THE EXCESS AREA BETWEEN IMAGES OF DOMAINS UNDER ANALYTIC MAPPINGS

Bergman space on the unit disk, $L_a^2(\mathbb{D})$, consists of complex analytic functions on the unit disk, $\mathbb{D}$ such that their square integral, $\int_{\mathbb{D}} |h|^2 dxdy$, is finite. Fock space, $\mathcal{F}^2(\mathbb{C})$, consists of complex analytic functions on $\mathbb{C}$ such that their weighted square integral with respect to the weight $e^{-|z|^2}$, $\int_{\mathbb{C}} |h|^2 e^{-|z|^2} dxdy$, is finite, [Zhu12]. Instead of $\mathbb{C}$, in $\mathbb{C}^n$, $D_j = \frac{\partial}{\partial z_j}$ is an unbounded linear operator on $\mathcal{F}^2(\mathbb{C}^n)$, known as annihilator operator, and M_k is the multiplication operator by z_k, known as creation operator. In fact, $D_j^* = M_j$ and $[D_j, M_k] = D_j M_k - M_k D_j = \delta_{jk}$. In quantum mechanics, position is defined by $A_j = \frac{D_j + M_j}{2}$ and momentum by $B_j = \frac{D_j - M_j}{2i}$ on $\mathcal{F}^2(\mathbb{C}^n)$, [D'A19]. If one considers operators on a Hilbert space as generalized complex numbers, then the adjoint of an operator T notated as T^* plays the role of the complex conjugate of a complex number.

In this REU project, students will work with versions of operators M and D on $L_a^2(\Omega)$, where Ω is a domain in the complex plane. When h and Dh are in $L_a^2(\Omega)$, that is, when h is in $W_a^{1,2}(\Omega)$, the squared L^2 norm, $\|Dh\|_{L^2(\Omega)}^2 = \int_\Omega |\partial h/\partial z|^2 dxdy$ equals the area of the image of h, notated as $\text{Area}(h(\Omega))$ [GK06, D'A19], with multiplicity counted [Ahl78]. The operator $M^* D^* D M - D^* D$ has been explicitly calculated on $W_a^{1,2}(\mathbb{D})$, with a geometric interpretation, as $\pi S^* S$, where S is the restriction of h to the circle. That is, $\text{Area}(Mh(\mathbb{D})) - \text{Area}(h(\mathbb{D}))$, is calculated as the average value of the module-square of h on the unit circle times π, [D'A19, Section 2 in Chapter 4]. In the case, where the domain is the unit disk, D and M are not adjoints. The project aims to explore the operator $M^* D^* D M - D^* D$ from different perspectives, such as a functional space of h and a domain on which the space is defined, [Zhu07]. The participants will also utilize computer algebra systems to visualize complicated mathematical concepts, generate conjectures, and collaborate to develop proofs or counterexamples for their conjectures. The participants will organize their results and present them at different venues in various formats, such as posters, research talks, and research articles. For examples of previous explorations of the operator, refer to [BÇGH22, ÇDTRRS23].

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