Experiment Instructions

WP130 Verification of Stress Hypotheses

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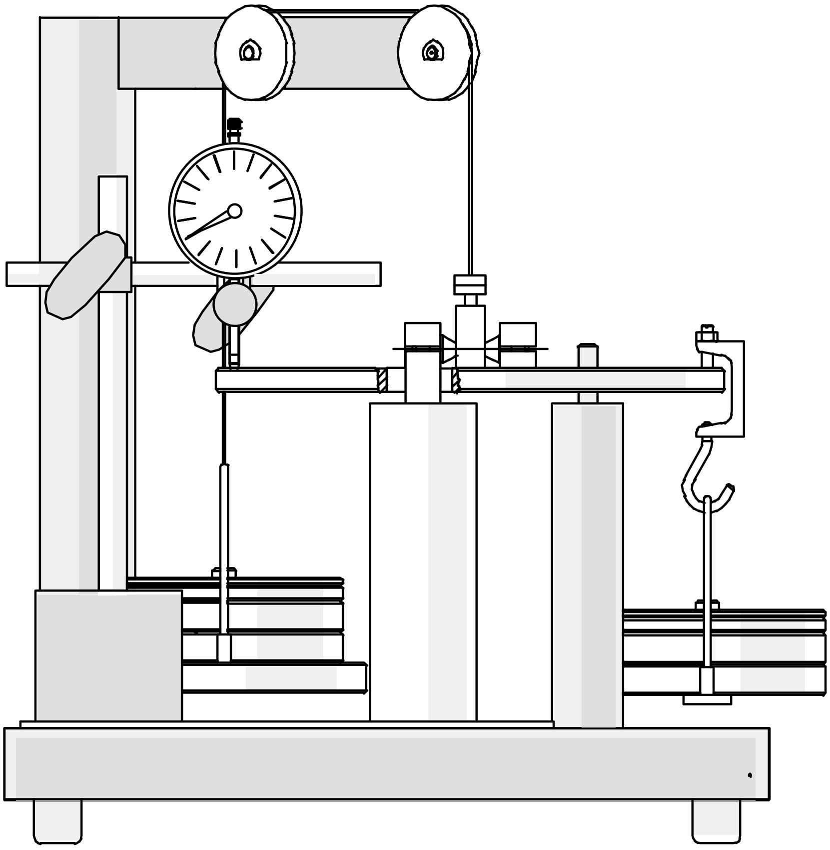

Experiment Instructions

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WP 130 Verification of Stress Hypotheses

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WP 130 Verification of Stress Hypotheses

1 Didactic objectives

Various **failure theories** with their correspondending **reference stresses** from the science of material strength can be tested experimentally for validity using the **WP 130 test unit**. The trainee's understanding of this theoretical and unclear area of material strength is promoted by the experiment.

Reference stresses are used in order to find a measure for the stress of a material by subjecting a component with combined shearing and direct loads. Since the characteristic material values are usually only available for single-axial stress (tensile strength, proof stress) a corresponding reference stress must be determined from the actual bi-or tri-axial stress.

Direct and shear stresses must be generated simultaneously at one point in the specimen, in order to prove the reference stress criterion. Shear forces cannot be applied here, since their maximum appears in the middle of the cross section and they cannot be combined with direct stresses reliably at one point. Of the remaining stresses -tension, bending and torsion - the latter, bending and torsion were selected for this test unit. They can be generated in sufficient magnitudes by simple mechanical means. The test unit is designed to allow both **pure bending or torsion or combined loads** to be applied to the specimen.

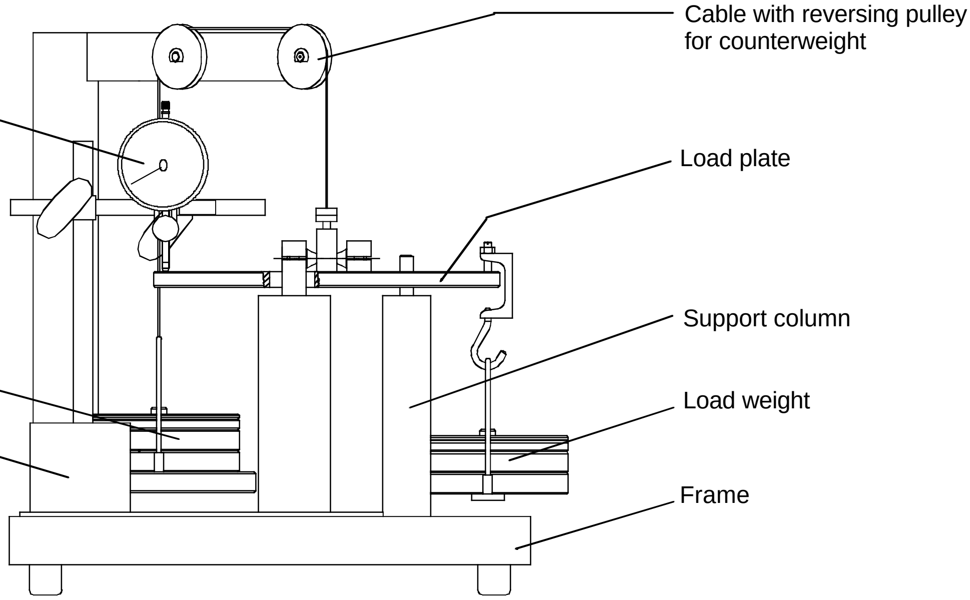
The WP 130 test unit features a simple design and clear operation. Simple, favorably priced specimens are used.

