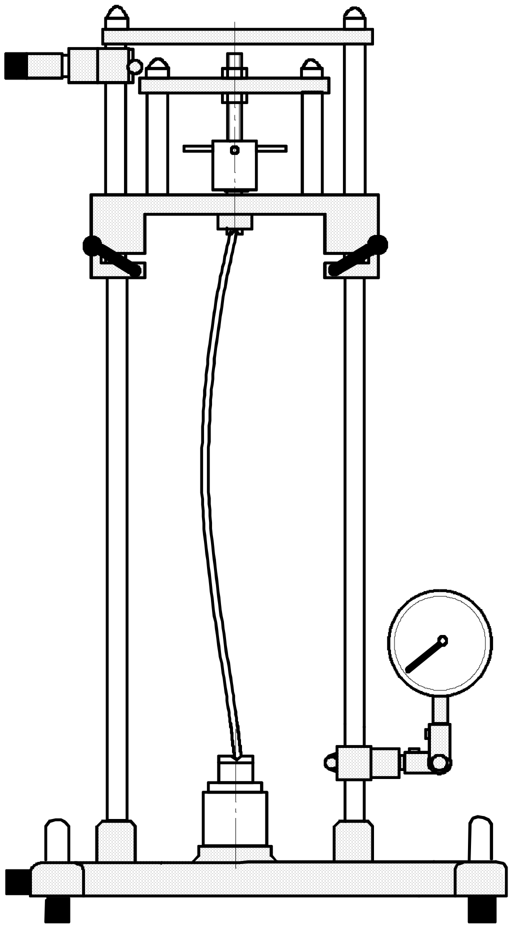
Experiment Instructions

WP 120 Buckling Test Device

WP 120 Buckling Tester



Experiment Instructions

Publication No. 912.000 00A 120 12 08/92

Table of contents

1. Didactic objectives

All relevant buckling problems can be demonstrated with the WP 120 test stand.

Buckling, as opposed to simple strength problems such as drawing, pressure, bending and shearing, is primarily a stability problem. Buckling problems number among the best known technical examples in stability theory.

Buckling plays an important role in almost every field of technology. Examples of this are:

- Columns and supports in construction and steel engineering
- Stop rods for valve actuation and connecting rods in motor construction
- Piston rods for hydraulic cylinders and
- Lifting spindles in lifting gear

All parts subjected to pressure are susceptible to buckling. Thus, the WP 120 buckling test device has a great number of potential applications.

The device can be used both by the instructor for demonstrations as well as by the trainee in laboratory experiments.

Main educational targets could include:

- Examination of the Euler theory of buckling
- The influence of different buckling rod mounting conditions
- Influence of the rod length and diameters
- Influence of material parameters

Furthermore, basic technical testing skills such as path and force measurement can be practiced.

Besides the actual instructions for performing tests, the instructional material accompanying the unit also contains a section on the basics which covers theoretical aspects. This often does away with the need for the user to conduct extensive literature research.

The structure of the instructional material allows users who are interested in theory to comprehend the basics of stability theory. However, the instructional material also allows practically minded users to immediately begin testing.

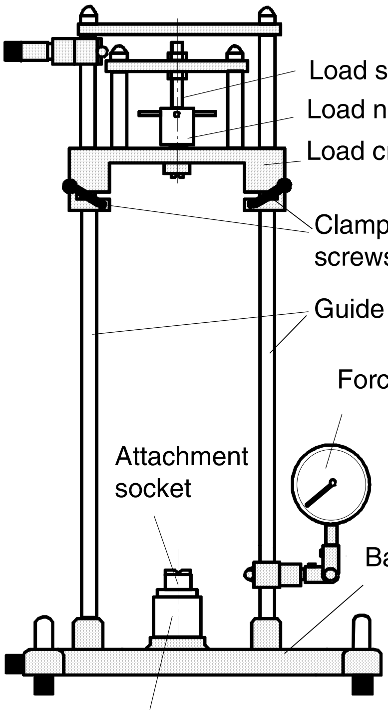
Emphasis as placed on ensuring that all phenomena which occur are accompanied by theoretical considerations and explanations. The opportunity for discussion promotes the development of a complete and clear picture of the problem of buckling in the trainee.

The individual sections of the instructional material are design to allow the respective topic to be dealt with individually at any time.

The instructional material is rounded off by a collection of the most important mechanical terms and relationships. This clearly arranged compilation offers the trainee valuable assistance even when not working with the device.

2. Description of Unit

2.1 Layout of Test Device



The test device mainly consists of a basic frame, the guide columns and the load cross bar

The basic frame contains the bottom mounting for the rod specimen, consisting of a force measuring device for measuring the testing force and an attachment socket which can hold different pressure pieces for realizing various storage conditions.

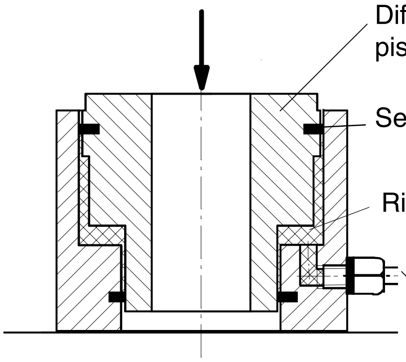
The height of the load cross bar can be adjusted along the guide columns and it can be clamped in position. This allows rod specimens with different buckling lengths to be examined.

The load cross bar features a load spindle for generating the test force. Using the load nut, the test force is applied to the rod specimen via guided thrust pieces. An axial mounting between the load nut and the thrust piece prevents torsional stresses from being applied to the rod specimen.

Two different thrust pieces are available for different storage conditions.

The device can be used both vertically as well as horizontally. The device is equipped with a base foot on one of the guide columns for horizontal set-up. The display instrument of the force measuring device can be turned 90° for easy readability.

2.1.1 Force Measurement



The test force is measured using a hydraulic force measuring device.

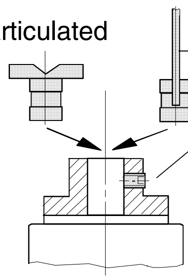
Here, the test force produces a pressure in a ring cylinder via a differential piston. This pressure is measured by a pressure gauge which acts as a display instrument.

The measuring path is very small due to the hydraulic transmission (max. 0.3 mm).

The display is well damped by a hydraulic throttle.

Disturbing influences cause by friction are prevented by direct support of the rod specimen on the force measuring cell.

2.1.2 Specimen Holders

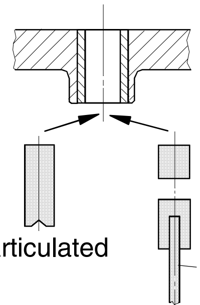


Bottom specimen holder

Two different mounting options are available:

- For articulated mounting
  Thrust piece with V notch for knife-edge mounting
- For clamped mounting
  A thrust piece which is firmly connected to the rod specimen

The thrust pieces are inserted in the attachment socket and are clamped firmly with a screw.



Top specimen holder

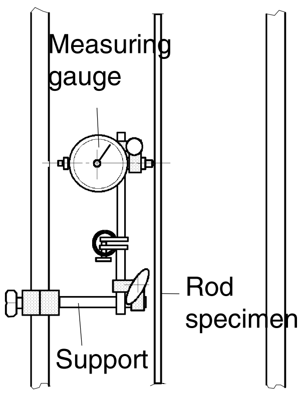
Two different mounting options are available:

- for articulated mounting
  Long thrust piece with V notch for knife-edged mounting

- For clamped mounting
  Short adapter and thrust piece firmly attached to the rod specimen

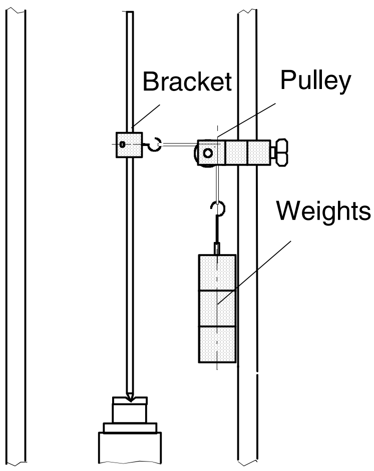
The thrust pieces are inserted into the guide bush of the load cross bar

2.1.3 Deformation Measurement



The measuring gauge for measuring the lateral deflection of the rod specimen is fasted to a guide column with the supplied support.

2.1.4 Lateral Load Device



The lateral load device can only be used when the test stand is in vertical position.

The lateral load device consists of a rope, a pulley, a bracket and a set of weights. The pulley is clamped to one of the guide columns. The bracket holds the rod specimen and is locked in place with a cotter pin.

A lateral force of 0-20 N can be produced in 5 N increments.

2.2 Rod Specimens

G.U.N.T. supplies two sets of rod specimens. The rod specimens have been dimensioned in accordance with the WP 120 test device, in order to best exploit its performance range.

The rod specimens contained in the standard set can be used to conduct tests on the influence of mountings, length and material.

The influences of eccentric mounting and different cross sectional shapes can be studied with the WP 120.01 accessories set.

Standard set WP 120				
No..:	Material	Diameter mm	Length mm	Mounting
S1	Tool steel 1.2842	20 x 4	350	knife-edge/knife-edge
S2	Tool steel 1.2842	20 x 4	500	knife-edge/knife-edge
S3	Tool steel 1.2842	20 x 4	600	knife-edge/knife-edge
S4	Tool steel 1.2842	20 x 4	650	knife-edge/knife-edge
S5	Tool steel 1.2842	20 x 4	700	knife-edge/knife-edge
S6	Tool steel 1.2842	20 x 4	650	clamped/knife-edge
S7	Tool steel 1.2842	20 x 4	650	clamped/knife-edge
S8	Alu. AlMgSi0.5 F22	25 x 6	600	knife-edge/knife-edge
S9	Brass CuZn40Pb2	25 x 6	600	knife-edge/knife-edge
S10	Copper E-Cu	25 x 6	600	knife-edge/knife-edge
S11	Fieberline	20 x 10	600	knife-edge/knife-edge

Accessories set WP 120.01				
No.:	Material	Diameter mm	Length mm	Mounting
SZ1	Alu. AlMgSi0.5 F22	25 x 6	500	knife-edge/knife-edge e=0mm
SZ2	Alu. AlMgSi0.5 F22	25 x 6	500	knife-edge/knife-edge e=1mm
SZ3	Alu. AlMgSi0.5 F22	25 x 6	500	knife-edge/knife-edge e=3mm
SZ4	Alu. AlMgSi0.5 F22	40 x 6	500	knife-edge/knife-edge
SZ5	Fieberline	25 x 10	700	knife-edge/knife-edge
SZ6	PVC	ø16 x 2	400	knife-edge/knife-edge
SZ7	PVC	ø20 x 1.5	400	knife-edge/knife-edge
SZ8	Alu. AlMgSi0.5 F22	20 x 10 x 2	700	knife-edge/knife-edge
SZ9	Alu. AlMgSi0.5 F22	ø15 x 2	700	knife-edge/knife-edge
SZ10	Alu. AlMgSi0.5 F22	ø14	700	knife-edge/knife-edge

2.3 Safety

DANGER!

