Equipment for Engineering Education

Experiment Instructions

WP950 Deformation of Straight Beams

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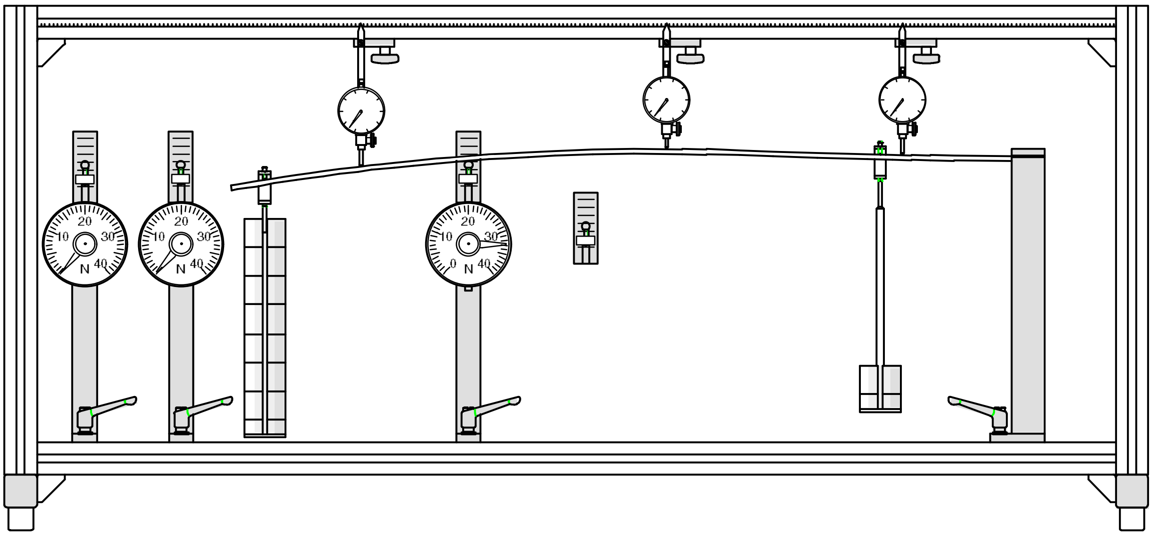
WP950 Deformation of Straight Beams



Experiment Instructions

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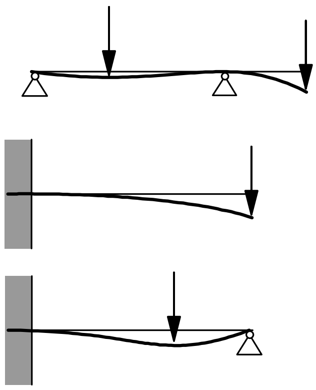
1 Introduction



The WP 950 device for deformation of straight beams permits a broad spectrum of experiments on the deformation of a bending bar.

The experiments include

- Elastic line under different support conditions
- Elastic line under different loads
- Demonstration of the *Maxwell-Betti* law
- Supporting forces in statically undetermined systems

The loads are applied in a visual manner using sets of weights.

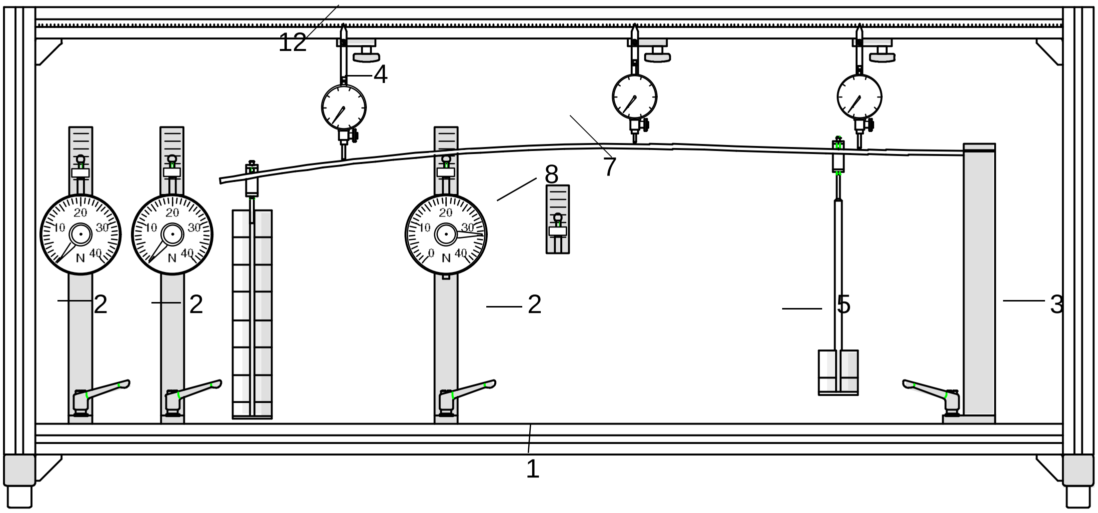
Deformation of the bar is measured using dial gauges.

Supporting forces can be determined via the dynamometers integrated in the supports.

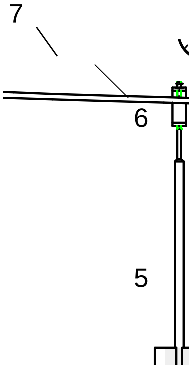
Bars of various materials are available in order to demonstrate the influence of the modulus of elasticity on deflection.