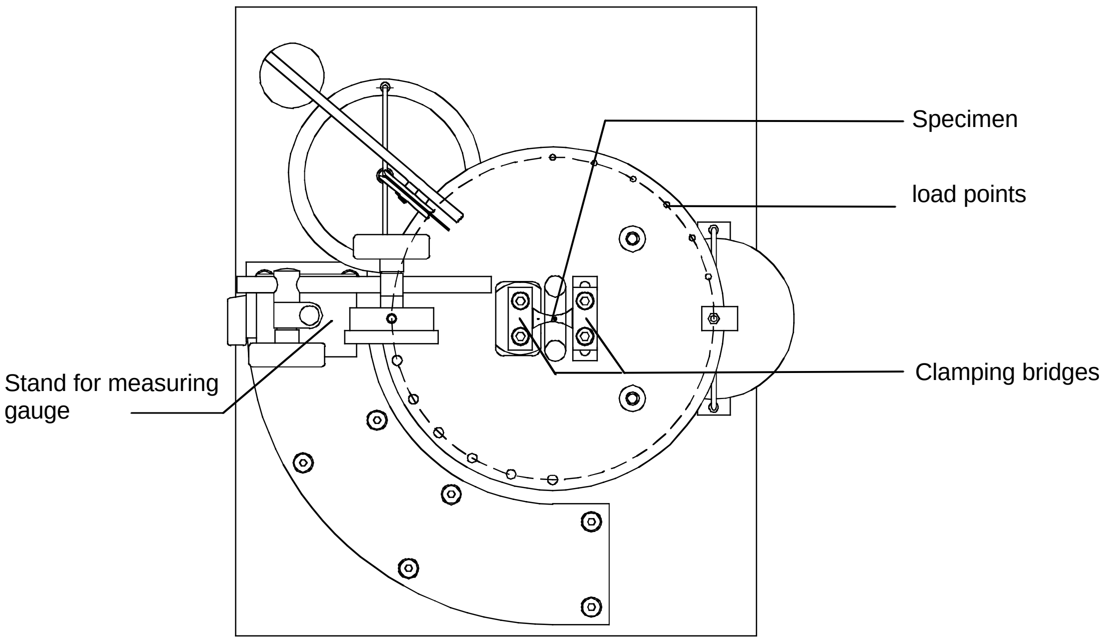
The specimens are clamped at one end in the stationary frame, at the other end they are clamped to a circular load plate. A load weight can engage with the circumference of the load plate at any required angle. This produces the desired multi-axial stress in the specimen cross section.

The deformation of the specimen is displayed by a measuring gauge. The trainee can thus learn how to use mechanical equipment, measure deformation and how to conduct series of tests.

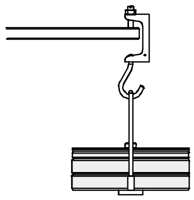
2 Unit description

2.1 View of test unit

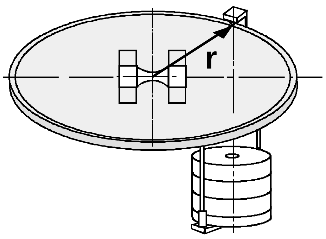




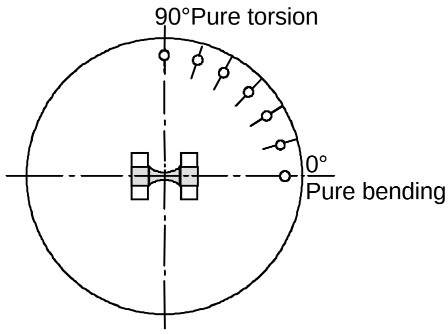
2.2 Test unit functioning



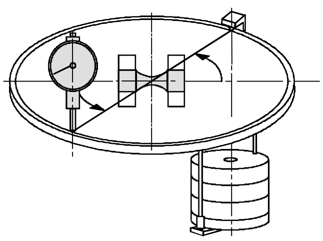
- The load moment is generated via a lever and weights. Each load moment can be set between 0 and 3.0 Nm with a resolution of 0.1 Nm thanks to a corresponding division in weight discs.



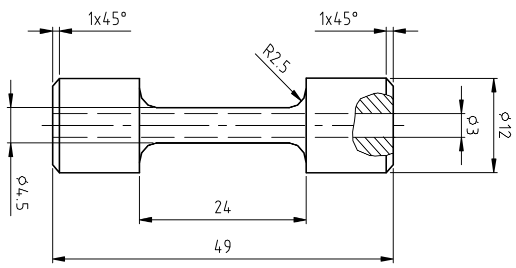
- A circular load plate is used as a lever for the load moment. The load weight engages around its circumference. The specimen is clamped into the center of the plate.



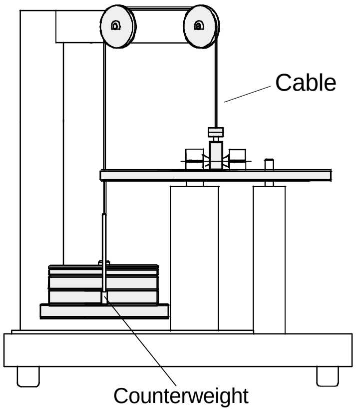
- The combined bending and torsion load can be generated using defined force engagement points. In this case, the angle position 0° is equivalent to a pure bending moment load, the 90° angle position is equivalent to a pure torque load. The load weight can be adjusted in 15° increments.