The load cross arm can drop of the clamping screws are loosened!

A drop could damage parts of the testing machine located underneath the cross arm.

Carefully support the cross arm by hand when loosening the clamping screws!

Before removing a rod specimen make sure that the clamping screws are tightened securely!

Pay attention to the top thrust piece when removing the rod specimen!

The hazards mentioned do not apply when the test device is set up horizontally.

DANGER!

Caution when working with brittle materials!

The rod specimen could breaks suddenly in this case. Pieces of specimen could fly around and cause injuries!

This hazard is not posed with original G.U.N.T. rod specimens, since they are made of ductile material.

CAUTION

Do not overload device!

The maximum testing force is 2000 N.

Overloads can occur if attempts are made to force a loaded rod specimen in the direction opposite that of deflection.

2.4 Technical Data

Dimensions

Length:	620 mm
Width:	450 mm
Height:	1150 mm
Weight:	35 kg

Max. test force:	2000 N
Max. lateral load:	20 N
Max. lateral deflection:	±20 mm
Max. rod specimen length:	700 mm
Max. load spindle stroke:	10 mm
Rod specimen hole:	20 mm dia.

3 Theory

3.1 Derivation of Buckling Theory

Whereas the maximum load in the case of rods subjected to tensile forces is limited by the tensile strength of the material alone, rods subjected to pressure can fail long before the corresponding compressive strength has been attained. They bend suddenly, deflect laterally and buckle - even if the load is centered. The danger of buckling is much greater in the case of slender rods, i.e. long and relatively thin rods, than it is with short, stout rods. Sudden buckling is a characteristic of instability and, therefore, buckling under pressure is considered to be a stability problem.

First, the term stability should be explained more precisely. In mechanics, the state of a system is considered to be stable when it returns to its original state of equilibrium following a minor disturbance. In unstable systems on the other hand, a minute disturbance is all that is needed to consistently bring the system out of its original state of equilibrium.

There is yet another state of stability classified as indifferent. Here, the system can assume any number of states of equilibrium. These states of equilibrium are described in the illustrations

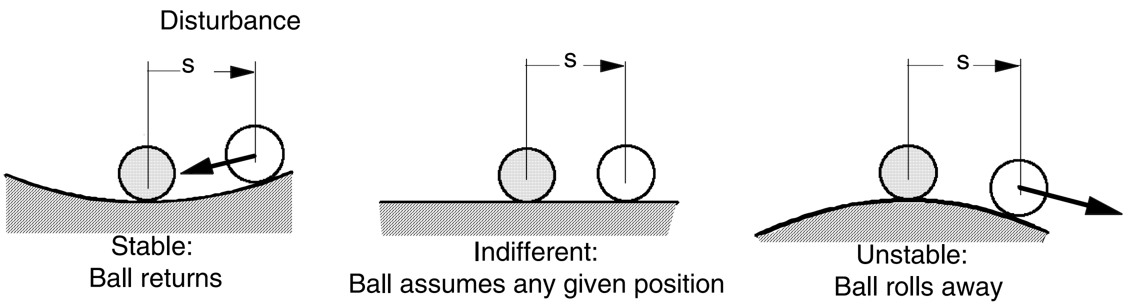


Fig. 3.1: States of stability using a ball as an example

3.1.1Simple Example for Studying Stability

The procedure used for calculating stability is explained using the example of an inverted pendulum.

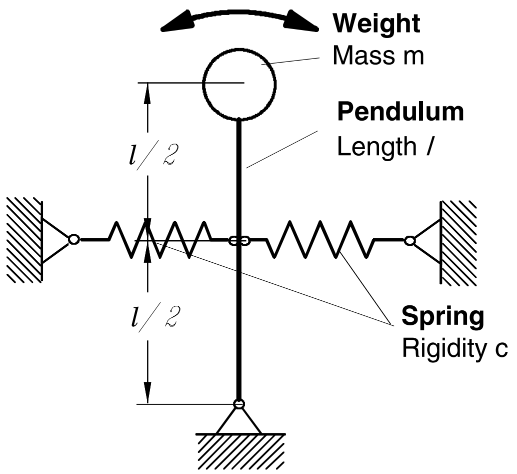


Fig. 3.2: Example for stability calculation: spring suspended inverted pendulum

The massless pendulum bears the weight on its top end. The pendulum is help in a vertical position by two pretightened springs. The question now is:

How great can the weight be before the pendulum becomes unstable and tips over? As mentioned at the beginning, the system is stable when it returns to its original state of equilibrium. To this end, the pendulum is slightly deflected and is observed to see it swings back into place. The pendulum is shown in deflected position in Figure 3.3 and all of the external forces acting upon it are denoted.

The deflection is shown in a greatly exaggerated manner for the sake of clarity. It should be very small, so that linear geometry can be reckoned with. The moment of equilibrium is established around the pivot point o for the deflected position.

$$\sum M_o = J\,\ddot{\varphi} = G\,f - F\,\frac{l}{2} \quad \text{mit} \quad G = m\,g$$

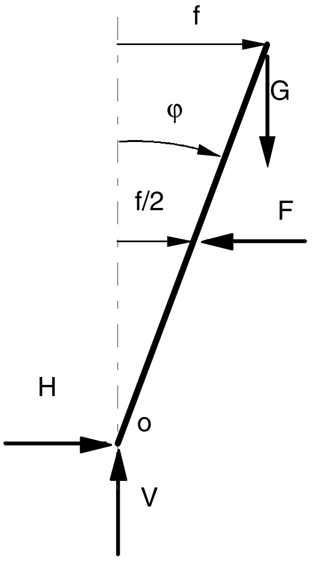


Fig. 3.3: Forces and deflection on the inverted pendulum in deflected position

The return spring force F is calculated from the spring path f /2 and the spring rigidity c.

$$F = \frac{f}{2} c$$

If the pendulum is to return to its original position, the angle acceleration must be negative. The pendulum accelerates to the left. Thus, the following is required with the consistently positive mass moment of inertia:

$$J \phi < 0$$

Therefore, the following applies to the right side of the moment of equilibrium:

$$0 > G f - \frac{f \ / \ c}{4}$$

As a result, the following maximum weight G is required:

$$G < \frac{c \ /}{4}$$

If the weight is greater, the pendulum will be unstable. It will accelerate in the positive direction to the right and tip over.

The borderline case between stability and instability is $G = c\,/\,/4$. Here, the state of equilibrium is indifferent, the system can theoretically assume any position. Due to the simplifications made at the beginning, this only applies to small deflections near the zero position. In this case, the moment of equilibrium will have the following appearance:

$$0 = f\,(\,G - \frac{c \ /}{4}\,)$$

As can be seen, the formula can only be fulfilled for random deflections if the expression in brackets is eliminated.

The critical load G_{crit}, up to which the arrangement is stable, can be calculated from this.

$$G_{crit} = \frac{c\ /}{4}$$

3.1.2 Buckling of a Bent Bar

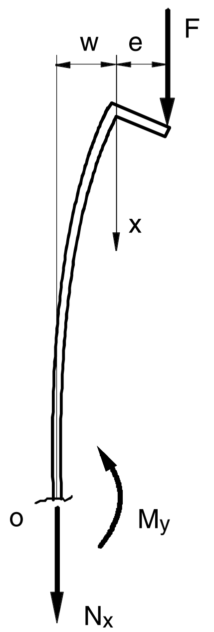


Fig. 3.4: Forces and moments on an eccentrically loaded bent bar

The procedure demonstrated with the inverted pendulum will now be applied to a bent bar which is subjected to an axial compressive force. It serves as a model for the rod specimens used in the test.

The equilibrium for the deformed state is formulated here, as well (theory 2nd order). In addition to deformation the load is to be applied with a slight eccentricity. In reality it is never possible to avoid this eccentricity and the model capable of accurately representing the influence of an eccentric load.

The moment of equilibrium around the point of intersection 0 results in:

$$\sum M_o = 0 = M_y - F\ (\ w + e\ )$$

The following relationship exists between the moment of bending My and the deflection w:

$$M_y = E\ I_y\ w\,''$$

Here, $w\,''$ is the 2nd derivation of the deflection according to the path or the curvature of the bending line. If M_y is used as the moment of equilibrium, the non-homogeneous differential equation 2nd order results.

$$w'' - \frac{F}{E\,I_y}\,(w + e) = 0$$

By simple substitution $u = w + e$ and $u'' = w''$ it can be converted into a homogeneous differential equation with the abbreviation $\kappa = \sqrt{\frac{F}{E\,I_y}}$

$$u'' - \kappa^2\,u = 0$$

The general solution is

$$u(x) = C_1\,e^{\kappa x} + C_2\,e^{-\kappa x}$$

or

$$w(x) = C_1\,e^{\kappa x} + C_2\,e^{-\kappa x} - e$$

The following can be written using the Euler formula.

$$w(x) = A_1 \sin \kappa x + A_2 \cos \kappa x - e$$

with $A_1 = i\,(C_1 + C_2)$ and $A_2 = (C_1 + C_2)$

The yet unknown constants A_1 and A_2 are determined by applying the boundary conditions from the bilateral mounting of the beam $w(0) = 0$ and $w(l) = 0$. This results in the following constants.

$$A_1 = e\,\tan\frac{\kappa\,l}{2}\quad,\quad A_2 = ew(0)=0$$

Thus, the equation for deflection is:

$$w(x) = e\left(\frac{\cos \kappa\,(l/2 - x)}{\cos \kappa\,l/2} - 1\right)$$

The following results for maximum deflection in the middle of the bar $x = l/2$

$$w(l/2) = e\left(\frac{1}{\cos \kappa\,l/2} - 1\right)$$

For $\kappa\, l/2 = \pi/2$ the expression in brackets exceeds all limits. Even with a very small eccentricity e the deflection w is infinitely large. There is instability. The accompanying critical compressive force can be determined easily from the relation:

$$\kappa\, l/2 = \frac{l\,\pi}{4}\sqrt{\frac{F}{E\,I_y}}$$

$$F_{krit} = F = \frac{\pi^2\, E\, I_y}{l^2}$$

It is called the critical load or Euler load. The bar buckles if this load is exceeded.