By using very thin, elastic bars, deformation of the bar under load can be seen very clearly, even without dial gauges. Accordingly, the device is not only suitable for practical experiments, but is also ideal for demonstration in the classroom. The dynamometers have large, clear scales and can easily be read from some distance away.

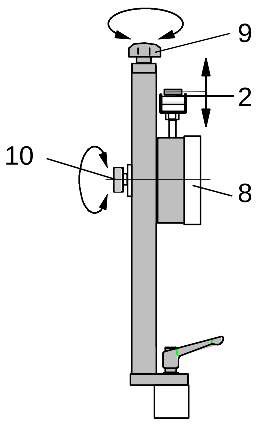
2 Unit description



The bar bending device consists of a **light, stable frame** (1) made of aluminium. The various supports (2,3) are fastened to the lower girder with clamping levers. The dial gauges (4) are fastened to the upper girder with holders.

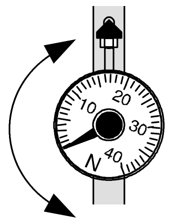


The **load weights** (5) are attached to the bar (7) via movable riders (6). The riders can be locked in position. The load can be adjusted in increments of 25 N using weight blocks.

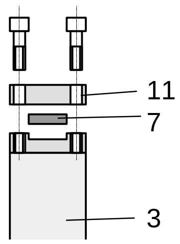


The **articulated supports** (2) are fitted with **dynamometers** (8). The height of the support can be adjusted using a threaded spindle (9). The support can be locked in position by the screw (10). This compensates deformation of the bar by its own weight or deflection of the support caused by spring excursion of the dynamometer.

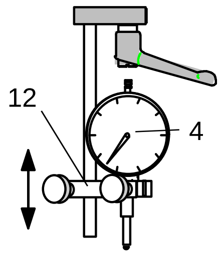
In statically undetermined systems, it is possible to demonstrate the influence of support deflection on load distribution.



The scales on the dynamometers (8) rotate to enable taring.



The bar (7) is fixed in the **support with clamp** (3) by means of a clamping plate (11).



The height of the **dial gauges** (4) can be adjusted on their holders (12).

3 Experiments

Below are descriptions of some of the experiments which can be performed with the WP950. They represent only a small proportion of the experiments which are possible with the device and should provide ideas for other experiments.

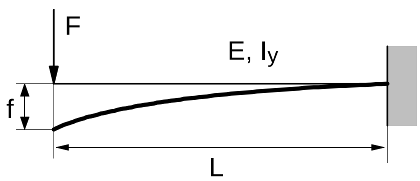
3.1 Bending on the cantilever bar

3.1.1 Deflection at the point of application of force

The aim of this experiment is to check the mathematically determined deflection of the cantilever bar.

In a cantilever bar, one side of the bar is fixed and the other side free. This is known as a tri-valent support which transmits normal force, transverse force and moment. The bar is therefore supported in a statically determined manner.

The equation for the **deflection** f of the bar at the point of application of force is



Cantilever beam

$$f = \frac{F\,L^3}{3\,E\,I_y}$$

Deflection is proportional to the load F and inversely proportional to the modulus of elasticity E and **planar moment of intertia** (PMI) I_y. The length of the bar is cubed.

The influence of the lenght L should be demonstated in this experiment.

For this purpose, the force should be constant.

The experimental bar is made of steel (modulus of elasticity E = 210000 N/mm^2) and has a cross-section of 20 x 6 mm^2. This produces a PMI of

$$I_y = \frac{b\,h^3}{12} = \frac{6 \cdot 20^3}{12} = 360\ mm^4$$

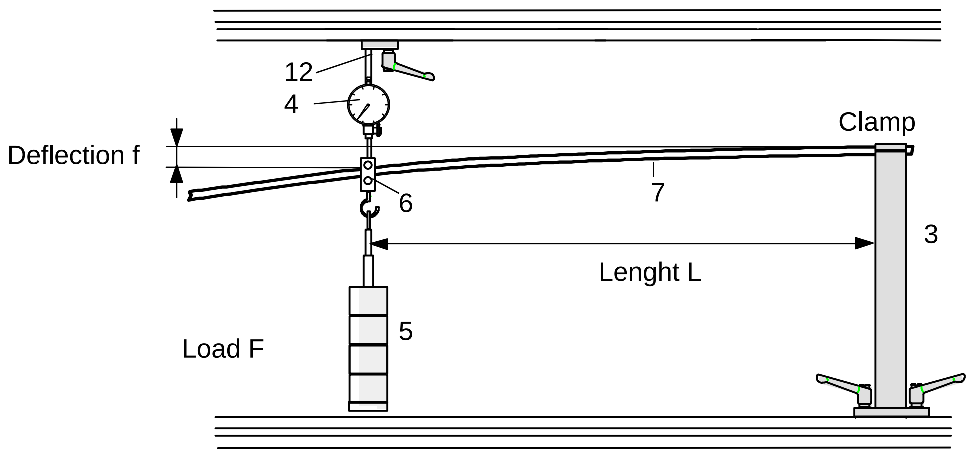
With these values and a load of 17.5 N (suspender 2.5N + 3 weights 5N), the following deflection values are achieved:

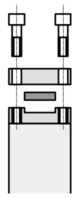
Deflection of the cantilever bar according to length	
Lenght L in mm	Deflection f in mm
300	2.08
400	4.94
500	9.64