- There are measuring points for measuring the deformation diametrically opposite the force engagement point. This allows the deformation to be measured at the point of greatest deflection. Since measurement is performed at an unloaded part of the plate in this case, the influence of errors due to a deformation of the test unit under the load is minimized.



- The samples are held by friction lock using clamping bridges. The specimen cross section for clamping is cylindrical.



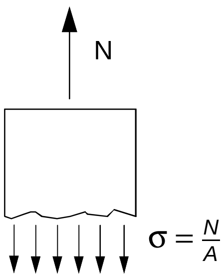
- Loading of the specimen free of transverse forces is possible due to the compensation of the load plate by its own weight and compensation of the load weight. In order to ensure statically determined and, thus, previous load free clamping of the specimen, the counterweight is coupled to the load plate flexibly and free of transverse forces via a cable.

2.3 Technical data

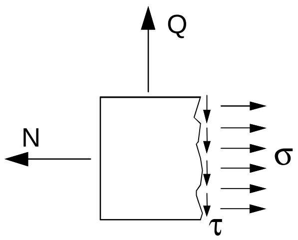
Dimensions			
	Length	390	mm
	Width	325	mm
	Height	325	mm
	Weight	approx. 17	kg
Load device			
Load moments		0...3.0	Nm
Increments		0.1	Nm
Direction:	Adjustable between pure bending moment and pure torque		
Lever arm		100	mm
Load weights			
	graduated 1 N, 2 N, 4 N, 8 N		
max. load		30	N
Specimens			
Specimen length		49	mm
Clamping diameter		12	mm
Clamping length		11.5	mm
Measured cross section			
Diameters d; D		3 mm; 4,5	mm
Cross sectional area		8.8	mm^2
Specimen material			
	Steel	S 235 JR	
	Copper	Cu-ETP	
	Brass	CuZn39Pb3	
	Aluminium	AlMgSi 0,5 F22	
Deformation measurement via measuring gauge			
Measuring range		0...10	mm
Resolution		0.01	mm

3 Theory

3.1 Reference stress hypotheses



Single-axial stress state



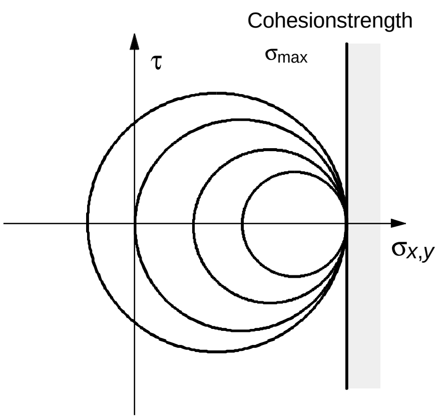
Multi-axial stress state

With **single-axial stress states** such as pure tensile or thrust loads the stress is to be determined in the case of yielding. It directly corresponds to the tensile or thrust stress. The admissible limit stress can be measured easily in a tensile or thrust test.

Multi-axial stress states featuring direct stresses σ and shear stresses τ arise with combined loads such as direct and transverse forces or bending along with simultaneous torsion. In combination, these various stresses can lead to component failure.

Since, however, material strength is generally only determined in a single-axial stress state such as in a tensile test, a suitable **reference stress** must be found which adequately represents the effect of a multi-axial stress state.

In the course of time various **failure hypotheses** have been postulated and corresponding reference stresses have been derived. The three most important hypotheses are:



Mohr's circle

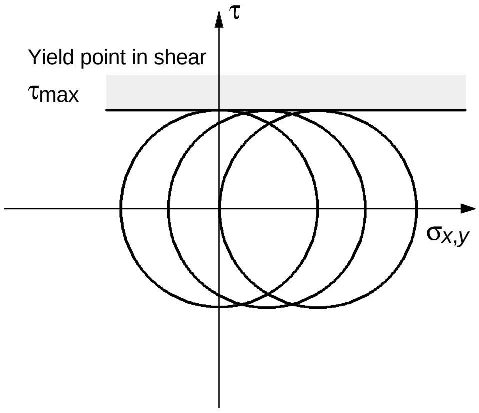
- **Maximum principal stress criterion** (RANKINE)

$$\sigma_v = \frac{1}{2}(\sigma_x + \sigma_y) + \frac{1}{2}\sqrt{(\sigma_y - \sigma_x)^2 + 4\tau^2}$$

The assumption here is that the material fails as a result of the greatest direct stress. The term used is cleavage fracture. The break plane is vertical with regard to the load axis.

Experiments have demonstrated that this criterion is accurate for **brittle materials** such as tempered steel of cast iron.

- **Maximum shear stress criterion** (GUEST, MOHR)



Mohr's circle

$$\sigma_v = 2\,\tau_{max} = \sqrt{(\sigma_y - \sigma_x)^2 + 4\,\tau^2}$$

The assumption here is that the material fails as a result of the greatest shear stress. A shear fracture is formed. Since the greatest shear stress occurs at an angle of 45° with refence to the direction of the greatest principal stress, the break plane is at an angle of 45° to the main load axis.

This criterion primarily applies to **tough, ductile materials** such as copper, soft steel or aluminium.