The course of the deflection over compressive force is demonstrated for various eccentricities in the following diagram. Especially when the eccentricity e=0, deflection occurs suddenly. The bar buckles suddenly. As a rule, the bar cannot transfer any forces greater than the critical load.

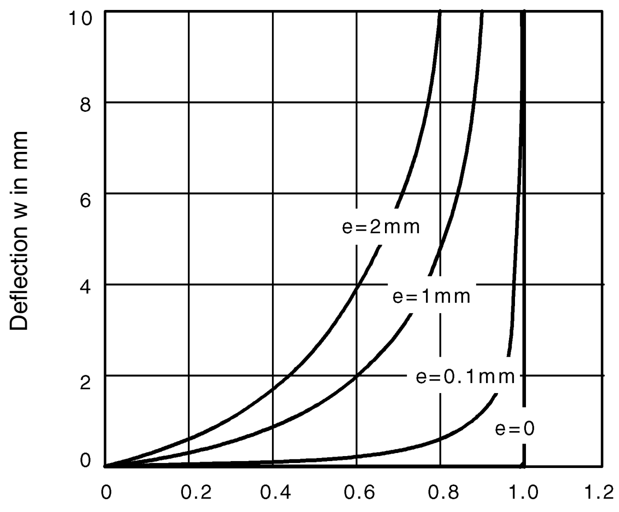


Fig. 3.5: Deflection of the bar for various eccentricities

In practical applications e > 0 will be measured since the curves always have some degree of eccentricity. They only approach the asymptote at F_{crit}. The value at which the force does not noticeably increase with increasing deformation is taken as the value for the critical load. As opposed to the eccentricity e = 0, at which the direction of buckling cannot be predicted, in the case of eccentricity it is predetermined.

3.1.3 Influence of Mounting Conditions

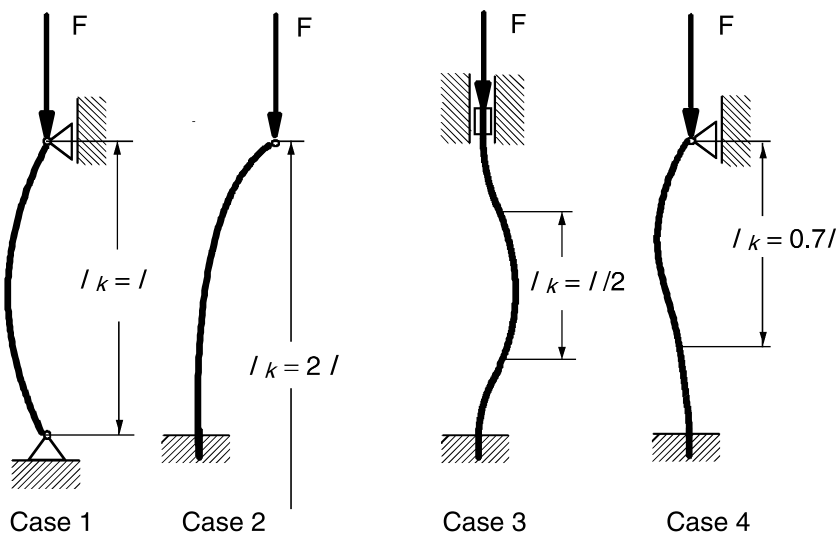


Fig. 3.6: Euler cases of buckling

Up to now only a rod with articulated mountings at both ends have been dealt with. From now on it will be referred to as buckling case 1. Other mounting conditions can be attributed to this reference case by introducing a so-called buckling length.

The definition of buckling length can be explained graphically in buckling cases 2 and 3. In buckling case 2 the buckling rod is practically half the length of the one in buckling case 1. Thus, the buckling length l_K is equivalent to twice the rid length. In buckling case 3 the section between the turning points in the curve is equivalent to bending case 1. Consequently, half the length of the rod can be taken as the buckling length here. In buckling case 4 the buckling length cannot be derived from simple analogous observations. Here it is $l_k = 0.7l$.

Except for buckling case 2, all of the buckling cases can be studied experimentally with the WP 120 buckling test device.

3.1.4 Influence of Lateral Loads

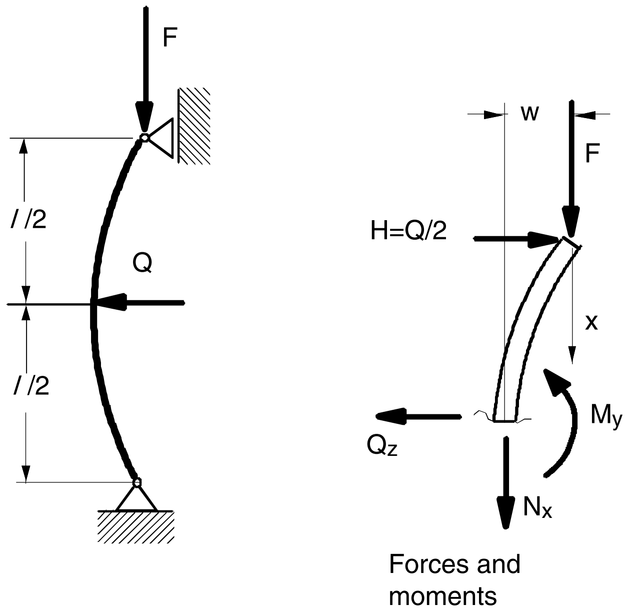


Fig. 3.7: Buckling and lateral force influence

Often, a compressed rod is also subjected to aggressive lateral forces. Now it is possible to study how lateral loads effect the buckling behavior of a rod. The rod mounted in articulated mountings at both ends from buckling case 1 will again be used as an example. This time, however, it will be without the introduction of eccentric forces.

The moment of equilibrium around the point of intersection produces:

$$\sum M_o = 0 = M_y - F\,w - \frac{Q}{2}\,x$$

The bending differential equation $M_y = E\,I_y\,w''$ results in

$$w'' - \frac{F}{E\,I_y}\,w = \frac{Q}{2\,E\,I_y}\,x$$

After introducing the abbreviation $\alpha = Q/2\,F$ and $\kappa = \sqrt{\frac{F}{E\,I_y}}$

we arrive at the non-homogeneous differential equation for deflection.

$$w'' - \kappa^2\,w = \kappa^2\,\alpha\,x$$

The solution comprises a part for the homogeneous differential equation and a particular solution.

$$w(x) = w_h(x) + w_p(x)$$

The homogeneous part has the same structure as was the case with an eccentric rod.

$$w_h(x) = C_1\, e^{\kappa x} + C_2\, e^{-\kappa x}$$

The particular solution is determined by varying the constants. Here, functions dependent on x replace the constants

$$w_p(x) = C_3(x)\ e^{\kappa x} + C_4(x)\ e^{-\kappa x} - e$$

After extensive calculation the following equations are found for the functions $C_3(x)$ and $C_4(x)$

$$C_3 = -\frac{\alpha}{2}\left(\frac{1}{\kappa} + x\right)e^{-\kappa x} \quad \text{und} \quad C_4 = \frac{\alpha}{2}\left(\frac{1}{\kappa} - x\right)e^{\kappa x}$$

Applying the constants in the solution produces

$$w(x) = C_1\, e^{\kappa x} + C_2\, e^{-\kappa x} - \alpha\, x$$

The following form results if the Euler formula is used

$$w(x) = A_1 \sin \kappa x + A_2 \cos \kappa x - \alpha\, x$$

The yet unknown constants A_1 and A_2 are determined by applying the boundary conditions of rod with articulated mountings on both sides $w(0) = 0$ and $w(l) = 0$

$$A_1 = \frac{\alpha\, l}{\sin \kappa\, l} \quad \text{and} \quad A_2 = 0$$

The following results for deflection

$$w(x) = \alpha\left(l\ \frac{\sin \kappa x}{\sin \kappa l} - x\right)$$

The equation below applies to the maximum deflection at $x = l/2$ and $\alpha = Q/2F$

$$w(l/2) = \frac{Q\ l}{4\,F}\left(\frac{1}{\cos\frac{\kappa\, l}{2}} - 1\right)$$

Here the expression in brackets tends towards ∞ if

$$\frac{\kappa\, l}{2} = \frac{\pi}{2}$$

In this case there is instability. The expression $\frac{Q\ l}{4\,F}$ has the same function here as the eccentricity e had in the preceding section.

The buckling behavior in the case of lateral stress can be compared to that of the rod subjected to eccentric compressive force.

3.2 Applying the Buckling Theory

If a rod is subjected to longitudinal forces, as implied in the sketch, it can fail in two ways. On the one hand, it can be plasticized and flattened if its admissible compressive strain is exceeded (see Fig. 3.8). On the other hand, it is possible that it will suddenly shift to one side and buckle before attaining the admissible compressive strain. This effect is called buckling. The shape of the rod is the factor which determines which of the two cases of failure will occur. A slender, thin rod is more likely to buckle than a thick, stout rod.

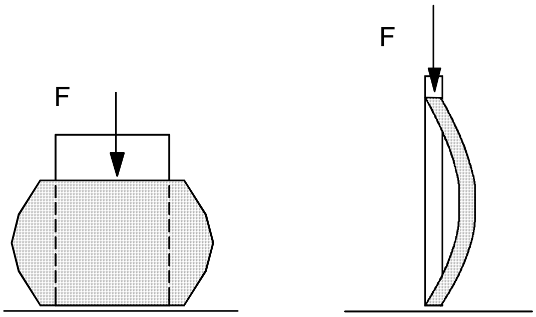


Fig. 3.8 Stout and slender rod under compressive force

3.2.1 Euler Formula

Buckling occurs suddenly and without warning when a certain limit load is attained. It is therefore an extremely dangerous type of failure, which must be avoided by all means. As soon as a rod begins to buckle, it will become deformed to the point of total destruction. This is typical unstable behavior. Buckling is a stability problem. The critical limit load F_{krit},above which buckling can occur, is dependent on both the slenderness of the rod, i.e. influence of length and diameter, and the material used. In order to define slenderness the slenderness ratio λ will be introduced here.

$$\lambda = \frac{l_k}{i}$$

In this case l_k is the characteristic length of the rod. It takes both the actual length of the rod and the mounting conditions into consideration.