3.1.2 Performing the experiment

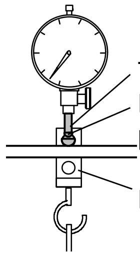
The experiment is set up as shown in the diagram. The following equipment is required:

- Steel bar 6 x 20 x 1000 mm (7)
- Rider for weight (6)
- Suspender for weights (5)
- 3 weights 5N
- Dial gauge with holder (4, 12)
- Support pillar with clamp (3)

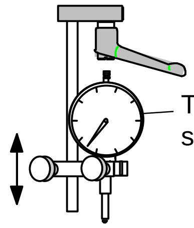




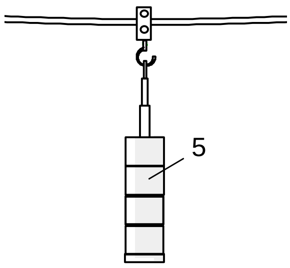
- Fasten the support pillar to the frame
- Clamp the bar in the support pillar



- Place the rider on the bar and lock in the required position
- Fasten the dial gauge to the frame with the holder in such a way that the tracer pin is touching the flattened part of the rider bolt



- Set the dial gauge to zero with the bar unloaded. To do so, adjust the holder and rotate the scale for precise adjustment



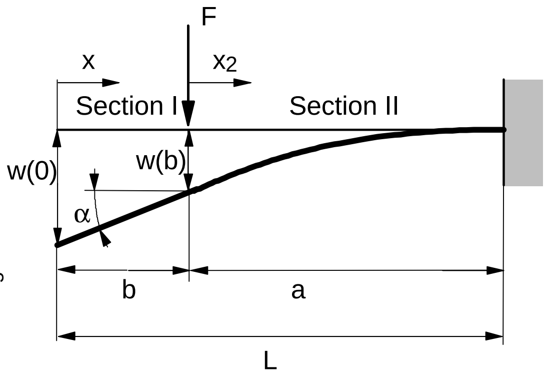
- Suspend the load weight, read the deflection on the dial gauge and record

The following table compares the results of the experiment with the results of the mathematical calculation.

Comparision between measured and calculated deflections		
Lenght L in mm	Measured deflection in mm	Calculated deflection in mm
300	2.45	2.08
400	5.4	4.94
500	10.2	9.64

The consistency can be described as good. The measured deflection values are all slightly too high. It is possible that the modulus of elasticity of the material itself does not match the one assumed for the calculation.

3.1.3 Determining the elastic line



Elastic line of a cantilever bar

This experiment measures the elastic line of a cantilever bar and compares it with the result of the mathematical calculation.

The **equation for the elastic line** of a cantilever bar loaded with a single force is as follows for the loaded section II with $0 \le x_2 \le a$

$$w(x_2) = \frac{F\,a^3}{6\,E\,I_y}\left[\,2 - 3\,\frac{x_2}{a} + \frac{x_2^3}{a^3}\right].$$

In the unloaded section I between the point of application of the force and the free end, the deflection is a linear function of the length and the inclination α in the point of application of force. This is not bending, but slanting

$$w(x) = w(b) + (b - x)\,\alpha\,.$$

Where

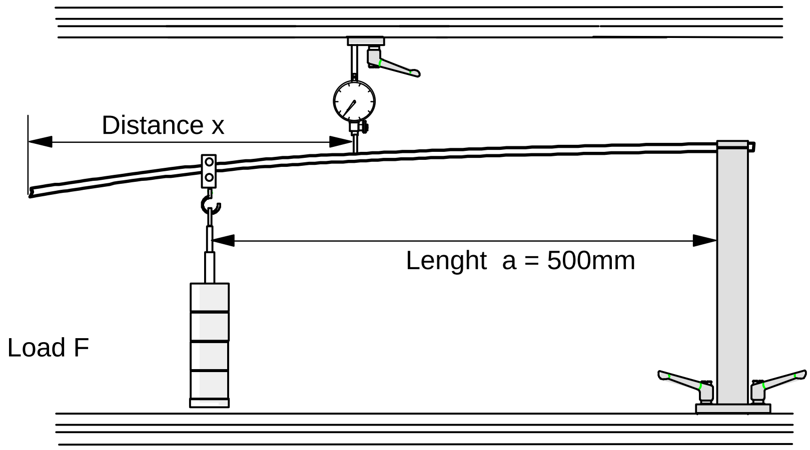
$$w(b) = \frac{F\,a^3}{3\,E\,I_y}\ \text{ und }\ \alpha = \frac{F\,a^2}{2\,E\,I_y}\,.$$

For a load of F = 17.5 N where a = 500 mm, the following deflection values are achieved:

Elastic line of a cantilever bar	
x in mm	Deflection f in mm
0	18.33
100	15.43
200	12.74
300	9.65
400	6.79
500	4.17
600	2.00
700	0.54
800	0

3.1.4 Performing the experiment

The experiment is set up as described in 3.1.2.



The load remains constant and is applied at a= 500 mm.