- **Maximum shear strain energy criterion** (v.MISES and HENKY)

$$\sigma_v = \sqrt{\sigma_x^2 + \sigma_y^2 - \sigma_x\,\sigma_y + 3\,\tau^2}$$

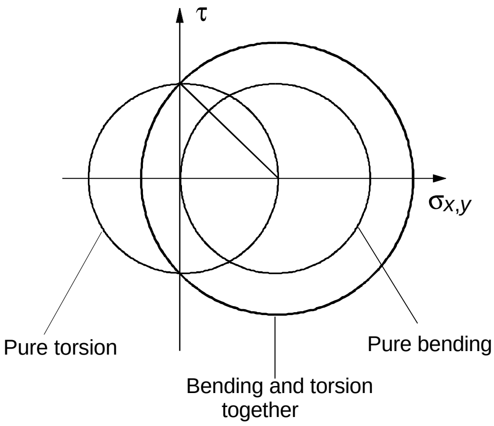
This criterion assumes that the material fails when the strain energy exceeds a certain level. If a component is subjected to a load, it is deformed.

In order to cause this strain, the external load performs work - the so-called energy of strain. The strain can be divided into a volume change and a change in shape.

According to the strain criterion, failure occurs when the strain of shape is too great. This criterion mainly applies to **dynamic, alternating loads** no matter what the type of material is.

The **reference stresses** are formed from the **principal stress** and the **shear stress**. The connection between these two stresses can be graphically represented with **Mohr's circles** (please refer to the pertinent specialized literature for more detailed explanations of Mohr's analogies).

Here, the referenced shear stress τ is represented over the principal stress σ.



Mohr's circles with combined load

- In the case of a **pure bending** , the stress circle is just to the right of the coordinate origin. The shear stress is zero just like the second principal direct stress.
- In the case of a **pure torsion**, both principal direct stresses are equal to zero. The center point of the stress circle is located in the origin of the coordinate system.
- In the case of a **mixed load with bending and torsion**, the stress circle is between the two positions of the pure loads.

3.2 Combined load featuring bending and torsion

The following is intended to describe the connection between the reference stress and the external load in the case of combined bending and torsion. Since tough materials are used for the specimens, it can be assumed that the shear stress criterion is true.

$$\sigma_v = 2\,\tau_{max} = \sqrt{(\sigma_y - \sigma_x)^2 + 4\,\tau^2}$$

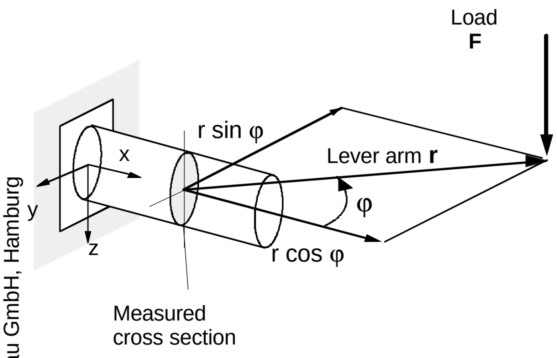
In the case of a bending load no direct stresses arise perpendicular to the longitudinal axis of the specimen.

Therefore, a stress of $\sigma_y = 0$ can be set

$$\sigma_v = 2\,\tau_{max} = \sqrt{\sigma_x^2 + 4\,\tau^2}\ .$$

The max. direct bending stress in the edge fiber is calculated from the bending moment and the geometric moment of inertia

$$\sigma_x = \frac{M_b}{I_b}\,\frac{d}{2}\,.$$



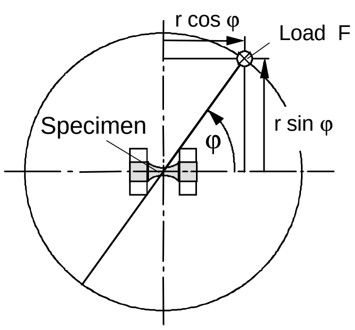
Clamped specimen subjected to combined bending and torsional loads

The max. shear stress as a result of torsion also appears in the edge fiber

$$\tau = \frac{M_t}{I_t}\,\frac{d}{2}\,.$$

The geometric moments of inertia for a circular cross section are

$$I_b = \frac{d^4\,\pi}{64}\ , \qquad I_t = \frac{d^4\,\pi}{32} = 2\,I_b\,.$$



Clamping of specimen in WP 130 test unit

The external bending moment M_b and the torque M_t can be calculated with the angle position φ of the point of contact and the load weight F to

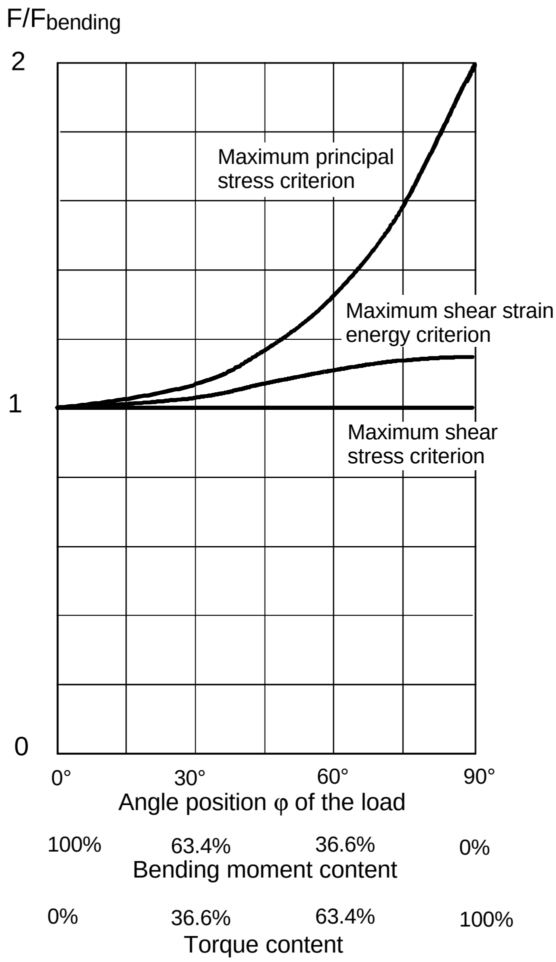
$$M_b = F\,r\ \cos\varphi\ , \qquad M_t = F\,r\ \sin\varphi\,.$$