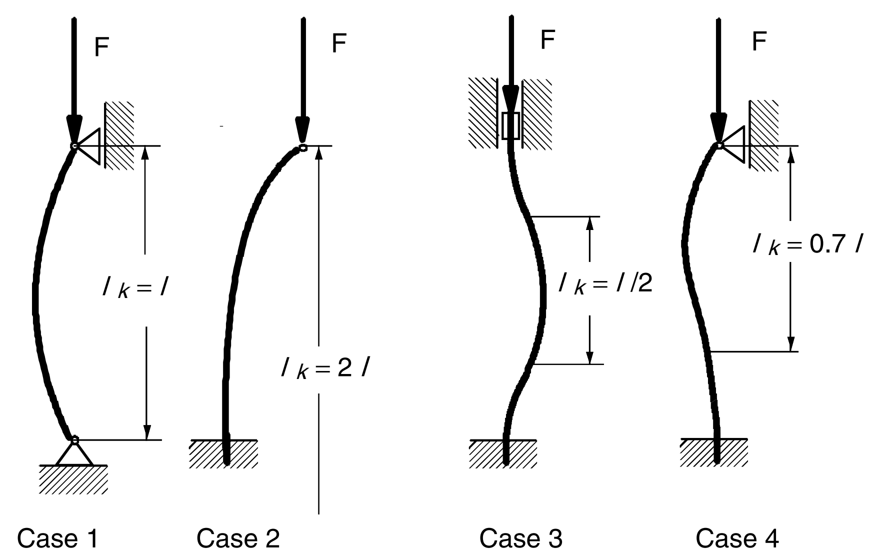


Fig. 3.9: Euler cases of buckling

For example, clamping the ends of the odds causes rigidity. The buckling length decisive for slenderness is shorter than the actual length of the rod. Altogether, a differentiation is made between four types of mountings, each having a different buckling length.

The influence of diameter in the slenderness ratio is expressed by the inertia radius i. It is calculated using the minimum geometrical moment of inertia I_y and the cross-sectional area A.

$$i = \sqrt{I_y / A}$$

The influence of material is taken into consideration by the longitudinal rigidity of the rod EA. Here, E is the modulus of elasticity of the respective material and A is the cross-sectional area.

The influence of various factors on the critical load are summarized in the so-called "Euler formula":

$$F_{crit} = \pi^2 \frac{E\,A}{\lambda^2}$$

or expressed in a different form:

$$F_{crit} = \pi^2 \frac{E\,I_y}{l^2}$$

3.2.2 Influencing Factors

Below the influence of various characteristic values such as the E modulus, geometric moment of inertia, length and the type of mounting on buckling behavior will be examined using the Euler formula.

E modulus

The E modulus is a measure of the rigidity of a material. A stiff material is sensible for high resistance to buckling. Since strength has no influence on buckling, materials with as high an E modulus as possible should be used. For example, in the case of buckling strength a simple constructive steel St37 with a tensile strength of only 370 N/mm should be given preference over a high strength titanium allow TiAl6Zr5 with 1270 N/mm.

Whereas the constructive steel has an E modulus of 210 kN/mm, the titanium alloy only features 105 kN/mm.

Geometric moment of inertia

The geometric moment of inertia indicates the resistance against deflection resulting from the cross-sectional shape of the rod. Since a rod buckles in the direction of least resistance, the minimum geometric moment of inertia is the decisive factor. The table contains the geometric moment of inertia for several cross-sectional shapes. Here, hollow sections with small wall thicknesses are more favorable at the same weight as solid cross sections. For example, the ratio of the geometric moment of inertia of a thin tube (dia. 52 x 2) to that if a solid rod (dia. 20 mm) with the same cross-sectional area is 12.5 to 1. In addition, double symmetrical cross-sections such as tubes or quadratic cross sections should be used since their geometric moment of inertia is the same in every direction.

Buckling length

The length of the rod as well as the type of mounting determine the buckling length l_K .The influence of the length is quadratic. At twice the length the admissible load is only one-fourth the original value.

3.2.3 Tensions in Buckling Rod

In order to determine whether a rod has failed due to exceeding the admissible compressive strain or by buckling, the normal compressive strain in the rod, which is part of the critical load, must be calculated.

$$\sigma_k = \frac{F_k}{A} = \pi^2 \frac{E}{\lambda^2}$$

If this normal compressive strain is lower than the admissible compressive strain, the rod will fail due to buckling. If the admissible compressive strain is used as the normal compressive strain, the critical slenderness ratio λ_{crit} at which buckling occurs can be calculated.

$$\lambda_{crit} = \sqrt{\pi^2 E / \sigma_p}$$

For constructive steel St37 with σ_p =192 N/mm the λ_{crit} = 104.

Above λ_{crit} buckling according to Euler can be expected. The buckling strain curve can be seen in Diagram 3.10.

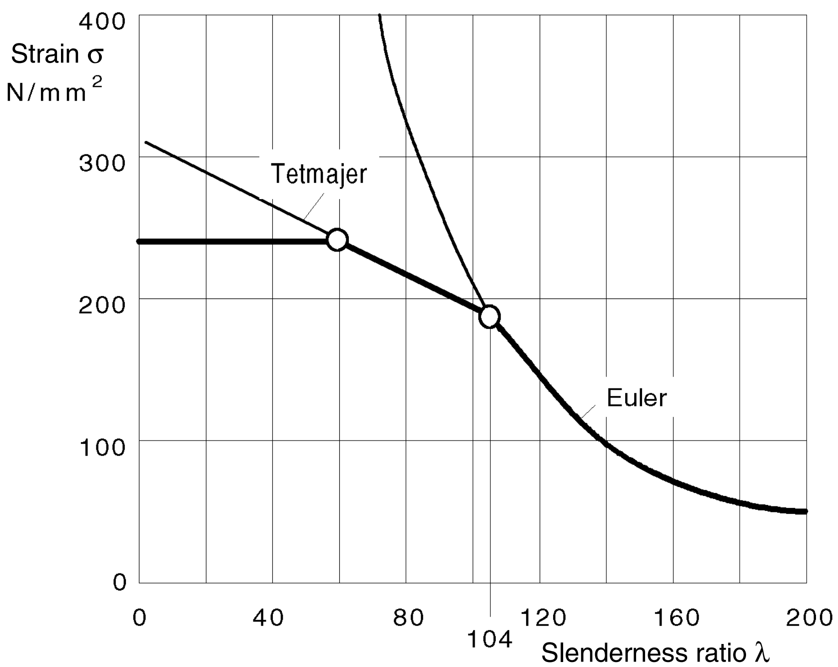
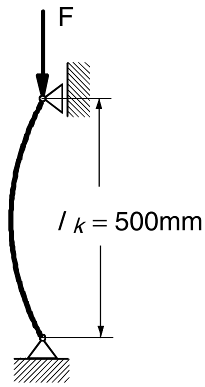


Fig. 3.10: Buckling strain σ as a factor of slenderness ratio λ

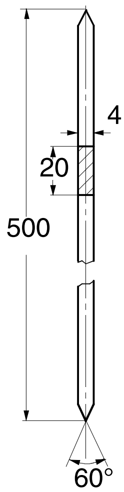
4 Tests

4.1 Introductory Test



In this test the operation of the WP 120 buckling test device and how to conduct a buckling test will be demonstrated.

A rod with articulated mounting at both ends according to Euler case 1 is slowly subjected to an axial force. Above a certain load it will buckle laterally. In this case the buckling (deformation) of the rod specimen will be measured in the middle of the rod and recorded in a table (appendix) along with the accompanying force. A force/deformation graph will be developed using these measured values. The results of the test should be compared with the buckling theory values.



The S2 rod specimen made of flat steel, dimensions 20 mm x 4 mm x 500 mm should be used. It has shaped edges at both ends which sit in corresponding V notched of the testing machine thrust pieces to form an ideal articulated mounting.

4.1.1 Estimation of Buckling Force and Deformation

It is expedient to calculate the expected buckling force prior to conducting the test. This is especially true with regard to rod specimens from other manufacturers with unknown behavior.

The buckling force can be determined according to the following formula (Euler formula).

$$F_{krit} = \frac{\pi^2 E I_y}{l_k^2}$$

The modulus of elasticity E for steel is 210000 N/mm. The geometric moment of inertia Iy is calculated as follows for a square cross section:

$$I_y = \frac{b\,h^3}{12} = \frac{20 \cdot 4^3}{12} = 106.6\ mm^4$$

The length of the rod $l = 500$ mm is taken as the buckling length l_k for Euler case 1. This results in the theoretical buckling force:

$$F_{krit} = \frac{\pi^2 E I_y}{l_k^2} = \frac{\pi^2 \cdot 210000 \cdot 106.6}{500^2} = 883\ N$$

Care should be taken not to exceed the elasticity limit during the buckling test, since the rod will otherwise be permanently bent and rendered unusable. The max. admissible deflection is reached when the combination of bending strain and compressive strain exceeds the admissible strain of the material at the elasticity limit σ_p . For the specimen steel σ_p = 300 N/mm^2. The corresponding deflection can be calculated as follows:

$$f_{zul} = \frac{(\sigma_p - F_k / A)\, I_y}{F_k\ z_{max}}$$

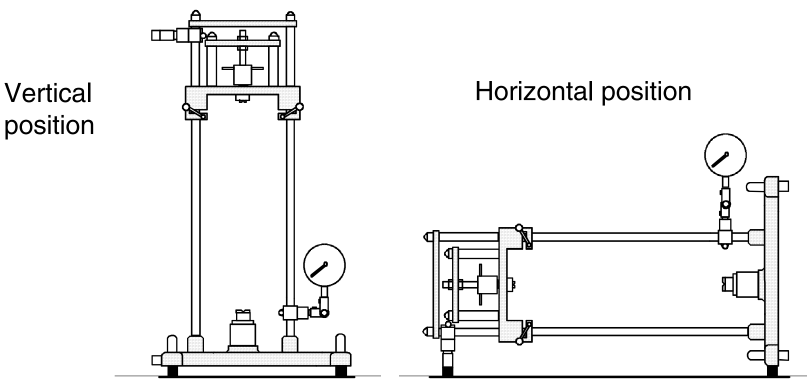
Where $A = 20 \cdot 4 = 80\ mm^2$, $F_k = 883\ N$ and $z_{max} = 2$ mm the following results for the deflection in the middle of the rod:

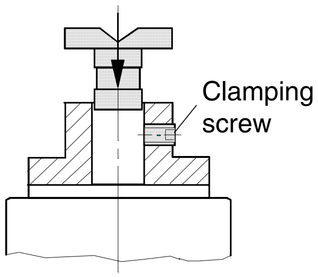
$$f_{max} = \frac{(300 - 883/80) \cdot 106.6}{883 \cdot 2} = 17.44\ mm$$

With a safety factor of 3, $17.44\ mm/3 = 5.81$ mm $\approx$6mm should not be exceeded! The deflection should be read off the measuring gauge during the test.