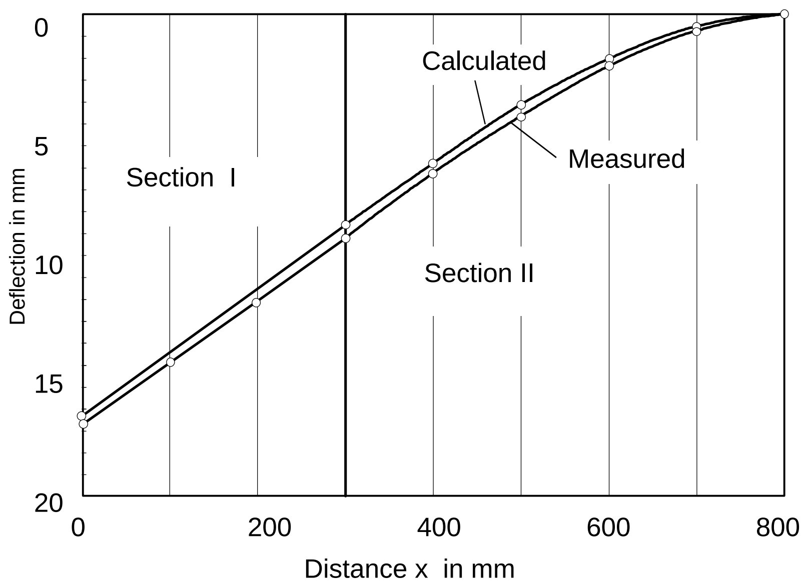
The deflection of the bar is measured at intervals of 100 mm with the dial gauge

- Relieve the bar
- Apply the dial gauge at the required position and set to zero
- Load the bar
- Read the deflection value and record
- Relieve the bar and move the dial gauge to the next position

Repeat the measuring procedure

Elastic line of cantilever bar		
x in mm	Measured deflection in mm	Calculated deflection in mm
0	18.7	18.33
100	15.9	15.43
200	13.0	12.74
300	10.2	9.65
400	7.3	6.79
500	4.65	4.17
600	2.3	2.00
700	0.8	0.54

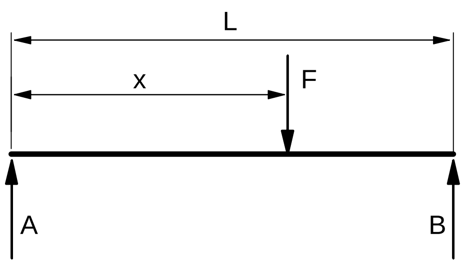
The values determined can be recorded in a graph as an elastic line. The calculation and the measurement are very consistent. The linear pattern is clearly recognisable in section I.



Elastic line of a cantilever bar

3.2 Bar on two supports

3.2.1 Measurement of the supporting forces



Supporting forces on the bar

The articulated supports are fitted with dynamometers to measure the **supporting forces**. This experiment determines the supporting forces for a bar depending on the point of application of the load x. The supporting forces A and B can be determined via balances of moments.

Balance of moments around support B

$$\Sigma M_B = 0 = F\,(L - x) - A\,L$$

Supporting force A

$$A = F\,\left(1 - \frac{x}{L}\right)$$

Balance of moments around support A

$$\Sigma M_A = 0 = B\,L - F\,x$$

Supporting force B

$$B = F\,\frac{x}{L}.$$

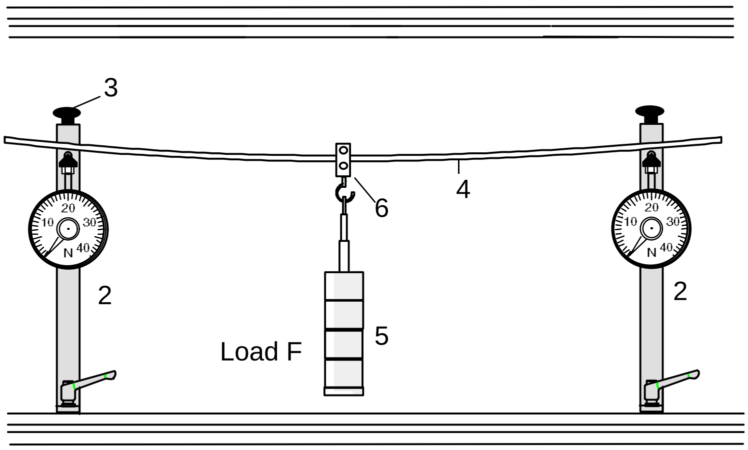
A bar with a length L=1000 mm and a load of F=20 N produces the following supporting forces A and B. The sum of the supporting forces must correspond to the load.

Supporting forces at a load of F = 20 N (values for one half only; the other half is symmetrical)		
Distance x from support A in mm	Supporting force A in N	Supporting force B in N
0	20	0
100	18	2
200	16	4
300	14	6
400	12	8
500 (Centre)	10	10

3.2.2 Performing the experiment

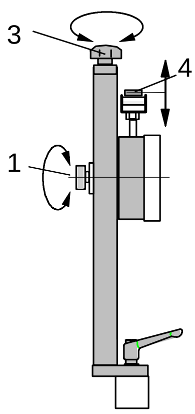
The experiment is set up as shown in the diagram. The following equipment is required:

- Steel bar 6 x 20 x 1000 mm (4)
- Rider for weight (6)
- Suspender for weights (5)
- 3 weights 5N, 1 weight 2.5 N
- 2 articulated supports (2) with dynamometer (7)

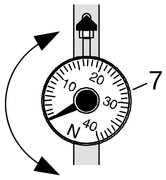


The load of 20 N is applied in the centre at x= 500 mm.

