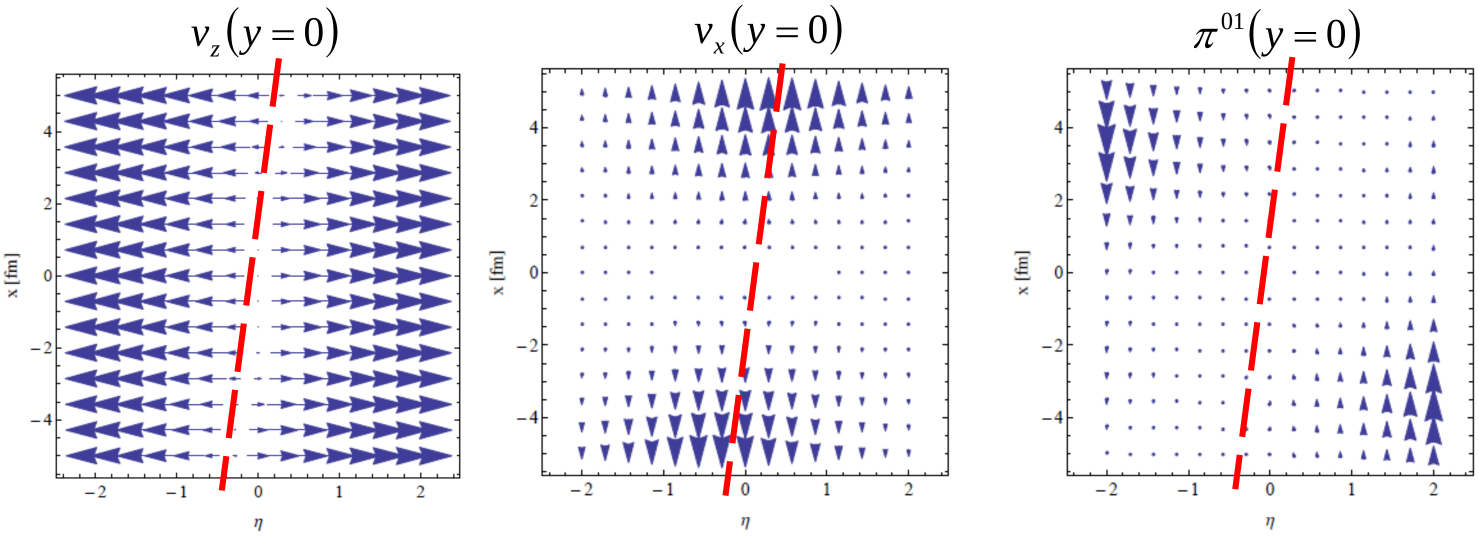
Matching to Hydrodynamics

- Instantaneous matching to viscous hydrodynamics using in addition

$$\partial_\mu M^{\mu\nu\lambda} = 0 \qquad M^{\mu\nu\lambda} = x^\mu T^{\nu\lambda} - x^\nu T^{\mu\lambda}$$

$$T^{\mu\nu}_{viscous} = (e + p + \Pi)u^\mu u^\nu - (p + \Pi)g^{\mu\nu} + \pi^{\mu\nu}$$

  - Mathematically equivalent to imposing smoothness condition on all components of $T_{\mu\nu}$.
- Numerical solution of the matching:



$v_z(y=0)$ $\quad$ $v_x(y=0)$ $\quad$ $\pi^{01}(y=0)$

- Tilting and odd flow terms translate into hydrodynamics fields.

Effect on Particle Spectra

- Need to run viscous 3+1-D hydro with large viscous corrections.
- Viscous freeze-out.

- Work in progress.

Summary

- The QCD phase transition has been established in nuclear collisions at RHIC and LHC. Matter at high *T* is
  - partonic
  - rapidly thermalizing
  - very opaque to colored probes
  - flowing with small η/s, little dissipation
- Near future: try to extract more quantitative properties of QGP. E.g. *T*-dependence of transport coefficients
- Low energy program: back to larger μ_B; critical point?
- Heavy quark program: study particles at the verge of thermalization. Heavy quark recombination is important.
- Global event dynamics from classical gluon fields: a promising attempt to describe the pre-equilibrium phase.