If the bending moment and the torque are used the following results

$$\sigma_v^2 = \frac{d^2}{4}\left[\left(\frac{F\,r\ \cos\varphi}{I_b}\right)^2 + 4\left(\frac{F\,r\ \sin\varphi}{2\,I_b}\right)^2\right]$$

$$\sigma_v = \frac{F\,r\,d}{2\,I_b}\ \sqrt{\cos^2\varphi + \sin^2\varphi}\ .$$

The $\sin^2$ and $\cos^2$ expressions can be combined to 1.



Load **F** along the yield limit for various reference stress criterions

The expression for the reference stress is not dependent on the angle φ and is, therefore, **independent of the ratio of bending moment to torque**

$$\sigma_v = \frac{F\,r\,d}{2\,I_b} = constant$$

For the experiment this means that the load F, which appears upon material yield, is not dependent on the ratio between bending and torsion. This is valid when the shear stress criterion applies. In the case of the principal stress criterion and shear strain energy criterion there are other dependencies of the yield limit on the ratio bending to torsion.

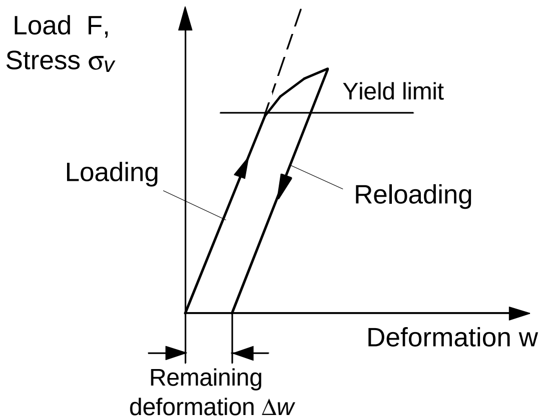
The corresponding dependencies are displayed graphically in the adjacent diagram. Here, the loads F were referenced to the yield limit for pure bending.

4 Tests

Since the yield limit has to be determined using the specimens, a load up to or exceeding the elasticity limit is required. In doing so, the specimen is subjected to plastic deformation. In the case of several successive load tests this leads to **work-hardening** and displacement of the **yield limit to higher stresses**. There are two methods for minimizing these error influences.

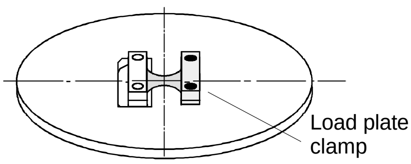
- A **new specimen** is used for **each load test**. Here, care must be taken that the specimens have absolutely the same dimensions and strength characteristics. If possible, they should be made from the same round material blank.
- The specimen remains the same for a series of tests, however, the **series of tests is repeated in reverse order** with a new specimen.
  One series is started with a pure bending moment and concluded with a pure torque and the other is started with a pure torque and concluded with a pure bending moment. By determining the measured values from the two series of tests the influences of increasing work-hardening are cancelled out.

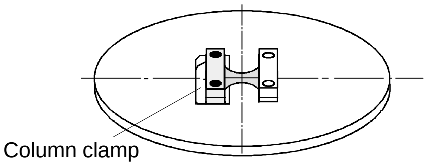
By arranging the engagement points for the load weight along a quarter circle at 15° increments, a total of 7 different load combinations with bending and torsion are possible. In each load test the load is increased by adding or replacing weight discs until the admissible stress in the measured cross section is exceeded and material yield ensues.

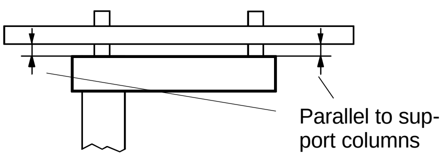


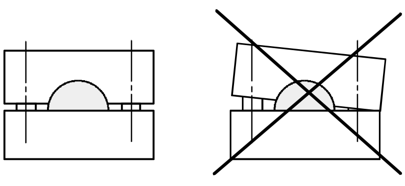
For reliable evidence of the yield limit the remaining deformation following unloading must be continuously measured during the load test. Stop the test as soon as the proportional characteristics turn into non-linear characteristics (yield limit). In order to keep the work-hardening effect as small as possible, the specimen should be plastically deformed as little as possible by yielding.