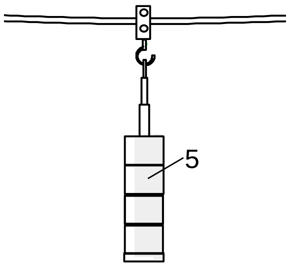
- Fasten the articulated supports (2) at a distance of 1000 mm
- Push the rider (6) for the weight suspender onto the bar (4) and place the bar on the supports



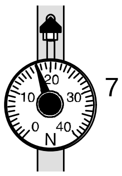
- Loosen the locking screw (1) on the support (2). Adjust the **height of the support** using the rotary knob (3) until the bar (4) is horizontal. Re-secure the support using the locking screw (1)



- Set the scale on the dynamometer (7) to zero by twisting



- Suspend the weight (5) and load the bar



- Read the supporting forces on the dynamometers (7) and record

The measured supporting forces are very consistent with the calculated values.

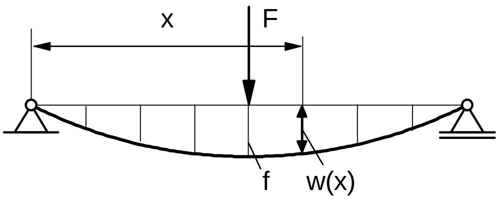
Measured suporting forces at a load of F = 20 N (values for one half only; the other half is symmetrical)		
Distance x from support A in mm	Support force A in N	Support force B in N
0	20	0
100	18.3	1.7
200	16	4
300	14.3	5.7
400	12	8
500 (Centre)	10	10

3.2.3 Elastic line for centre loading

This experiment measures the elastic line of a bar on two supports and compares it with the mathematically calculated result.

The **equation for the elastic line** of a bar loaded in the centre with a single force is as follows for the section between the left-hand support and the load with $0 \le x \le L/2$

$$w(x) = \frac{F\,L^3}{48\,E\,I_y}\left[3\,\frac{x}{L} - 4\,\frac{x^3}{L^3}\right].$$



Bar on two supports

The section between the load and the right-hand support is symmetrical to this.

The maximum deflection is at the centre of the bar where $x = L/2$ directly beneath the load

$$f = \frac{F\,L^3}{48\,E\,I_y}\,.$$

The following deflection values are achieved for a 1000 mm long bar with a load of F = 20 N at x = 500 mm:

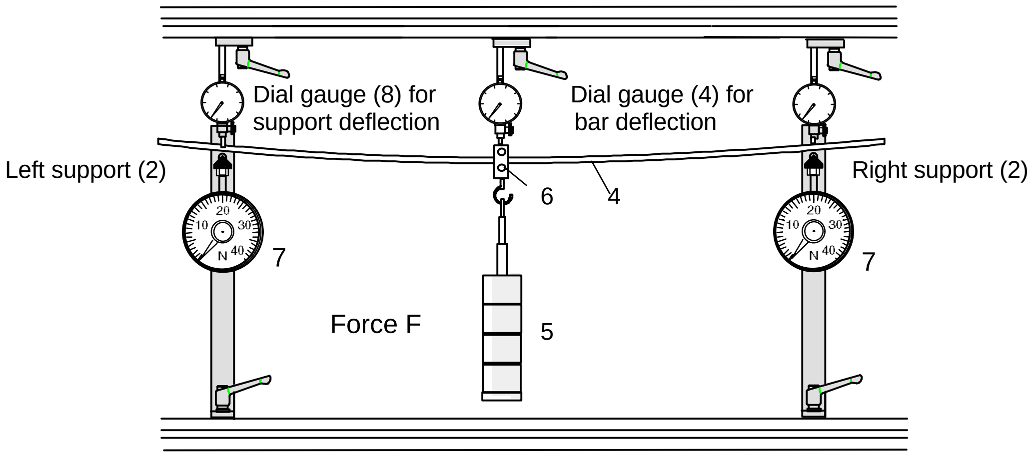
Elastic line of a bar on two supports (only one half; the other half is symmetrical)	
x in mm	Deflection w in mm
0	0
100	1.63
200	3.13
300	4.36
400	5.20
500 (Centre of bar)	5.51 (max. deflection)

3.2.4 Performing the experiment

The experiment is set up as shown in the diagram. The following equipment is required:

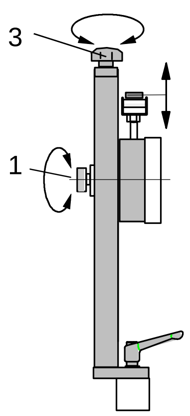
- Steel bar 6 x 20 x 1000 mm (4)
- Rider for weight (6)
- Suspender for weights (5)
- 3 weights 5N, 1 weight 2.5 N
- 2 articulated supports (2) with dynamometer (7)
- 3 dial gauges with holder (4,8)

The load remains constant and is applied in the centre at x= 500 mm.

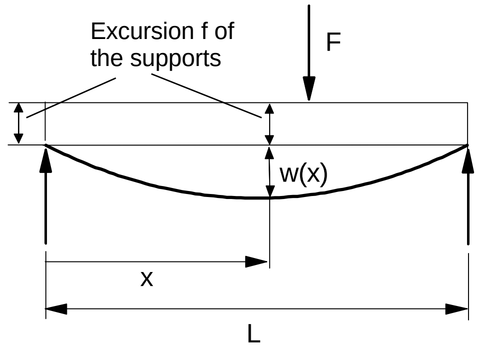


The deflection of the bar is measured with the dial gauge (4) at intervals of 100 mm.