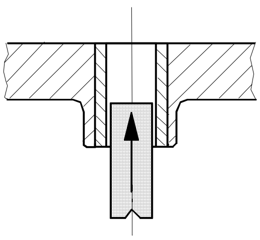
4.1.2 Conducting the Test

- Set up the test device in vertical or horizontal position. The force gauge can be turned 90 for this purpose

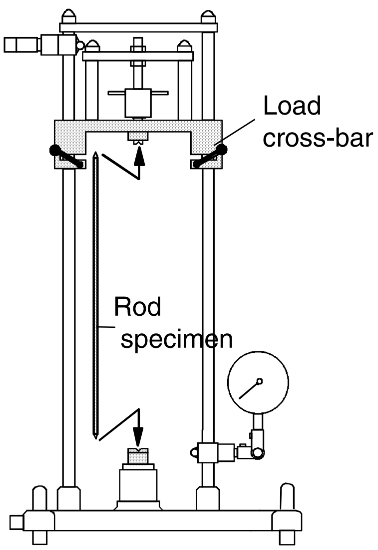




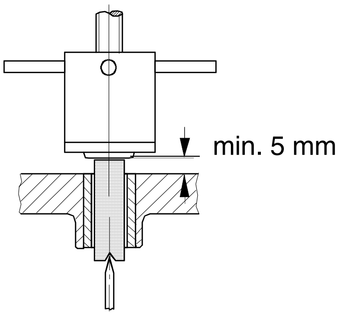
- Insert thrust piece with V notch into attachment socket and fasten with clamping screw



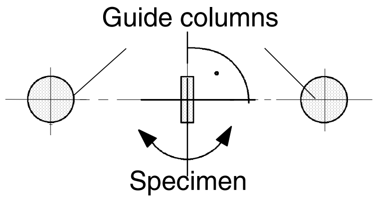
- Insert long thrust piece with V notch into the guide bush of the load cross-bar and hold it firmly



- Insert the S2 rod specimen with edges in the V notch

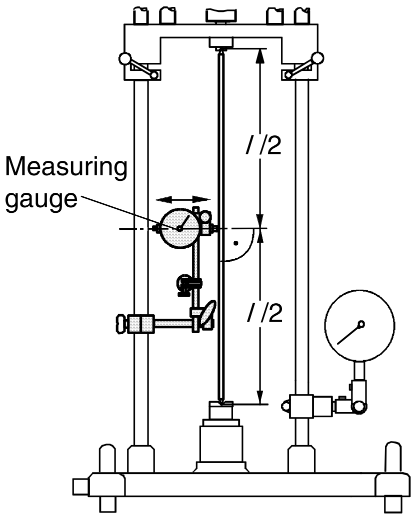


- The load cross-bar must be clamped on the guide column in such a manner that there is still approx. 5 mm for the top thrust piece to move.



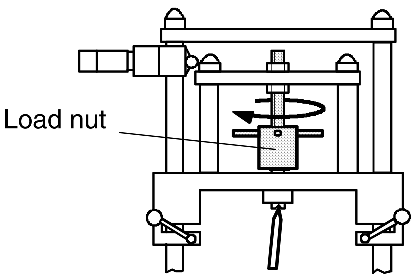
- Align the rod specimen in such a manner that its buckling direction points in the direction of the lateral guide columns. Here, the edges must be perpendicular to the load cross-bar.

- Pretighten the rod specimen with low, non-measurable force.



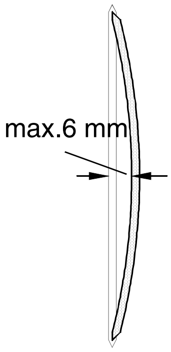
- Align the measuring gauge to the middle of the rod specimen using the support clamps. The measuring gauge must be set at a right angle to the direction of buckling.

- Pretighten the measuring gauge to 10 mm deflection with the adjustable support.

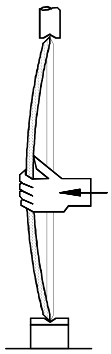


- Slowly subject the rod specimen load using the load nut.

- Read the deflection from the measuring gauge. Read and record the deflection every 0.25 mm up to 1 mm.



- Above 1 mm deflection, it suffices to record the deflection and force every 0.5 mm.
- **Caution!** Never deflect more than max. 6 mm, since there is a risk or plastic deformation and damage to the rod specimen.
- The test can be concluded when the force does not change, despite an increasing load (in the case of rod specimen S2 this as at approx. 4 mm).
- Slowly remove the tension from the rod specimen.
- As a control measure, repeat the test with the opposite buckling direction.



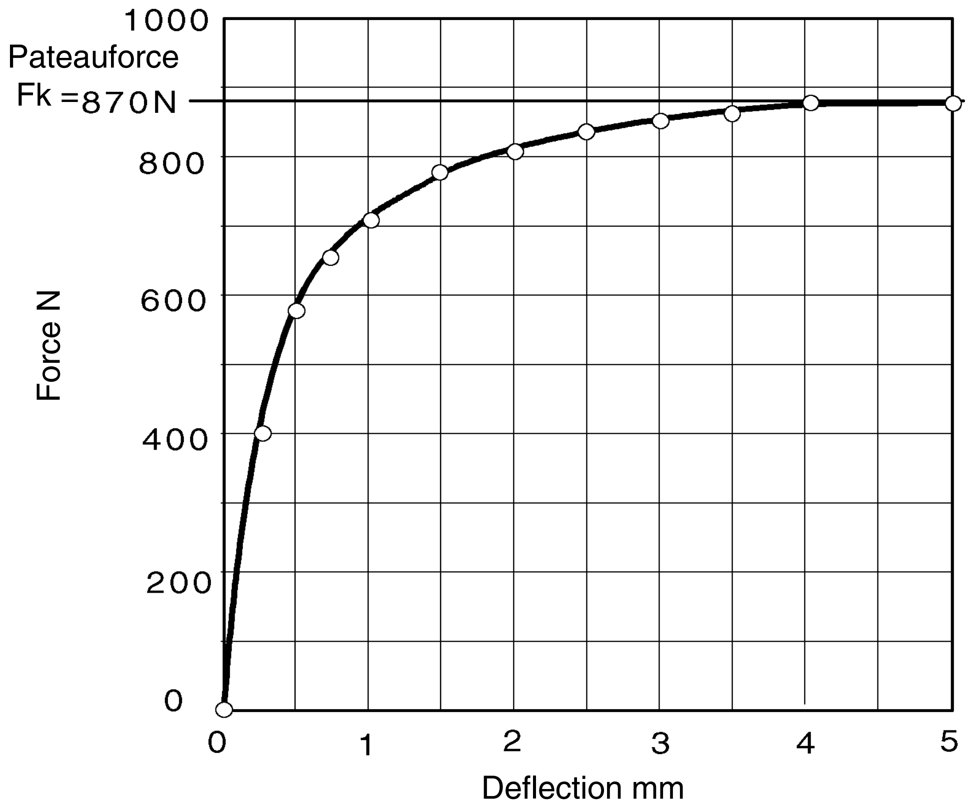
- To do this, set the buckling direction by initially guiding the rod by hand.
- It is not necessary to record the force and the deflection here.
- Increase load until the force no longer changes.
- Compare the buckling loads in both buckling directions.
- If the deviation is more than 10%, the rod could be strongly deformed. Then try to straighten the rod specimen. If this is not successful, replace the rod specimen.

4.1.3 Test Evaluation

- Present the valued recorded on the work sheet graphically by entering them in the graph in the bottom half of this page

Rod specimen: S2 steel
Length: 500 mm
Geometrical moment of inertia: 106.6 mm^4
Modulus of elasticity: 210000 N/mm^2

Deflection in mm	0	0.25	0.5	0.75	1.0	1.5	2.0	2.5	3.0	3.5	4.0	5.0		
Force N	0	400	570	650	710	780	810	840	850	860	870	870		

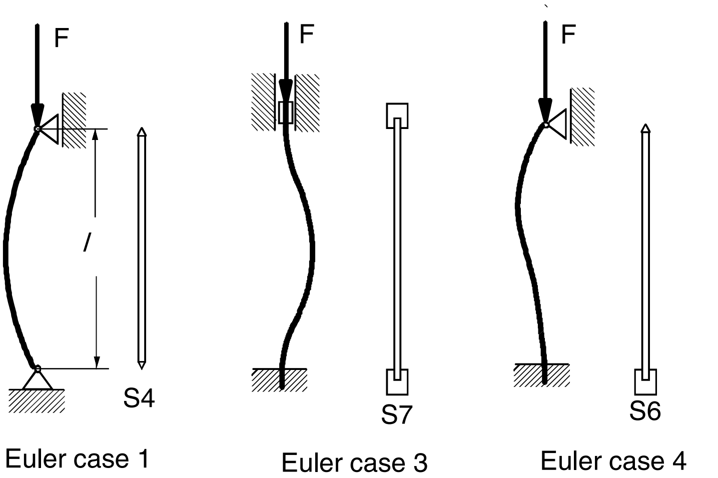


The plateau force can be clearly recognized. Here, no increase in force can be detected with increasing deflection. The critical load is equivalent to this maximum force and is F_k = 870 N for rod specimen S2.

A comparison with the theoretical value of 883 N shows a good correlation with regard to measuring accuracy (only 1.5% deviation).

4.2 Mounting Influence Test

This test demonstrates the influence of the mounting on buckling behavior. Euler cases 1, 3 and 4 are compared.



Rod specimens S4, S6 and S7 are used.

- Rod specimen S4 has edges at both ends for an articulated mounting on both sides (Euler case 1).
- Rod specimen S7 has thrust pieces at both ends for double ended clamping (Euler case 3).
- Rod specimen S6 has an edge at both sides and a firmly attached thrust piece at the other ends for single-ended clamping (Euler case 4).