4.1 Clamping the specimens



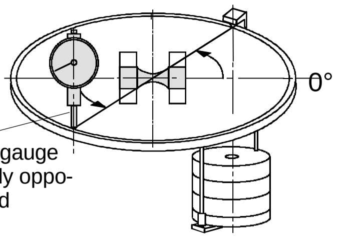






- Remove the measuring gauge with magnetic holder from the load plate.
- Take off the load weight.
- Unhook the counterweight and place the load plate on the support column.
- Clamp the specimen in the specimen holder of the **load plate**. Carefully tighten the clamping screws.
- Suspend the counterweight (no additional weights).
- Clamp the other end of the specimen in the **column** specimen holder.
- **Important!** Slide the specimen into the clamping bridges until it stops.
  Make sure that the load plate is exactly **horizontal**. The load plate must lie **parallel** to the support column.
  Clamping bridges and specimen must be **clean** and **free of grease** at the point of clamping.
  Tighten the clamping bridge **straight**.

4.2 Conducting the measurement



The yield limit should be determined twice for at least 3 different angles for a complete test series. Work-hardening can be eliminated by determining the values from the two test series.

- Compensate the planned average load weight with the **counterweight**. For specimens made of
  - Steel: 14 N
  - Copper: 8 N
  - Brass: 11 N
  - Aluminium: 12 N

  Please refer to Table 2 for the weight combination
- Place the **measuring gauge** diametrically opposite the position of the load weight and set to zero.
  To do this, lightly press against the load plate with a finger at the position of the load weight. The measuring gauge must point exactly to zero after slow, careful unloading.
- Increase the **load weight in increments** of 1 N (steel 2 N) beginning with the planned mean load weight (see above). (Please refer to Appendix, Table 1 for weight combinations).
  When doing so, remove the load weight and hook from the load plate, add the new weight combination and suspend the load weight again from the from the load plate **slowly and without jolting**.
- Record the **deformation** on the measuring gauge on Worksheet 1 after each load increment. After removing the load weight, read the remaining deformation from the measuring gauge (measuring gauge does not return to zero) and record it, as well.

When doing so depress the load plate again lightly with a finger and carefully unload it. This prevents influences from clearence and friction.

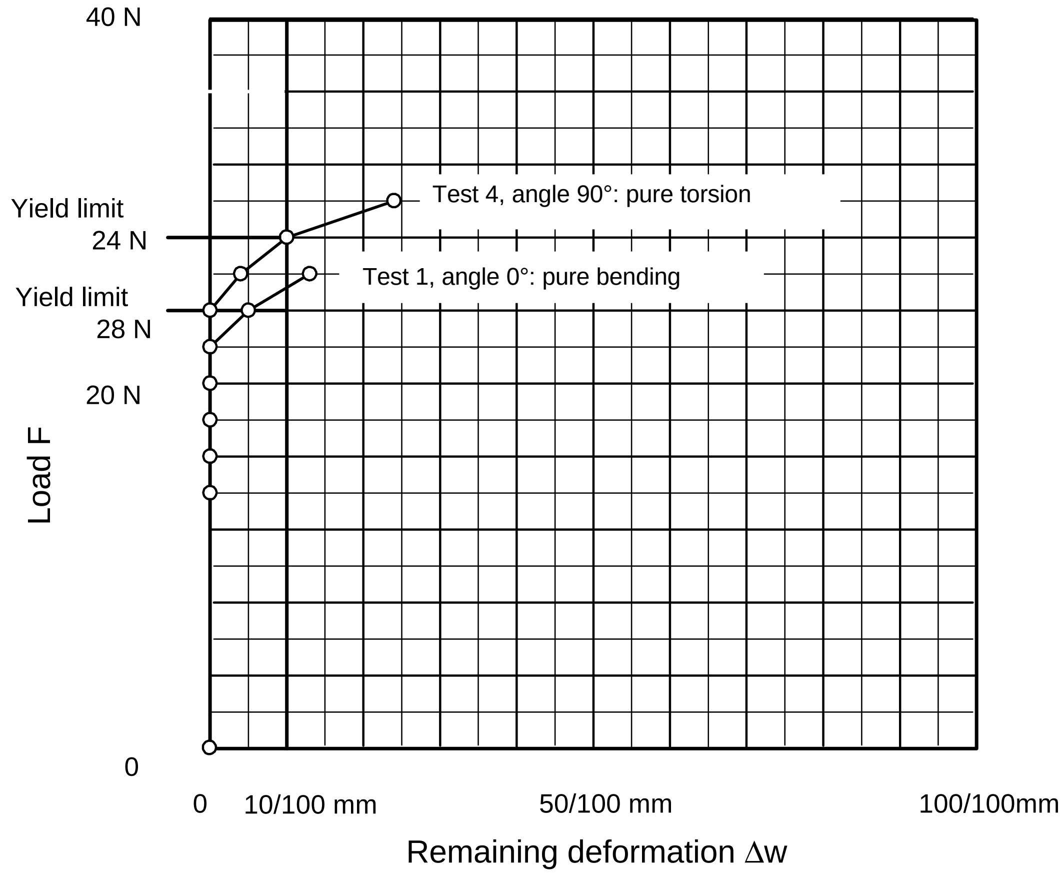
- As soon as a **deviation from proportional characteristics occurs** - remaining deformation greater than 10/100 mm - stop the test. The **yield limit** is then determined by graphing the load over remaining deformation at point = 10/100 (Worksheet 2). Here, the increase in strength as a result of strain hardening can be seen clearly.