Two dial gauges (8) on the supports (2) measure the deflection due to the dynamometer (7)



- Relieve the bar
- Loosen the locking screw (1) on the support. Adjust the height of the support using the rotary knob (3) until the dial gauges (8) read zero.
- Fasten the supports using the locking screw (1)
- Place the dial gauge (4) in the required position and set to zero
- Load the bar



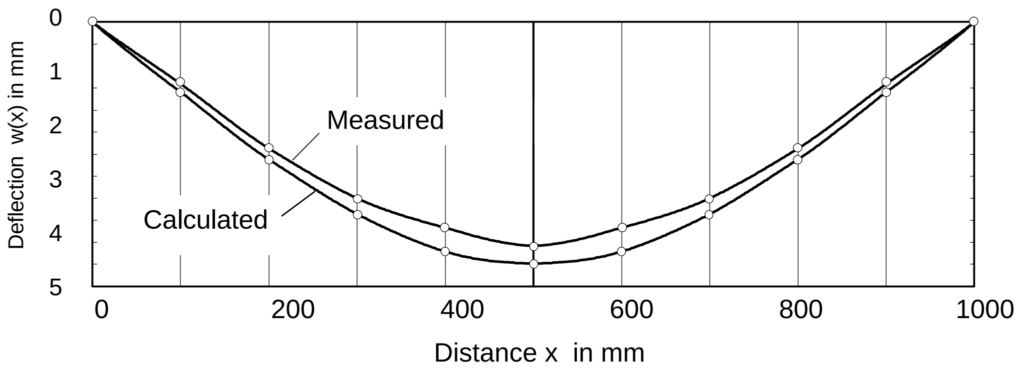
Measurement of the elastic line

The dynamometers experience spring excursion under load. In order to prevent measurement errors as a result of this additional deflection f, the supports should be returned to their original position.

- Loosen the locking screw (1) on the support. Raise the support using the rotary knob (3) until the dial gauges (8) read zero.
  Fasten the support using the locking screw (1)
- Read the deflection value from the dial gauge (4) and record
- Relieve the bar, move the dial gauge to the next position, and repeat the measurement.

Elastic line of a bar on two supports		
x in mm	Measured deflection w(x) in mm	Calculated deflection w(x) in mm
0	0	0
100	1.3	1.63
200	2.8	3.13
300	4.0	4.36
400	4.6	5.20
500	5.1	5.51

The measured deflections are slightly too low. The following graph compares the measured elastic line with the calculated elastic line:



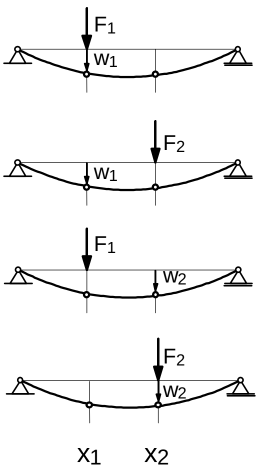
Elastic line of bar on two supports

3.3 Maxwell-Betti's influence coefficients and law

Influence coefficients link the deflection at a certain point in the bar to the loading of the bar. In general, the deflection w_i for a point x_i can be specified as the function of n forces F_j as follows:

$$w_i = \sum_{j=1}^{n} a_{ij}\, F_j.$$

This experiment is only intended to examine the effect of a force on points x_1 and x_2 on the deflection at points x_1 and x_2



$$w_1 = a_{11}\, F_1$$

$$w_1 = a_{12}\, F_2$$

$$w_2 = a_{21}\, F_1$$

$$w_2 = a_{22}\, F_2$$

According to the *Maxwell-Betti* transposition law, deflection at point x_1 as a result of the force on point x_2 is just as large as the deflection at point x_2 caused by an identical force on point x_1. This correlation is described by the following formula

$$w_1 = a_{12}\, F_2 = w_2 = a_{21}\, F_1$$

$$a_{21} = a_{12}$$

In general, according to *Maxwell-Betti*, the following applies

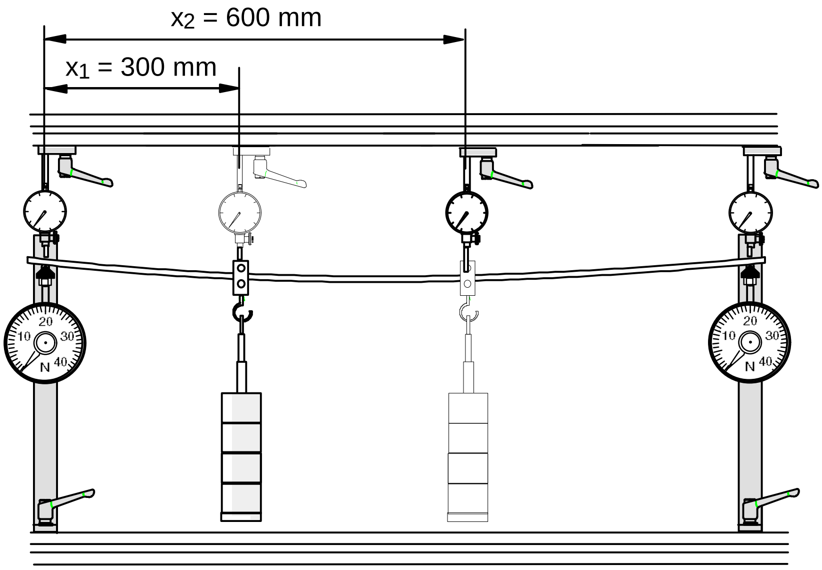
$$a_{ij} = a_{ji}$$

The two **influence coefficients** a_{11} and a_{22} indicate the deflection at the point of force.