The test results should be compared with the buckling theory values.

Test preparation

The expected buckling force for the various Euler cases can be calculated using the following equations:

- Euler case 1:

$$F_{krit} = \frac{\pi^2 E I_y}{\mathbf{l}^{\mathbf{2}}}$$

- Euler case 3:

$$F_{krit} = \frac{\mathbf{4}\,\pi^2 E I_y}{\mathbf{l}^{\mathbf{2}}}$$

- Euler case 4:

$$F_{krit} = \frac{\pi^2 E I_y}{(\mathbf{0.7\,l})^2}$$

The modulus of elasticity E for steel is 210000 N/mm. The geometric moment of inertia I_y for rectangular cross sections is:

$$I_y = \frac{b\,h^3}{12} = \frac{20 \cdot 4^3}{12} = 106.6\ \text{mm}^2$$

Thus, the theoretical buckling force

- For Euler case 1:

$$F_{krit} = \frac{\pi^2 E I_y}{l^2} = \frac{\pi^2 \cdot 210000 \cdot 106.6}{650^2} = 523\ \text{N}$$

- For Euler case 3:

$$F_{krit} = \frac{4\,\pi^2 E I_y}{l^2} = \frac{4\,\pi^2 \cdot 210000 \cdot 106.6}{650^2} = 2091\ \text{N}$$

- For Euler case 4:

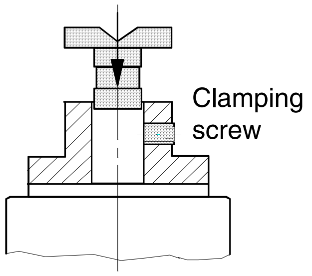
$$F_{krit} = \frac{\pi^2 E I_y}{(0.7\,l)^2} = \frac{\pi^2 \cdot 210000 \cdot 106.6}{(0.7\ \ 650)^2} = 1067\ \text{N}$$

The buckling forces must, therefore, must be in a 1 to 4 to 2.04 ratio.

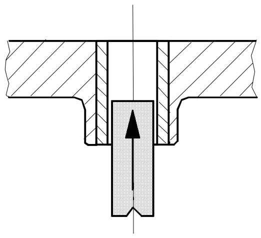
Conducting the test

- Set-up the test device in vertical or horizontal position. The force gauge can be turned 90 for this purpose.

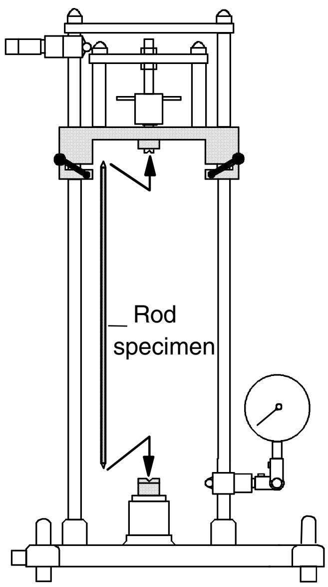
Euler case 1: articulated/articulated



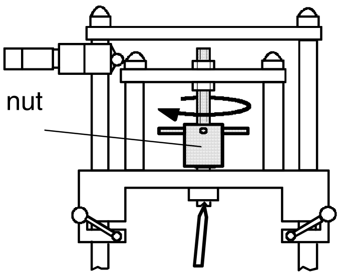
- Insert thrust piece with V notch in the bottom specimen holder and fasten with clamping screw.



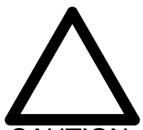
- Insert thrust piece with V notch in the guide bush of the load cross bar and hold firmly

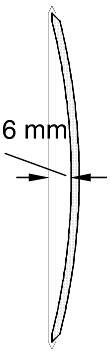


- Insert rod specimen S4 with edges in V notch.
- Alignment of the rod specimen and adjustment of the measuring gauge as described in Section 4.1.2.



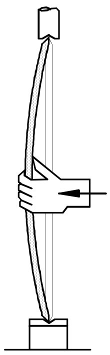
- Slowly subject rod specimen to load using the load nut.





- Caution! Never deflect more than max. 6 mm, since there is a risk or plastic deformation and damage to the rod specimen.

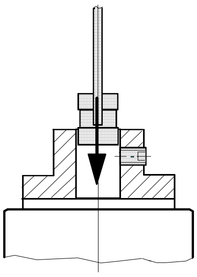
- The test can be concluded when the force does not change, despite an increasing load.
- Slowly remove the tension from the rod specimen.



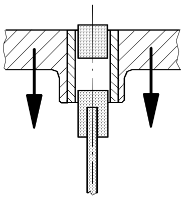
- As a control measure, repeat the test with the opposite buckling direction.
- To do this, set the buckling direction by initially guiding the rod by hand.

- Compare the buckling loads in both buckling directions. In the case of a non-deformed rod specimen the deviation should not be more than 10%.

Euler case 3: clamped/clamped



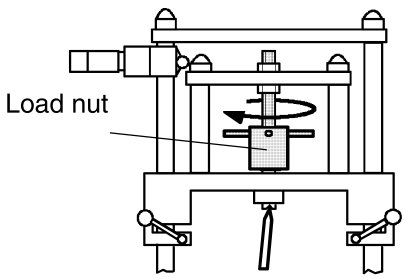
- Insert rod specimen S7 with thrust piece in the bottom specimen holder and fasten with clamping screw.



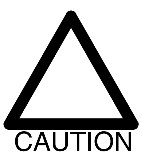
- Insert short thrust piece in the guide bush of the load cross bar and hold firmly.

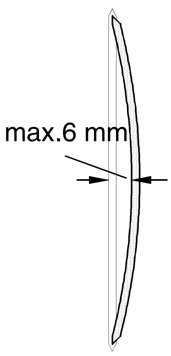
- Insert rod specimen S7 along with the top thrust piece in the guide bush of the load cross bar.

- Align the rod specimen and adjust the measuring gauge as described in Section 4.1.2.



- Slowly subject the rod specimen to load with load nut.

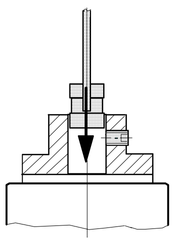




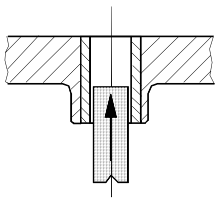
- Caution! Never deflect more than max. 6 mm, since there is a risk or plastic deformation and damage to the rod specimen.

- The test can be concluded when the force does not change, despite an increasing load.
- Slowly remove the tension from the rod specimen.
- As a control measure, repeat the test with the opposite buckling direction.

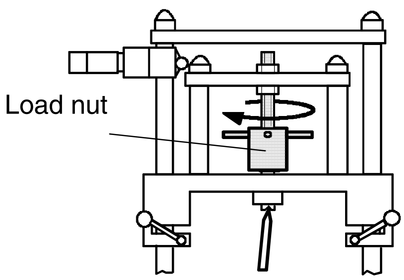
Euler case 4: clamped/articulated



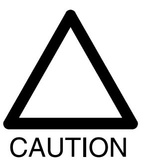
- Insert rod specimen S6 and thrust piece in the bottom specimen holder and fasten with clamping screw.

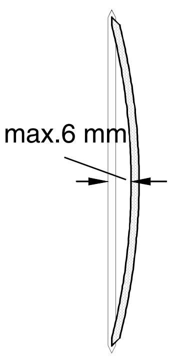


- Insert long thrust piece with V notch in the guide bush of the load cross bar and hold firmly.
- Align the rod specimen and adjust the measuring gauge as described in Section 4.1.2.



- Slowly subject the rod specimen to load with load nut.



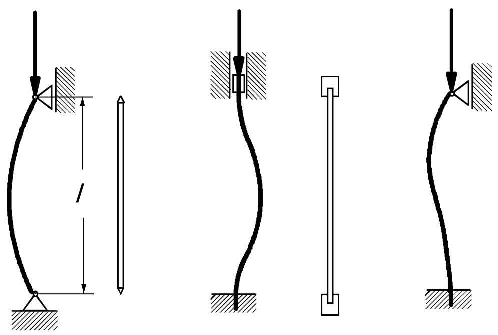


- Caution! Never deflect more than max. 6 mm, since there is a risk or plastic deformation and damage to the rod specimen.

- The test can be concluded when the force does not change, despite an increasing load.
- Slowly remove the tension from the rod specimen.
- As a control measure, repeat the test with the opposite buckling direction.

Test evaluation

The mean value from the two buckling directions has been measured (theoretical value in brackets):



Euler case 1:
470 N.
(610 N)

Euler case 3:
1750 N.
(1793 N)

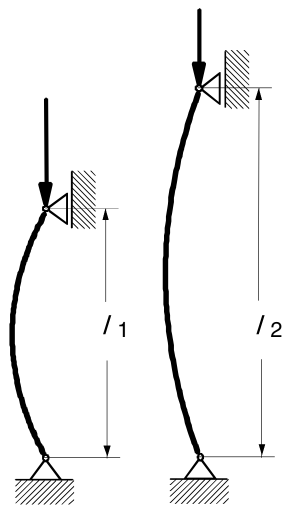
Euler case 4:
970 N.
(878 N)

Whereas the value for Euler cases 3 and 4 correlate well with the theoretical values, the test result for Euler case 1 deviates by 23%. This great deviation was traced to a slight deformation of the rod specimen and to the measuring inaccuracy of the force measuring device (25N), which is more significant at lower forces.

The buckling forces are in a 1 to 3.72 to 2.06 ratio.

The max. relative deviation from the theoretical ratio is 6.6%.

4.3 The Influence of Length



This test demonstrates the influence of rod length on buckling behavior. Rod specimens S1, S2, S3 and S5 made of steel are used. The rod specimens have edges at both ends for double-ended articulated mounting (Euler case 1).

The test results should be compared with the buckling theory values.