Test no.: 1	Angle position: 0°						Run: 1					Material: steel						
Load F in N	14	16	18	20	22	24	26	28	30									
Deformation w in 1/100 mm	198	223	248	274	300	325	351	385	430									
Remaining deformation Δ*w* in 1/100 mm	0	0	0	0	0	0	4	10	24									
Load at yield limit : 28 N																		

Test no.: 4	Angle position: 90°						Run: 1					Material: steel						
Load F in N	14	16	18	20	22	24	26											
Deformation w in 1/100 mm	252	284	316	348	381	414	456											
Remaining deformation Δ*w* in 1/100 mm	0	0	0	0	0	5	13											
Load at yield limit : 24 N																		

Example: Worksheet 1, steel specimen at 0° and 90°

- Once the yield limit has ben determined, move the load weight and measuring gauge by the desired angle and repeat the load series.



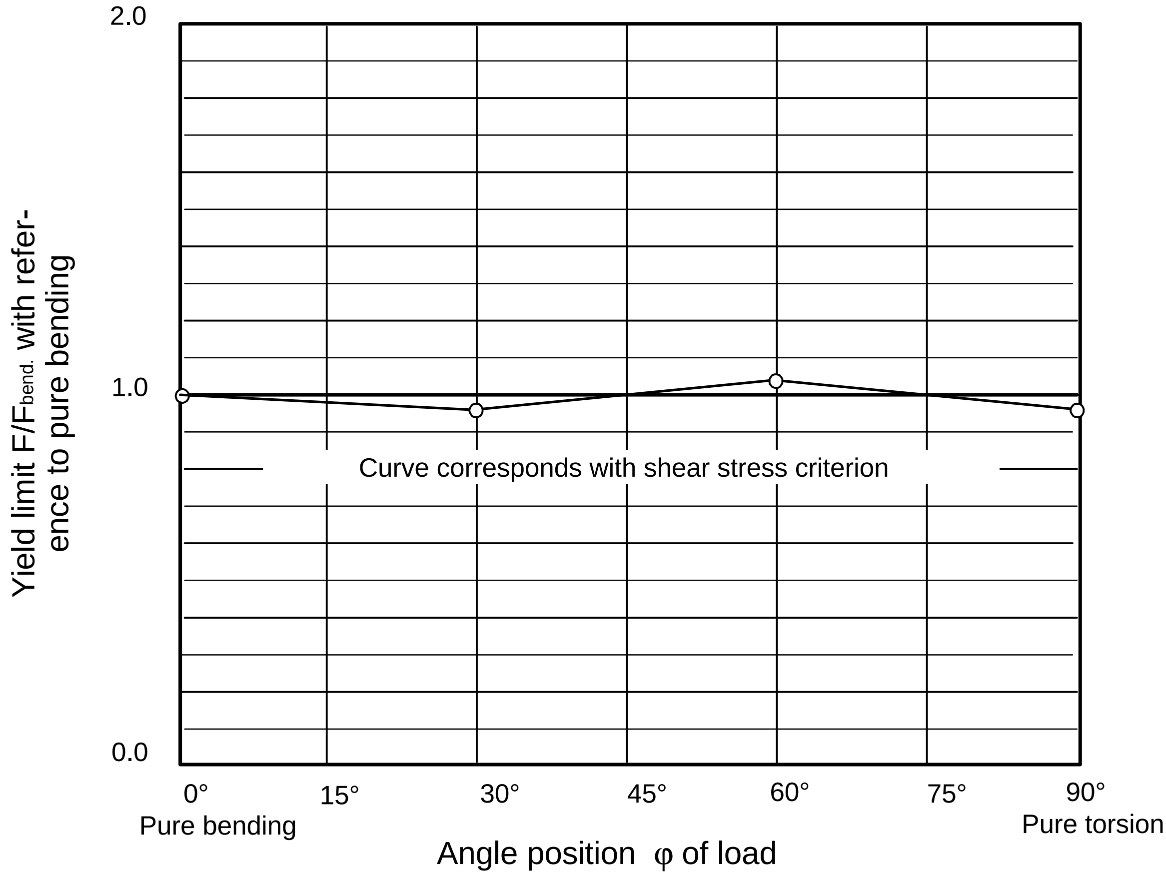
Example: Worksheet 2, steel specimen at 0° and 90°

After the measurement at 90° (pure torsion) exchange the specimen for a new specimen made of the same material and repeat the entire series of tests with the sequence of angles in the opposite order beginning at 90° and ending at 0°.

4.3 Evaluation

Once the yield limit has been determined for all angle positions, the mean of both runs is determined and referenced to the pure bending value (angle 0°). These values are then entered in a diagram (Worksheet 3). If this results in a horizontal curve, the shearing stress hypothesis applies as with the specimen used. If, however, the yield limit is twice as high for pure torsion as it is for pure bending, the direct stress hypothesis applies.

Material:				
Angle position of load	0° Pure bending	30°	60°	90° Pure torsion
Yield limit F_{l1} Run 1	28	24	28	26
Yield limit F_{l2} Run 2	24	26	26	24
Mean value from 1 and 2 $\overline{F} = (F_{l1} + F_{l2}) / 2$	26	25	27	25
Yield limit with reference to pure bending $\overline{F}/\overline{F}_{bend}$	1	0.96	1.04	0.96



Example: Worksheet 3, dependency of yield limit on type of load for a steel specimen