3.3.1 Performing the experiment

The experiment is set up as shown in the diagram. The following equipment is required:

- Steel bar 6 x 20 x 1000 mm
- Rider for weight
- Suspender for weights
- 3 weights 5N, 1 weight 2.5 N
- 2 articulated supports with dynamometer
- 3 dial gauges with holder



The distance between the supports is 1000 mm.

The load of 20 N remains constant and is applied at x_1= 300 mm and x_2 = 600 mm.

The deflection of the bar at points x_1 and x_2 is measured with the dial gauge.

Two dial gauges on the supports measure the deflection caused by the dynamometers and serve to compensate it. The procedure is the same as described in the previous experiment.

- Load the bar at point x_1 and measure the deflection at x_1 and x_2.
- Load the bar at x_2 and measure the deflection at x_1 and x_2

Influence coefficients for bars on two supports			
Deflection at point in mm	Force at point in mm	Deflection w in mm	Influence coefficient in mm/N
x_1= 300	x_1= 300	3.1	a_{11}= 0.155
x_1= 300	x_2= 600	3.5	a_{12}= 0.175
x_2= 600	x_1= 300	3.5	a_{21}= 0.175
x_2= 600	x_2= 600	4.3	a_{22}= 0.215

From the load and the deflections, it is possible to calculate the influence coefficients.

As predicted, the influence coefficients a_{12} and a_{21} are identical.

3.4 Statically undetermined systems

A statically undetermined system exists when the **valency of the support** is greater than the number of **degrees of freedom** of the system. A system in the plane, such as the bar, has 3 degrees of freedom:

- horizontal displacement
- vertical displacement
- rotation in the plane

A **movable support** (articulated support) is monovalent, as it only prevents vertical motion.

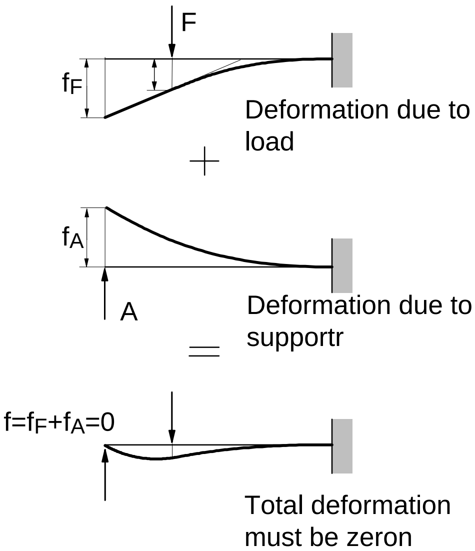
A **clamp** (support pillar) is tri-valent, since in addition to vertical motion, it also prevents horizontal motion and rotation.

If a bar is clamped on one side and supported by a movable support on the other side, the sum of all support valencies is 4, in other words, larger than the number of degrees of freedom (3). The bar is statically undetermined, or more precisely, it is statically overdetermined.

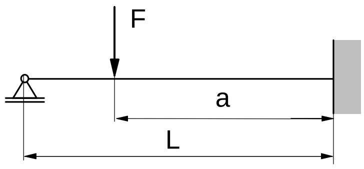
In such cases, the unknown support reactions, whose number corresponds to the sum of the support valencies, cannot be calculated solely by means of the equilibrium relations of the forces and moments. In such a case, there are only as many equations which are independent of one another as there are degrees of freedom. In order to be able to calculate the numerous unknown support reactions, equations with the deformation properties of the bar must be used.

By applying the **superposition principle**, the experimental approach is similar to the calculatory approach.

3.4.1 Calculation of the supporting force



Superposition principle



The superposition principle states that the sum of all deformations caused by individual loads corresponds to the deformation caused by a combined load from the individual loads.

Therefore, the system is loaded with the individual loads, one after the other, and from the calculatory summation of these individual results, it is possible to determine the overall deformation of a combined load occurring simultaneously.

In order to determine the unknown supporting force, the total deformation at the front end of the bar must be zero. Consequently, the deformation induced by supporting force A must be just as large as but opposite to the one induced by the load F.