Test preparation

The expected buckling force for various rod lengths **l** can be calculated in accordance with the following formula for Euler case 1.

$$F_{krit} = \frac{\pi^2 E I_y}{\mathbf{l}^2}$$

This results in the theoretical buckling force

- for **l** = 350 mm

$$F_{krit} = \frac{\pi^2 E I_y}{\mathbf{l}^2} = \frac{\pi^2 \cdot 210000 \cdot 106.6}{350^2} = 1793 \text{ N}$$

- for **l** = 500 mm

$$F_{krit} = \frac{\pi^2 E I_y}{\mathbf{l}^2} = \frac{\pi^2 \cdot 210000 \cdot 106.6}{500^2} = 878 \text{ N}$$

- for **l** = 600 mm

$$F_{krit} = \frac{\pi^2 E I_y}{\mathbf{l}^2} = \frac{\pi^2 \cdot 210000 \cdot 106.6}{600^2} = 610 \text{ N}$$

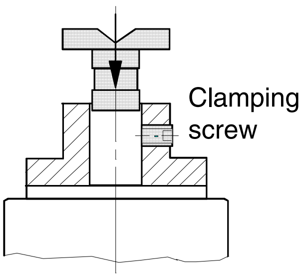
- for **l** = 700 mm

$$F_{krit} = \frac{\pi^2 \, E \, I_y}{\mathbf{l}^{\,\mathbf{2}}} = \frac{\pi^2 \cdot 210000 \cdot 106.6}{700^2} = 448 \text{ N}$$

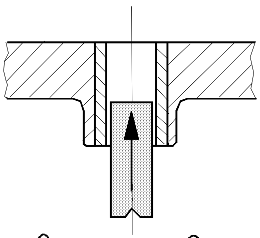
The buckling loads must be in a 4 to 2.19 to 1.33 to 1 ratio.

Conducting the test

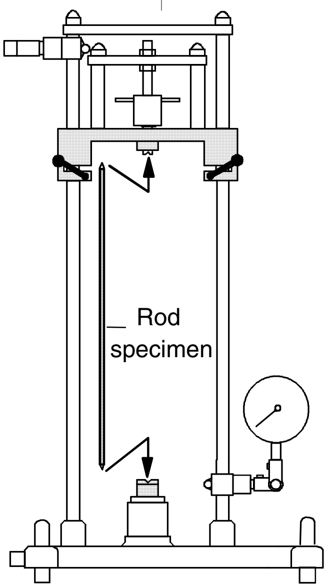
- Set-up the test device in vertical or horizontal position. The force gauge can be turned 90 for this purpose.



- Insert the thrust piece with a V notch in the bottom specimen holder and fasten with clamping screw.

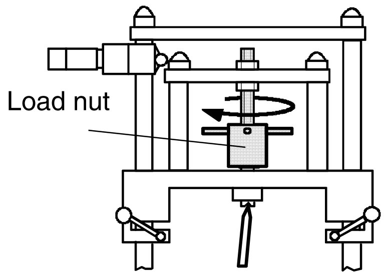


- Insert the long thrust piece with V notch in the guide bush of the load cross bar and hold firmly.

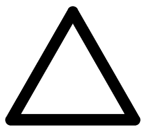


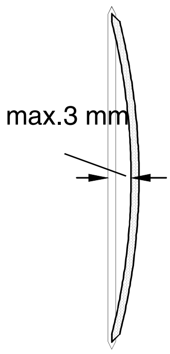
- Insert rod specimen S1 (350 mm) with edges in the V notches.

- Align the rod specimen and adjust the measuring gauge as described in Section 4.1.2.



- Slowly subject the rod specimen to load with load nut.


CAUTION



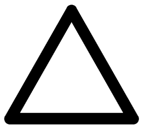
- Caution! Never deflect more than max. 3 mm, since there is a risk or plastic deformation and damage to the rod specimen.

- The test can be concluded when the force does not change, despite an increasing load.

- Slowly remove the tension from the rod specimen.

- As a control measure, repeat the test with the opposite buckling direction.

- Repeat test with rod specimen S2 (500 mm), S3 (600 mm) and S5 (700 mm).


CAUTION

The maximum deflection for these specimens may only be 5 mm (S2) and 6 mm (S3 and S5).

Test evaluation

The mean value from the two buckling directions has been measured (theoretical value in brackets):

- Length 350 mm (S1): 1660 N (1793 N).
- Length 500 mm (S2): 890 N (878 N).
- Length 600 mm (S3): 540 N (610 N).
- Length 700 mm (S4): 405 N (448 N).

The buckling loads are in a 4.09 to 2.19 to 1.33 to 1 ratio. Doubling the length reduces the load bearing capacity to 1/4.

The max. deviation from the theoretical value is 10% and is, therefore, within the range of precision which can be attained with this device.

4.4 Material Influence and E Modulus Test

This test examines the resistance to buckling offered by materials such as aluminium, brass and copper. In addition, the E modulus is to be determined for an unknown material. Rod specimens S8 (aluminium), S9 (brass) and S10 (copper) are used for the comparison. They can be compared directly since they have the same dimensions.

Different materials

The modulus of elasticity is:

- Aluminium $E_{Al} = 70 \cdot 10^3 N/mm^2$
- Brass $E_{Ms} = 104 \cdot 10^3 N/mm^2$
- Copper $E_{Cu} = 125 \cdot 10^3 N/mm^2$

According to the ratio for critical buckling force (Euler formula) the buckling force is directly proportional to the size of the **E** modulus.

$$F_{krit} = \frac{\pi^2 \mathbf{E}\, I_y}{l^2}$$

or the ratio

$$F_{Al} : F_{Ms} : F_{Cu} = E_{Al} : E_{Ms} : E_{Cu}$$

The buckling forces are in a 1 to 1.48 to 1.78 ratio.

The specimens are inserted and subjected to loads in accordance with test 4.1.

A measurement of deformation is not necessary.

The maximum deflection of the specimens should not exceed 10 mm.

The following buckling forces have been measured for the various materials: (theoretical values in brackets)

- Aluminium F_k = 888 N (863 N)
- Brass F_k = 1055 N (1283 N)
- Copper F_k = 1400 N (1542 N)

This results in a 1 to 1.18 to 1.57 ratio. Whereas aluminium and copper demonstrate good correlation with the theory, the value for brass deviates greatly (-17%). Here, it must be noted that the E moduli indicated are only average values and that there are significant deviations in the dependency of the alloy composition, the heat treatment and the strain hardening.

Determining the E modulus

The unknown E modulus of a fiberglass rod (S11) is determined in this test.

The rod is clamped and loaded as described in test 4.1.

A buckling force of 1400 N was measured.

The E modulus being sought for can be calculated using the Euler formula.

$$E = \frac{F_k \ l^2}{I_y \ \pi^2} = \frac{1400 \cdot 600^2}{2083.3 \cdot \pi^2} = 24.5 \cdot 10^3 \text{ N/mm}^2$$

The E modulus of the fiberglass rod is only around 1/8 the value of steel ($210 \cdot 10^3$N/mm).

The following tests require the test rods in the WP 120.01 accessory set

4.5 Cross Section Influence Test

According to the Euler formula the buckling load of a rod is also dependent on the geometrical moment of inertia and thus from the shape of the rod cross section.

The geometrical moment of inertia is a measure of the ability of the specimen to withstand bending loads. The geometrical moment of inertia reflects both the size of the cross sectional area as well as its shape.

Influence of width

First, two rectangular cross sections with the same height but different width **b** (rod specimens SZ1 and SZ4) are compared. The geometric moment of inertia for a rectangular cross section is:

$$I_y = \frac{\mathbf{b} \cdot h^3}{12}$$

Applied in the Euler formula this results in:

$$F_{krit} = \frac{\pi^2 E \mathbf{b} h^3}{12 \, l^2}$$

The buckling load is also in linear dependence to the width **b**.

Clamp and subject the rod specimens to loads as described in test 4.1.

The measurement reveals for

- Width 25 mm: 1250 N
- Width 40 mm: 2050 N

The buckling forces are in a 1 to 1.64 ratio. The theoretical ratio is 40/25=1.6. The measuring results correlate very well with the theoretical values.

Influence of diameter

Two rod specimens (SZ6 and SZ7) with tubular cross sections are compared in this test. They have nearly the same cross sectional area and thus the same weight with different areas.

The geometrical moment of inertia for tubular cross sections is:

$$I_y = \frac{\mathbf{D}^4 - \mathbf{d}^4}{64}\pi$$

The fourth power of the diameter is included in the buckling force. The ratio for the diameters of the rod specimens is

$$\frac{I_{sz7}}{I_{sz6}} = \frac{20^4 - 17^4}{16^4 - 12^4} = 1.70$$

i.e., the thicker tube (dia. 20 mm) can support 1.7 times as much at the same weight than the tube with 16 mm dia.

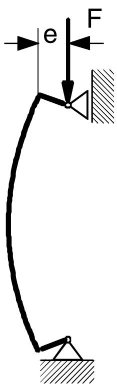
The rod specimen is clamped and subjected to loads as described in test 4.1.

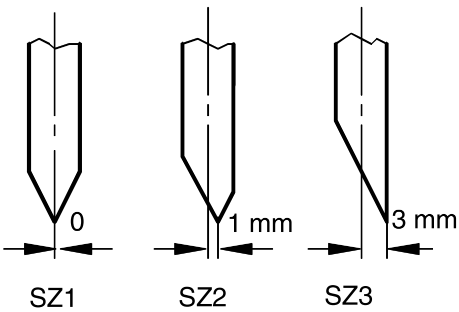
The measurement reveals for a

- 16 mm diameter: 335 N
- 20 mm diameter: 720 N

The buckling forces are in a 1 to 2.15 ratio. The theoretical ratio is 1.70. The large deviation can be traced to the greater influence of measuring inaccuracy.