5 Appendix

Table 1: **Load** weight combinations

Load in N	Hook	Basket	Disc 1N	Disc 2N	Disc 4N	Disc 8N
2 (18)	x	x				(x x)
3 (19)	x	x	x			(x x)
4 (20)	x	x		x		(x x)
5 (21)	x	x	x	x		(x x)
6 (22)	x	x			x	(x x)
7 (23)	x	x	x		x	(x x)
8 (24)	x	x		x	x	(x x)
9 (25)	x	x	x	x	x	(x x)
10(26)	x	x				x (x x x)
11(27)	x	x	x			x (x x x)
12(28)	x	x		x		x (x x x)
13(29)	x	x	x	x		x (x x x)
14(30)	x	x			x	x (x x x)
15	x	x	x		x	x
16	x	x		x	x	x
17	x	x	x	x	x	x

Table 2: **Counterweight** weight combinations

Load in N	Disc 1N	Disc 2N	Disc 4N	Disc 8N
1	x			
2 (18)		x		(x x)
3 (19)	x	x		(x x)
4 (20)			x	(x x)
5 (21)	x		x	(x x)
6 (22)		x	x	(x x)
7 (23)	x	x	x	(x x)
8 (24)		(x x)	(x)	x (x)
9 (25)	x	(x x)	(x)	x (x)
10		x		x
11	x	x		x
12			x	x
13	x		x	x
14		x	x	x
15	x	x	x	x
16				x x
17	x			x x

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Worksheet 1

Test no.:	Angle position:								Run:					Material:					
Load F in N																			
Deformation w in 1/100 mm																			
Remaining deformation Δ*w* in 1/100mm																			
Load at yield limit:																			

Test no.:	Angle position:								Run:					Material:					
Load F in N																			
Deformation w in 1/100 mm																			
Remaining deformation Δ*w* in 1/100mm																			
Load at yield limit:																			

Test no.:	Angle position:								Run:					Material:					
Load F in N																			
Deformation w in 1/100 mm																			
Remaining deformation Δ*w* in 1/100mm																			
Load at yield limit:																			

Test no.:	Angle position:								Run:					Material:					
Load F in N																			
Deformation w in 1/100 mm																			
Remaining deformation Δ*w* in 1/100mm																			
Load at yield limit:																			

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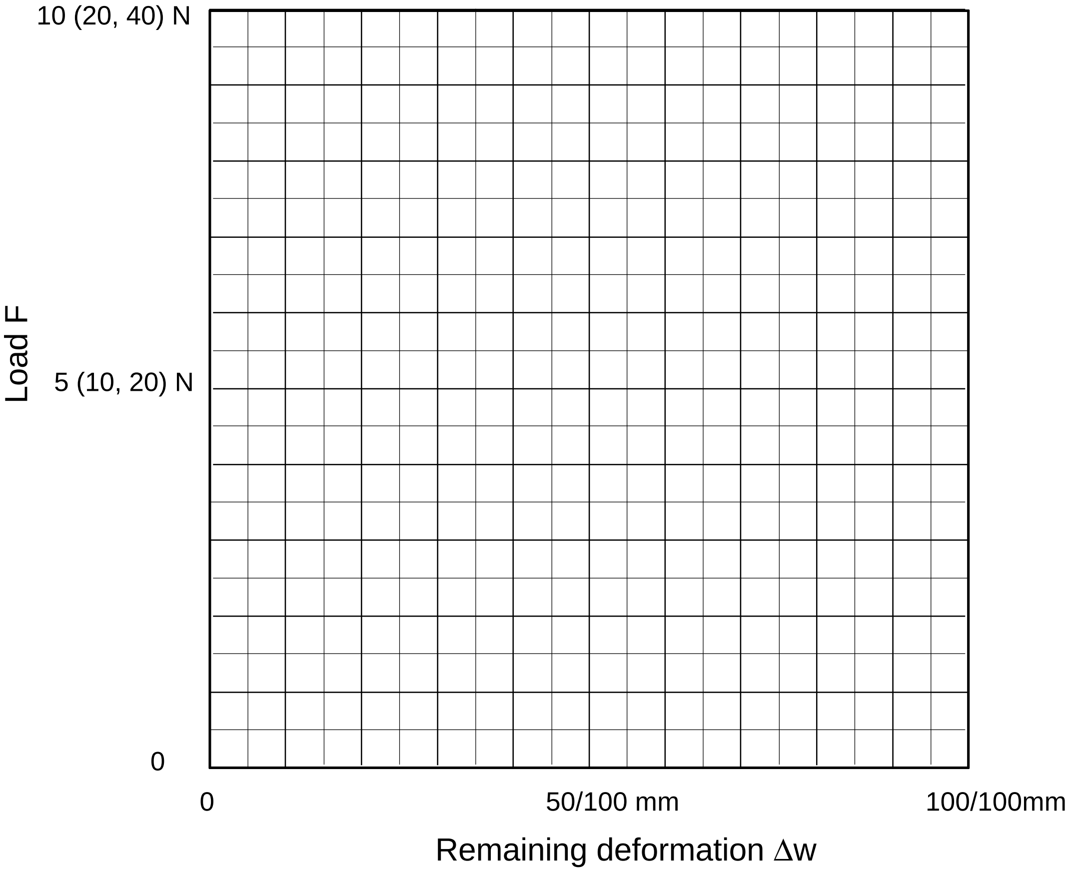
Worksheet 2



Worksheet 3

Material:							
Angle position of load	0°Pure bending	15°	30°	45°	60°	75°	90°Pure torsion
Yield limit F_{l1} Run 1							
Yield limit F_{l2} Run 2							
Mean value from 1 and 2 $\overline{F} = (F_{l1} + F_{l2}) / 2$							
Yield limit with reference to pure bending $\overline{F}/\overline{F}_{bend}$	1