According to experiment 3.1.3, the deformation at the end of the bar caused by the load F is

$$f_F = w(b) + b\,\alpha = \frac{F\,a^3}{3\,E\,I_y} + \frac{F\,(L-a)\;a^2}{2\,E\,I_y}.$$

The deformation at the end of the bar due to supporting force A is calculated as follows:

$$f_A = -\frac{A\,L^3}{3\,E\,I_y}\,.$$

The sum of both deformations must be zero

$$f_F + f_A = 0 = \frac{F\,a^3}{3\,E\,I_y} + \frac{F\,(L-a)\,a^2}{2\,E\,I_y} - \frac{A\,L^3}{3\,E\,I_y}.$$

Solving the equation with respect to the unknown supporting force produces:

$$A = \frac{F}{2\,L^3}\,(\,3\,L\,a^2 - a^3\,).$$

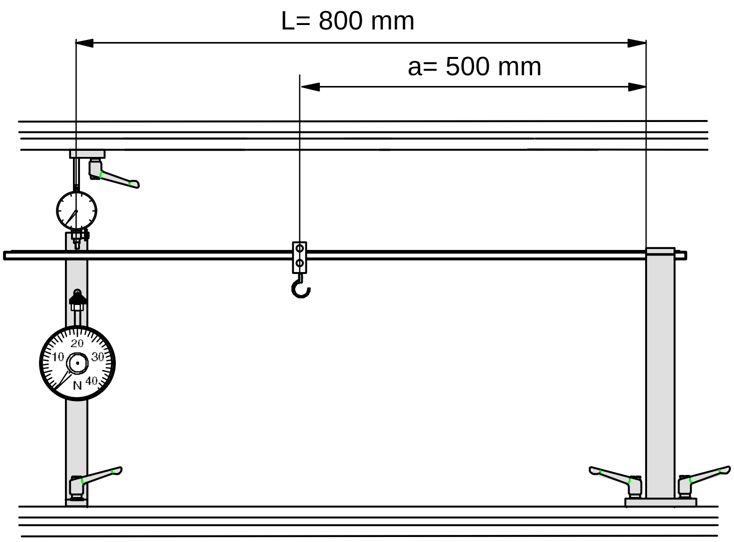
With a load of F= 15 N, a length L= 800 mm and a distance a= 500 mm, this produces the following for the unknown supporting force A:

$$A = \frac{15}{2 \cdot 800^3} (3 \cdot 800 \cdot 500^2 - 500^3) = 6.96\ N$$

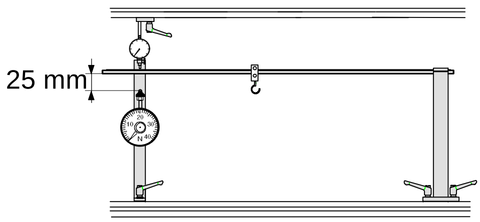
3.4.2 Performing the experiment

The experiment is set up as shown in the diagram. The following equipment is required:

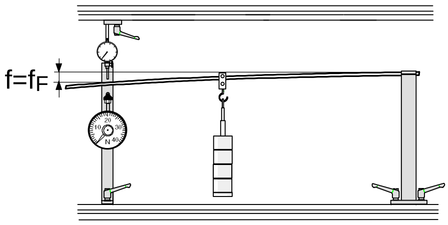
- Steel bar 6 x 20 x 1000 mm
- Rider for weight
- Suspender for weights
- 2 weights 5N, 1 weight 2.5 N
- Articulated support with dynamometer
- Support pillar with clamp
- Dial gauge with holder



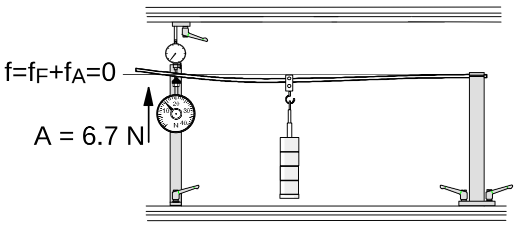
The articulated support has a distance of 800 mm from the clamp, and the load 500 mm from the clamp. The dial gauge is positioned above the articulated support



- Twist the articulated support downwards. The distance between the unloaded end and the support should be at least 25 mm
- Whilst unloaded, set the dial gauge at the end of the bar to zero



- Load the bar with 15 N. The bar will deflect downwards at the end by f_F= 16.2 mm



- Twist the support upwards to compensate the deflection by load. The dial gauge should return to zero. The dynamometer will now display the supporting force.

Reading produces a supporting force of 6.7 N. A comparison with the mathematically calculated result of 6.96 N indicates good consistency. To check, the unloaded bar can be raised by f_A= 16.2 mm via the support. The dynamometer should then likewise show 6.7 N.

4 Appendix

4.1 Technical data

Number of sets of weights:		4	
Weight graduation:		16 x 5	N
		4 x 2.5	N
Number of supports:	articulated	3	
	fixed	1	
Measurement range of dynamometers:		40	N
Measurement range of dial gauges:		20	mm

Test bars

Steel	90MnCrV8		
		3 x 20 x 1000	mm
		4 x 20 x 1000	mm
		6 x 20 x 1000	mm
Brass	CuZn39Pb3		
		6 x 20 x 1000	mm
Aluminium	AlMgSi0,5F22		
		6 x 20 x 1000	mm

Dimensions L x W x H:	1400x400x750	mm
Weight:	40	kg

4.2 Formulae and units used

Formulae:

a , a_{ij}:	Distance, influence coefficient
A :	Supporting force
b :	Width of cross-section, distance
B :	Supporting force
E :	Modulus of elasticity
f :	Deflection
F :	Force, load
h :	Height of cross-section
I_y:	Planar moment of inertia
L :	Length
M :	Bending moment
w :	Deflection downwards
x :	Longitudinal coordinate of bar
α :	Inclination

Units:

Force:	N
Length, width, height:	mm
Deflection, deflection downwards:	mm
Modulus of elasticity:	N/mm^2
Planar moment of inertia:	mm^4
Inclination:	rad

4.3 Index