4.6 Eccentric Force Application Test



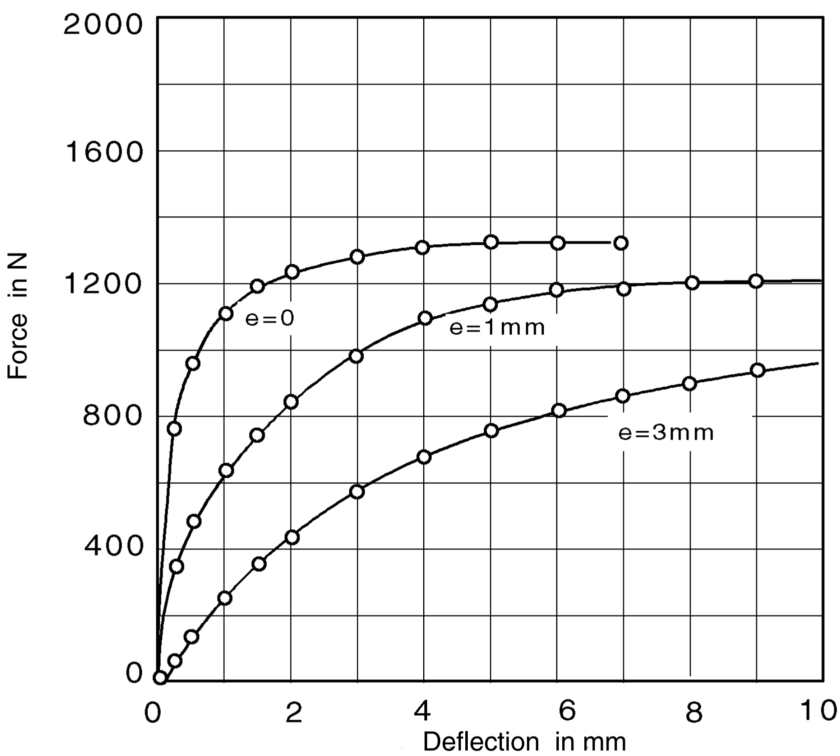


Knife edge bearing with eccentricity

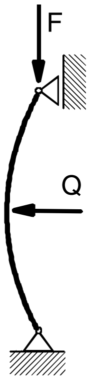
The influence of applied eccentric force can be tested with specimen sets SZ1, SZ2 and SZ3. Here, the rod deflects even at clearly lesser forces and the transition from a stable to an instable state tends to be continuous. Please refer to Fig. 3.5, Section 3.1.2.

The deformation curves for the three rod specimens are recorded a in test 4.1. SZ1 acts as a reference rod without eccentricity. All curves should be entered in a graph.

In the case of major eccentricity (SZ3 where e=3mm) the force does not attain a constant maximum value even at maximum deflection (10 mm). A buckling force can, therefore, not be defined here.

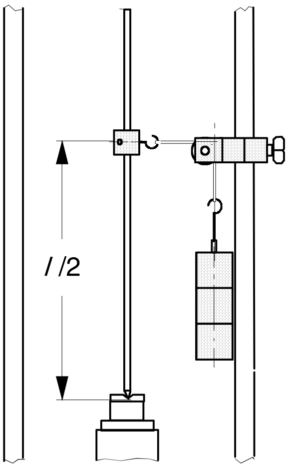


4.7 Lateral Load Test

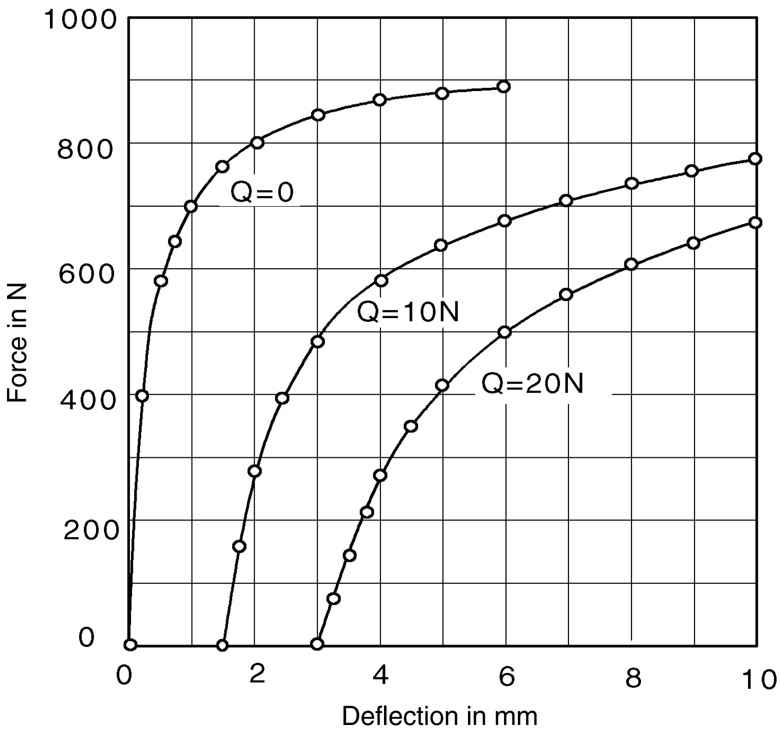


Similar to the influence of eccentricity, the influence of a lateral load can be studied in this test. According to Section 3.14 it results in similar buckling rod behavior.

The lateral load device must be attached for this test.



- SZ1 is used as a rod specimen
- The pulley is clamped to a guide column. the lateral load should act on the middle of the rod specimen
- The measuring gauge should be attached in the immediate vicinity of the lateral load device
- The clip is placed around the rod specimen and closed with a cotter pin
- The rope is fed over the pulley and the loading weights are suspended



- The deformation curves are recorded without lateral load and with 10 N and 20 N lateral loads. The results are entered in a single graph.

Just as in the case of eccentric loading, the transition from a stable to an instable state is continuous.

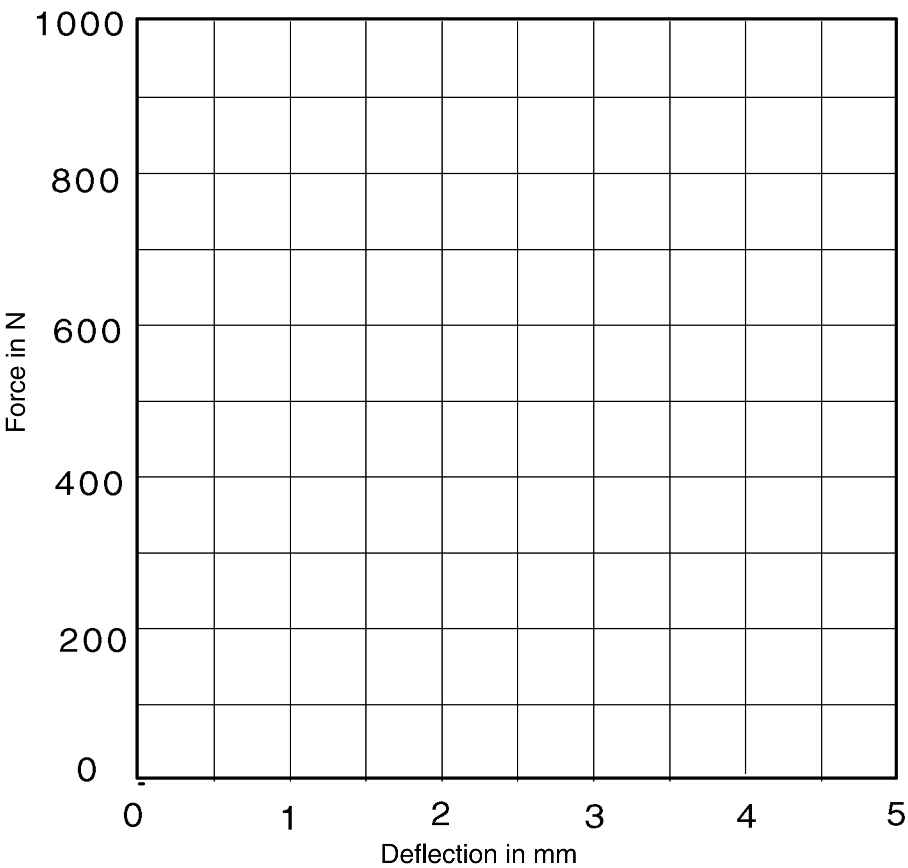
As opposed to eccentric loading, however, deflection is caused due to the curvature caused by the lateral load. This means that the deformation curves do not have the same origin in the zero point.

5. Appendix

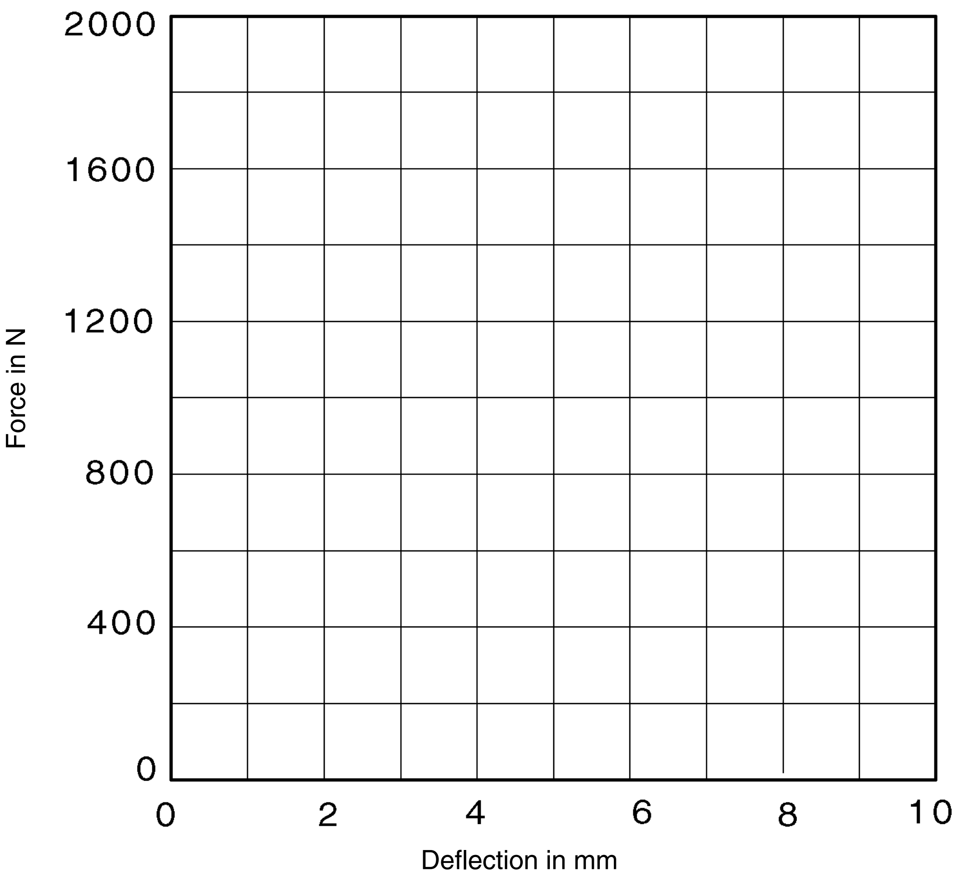
5.1 Work Sheets

Introduction Test
Rod: S2 steel
Length: 500 mm
Geometric moment of inertia: 106.6 mm^4
Modulus of elasticity: 210 000 N/mm

Deflection mm															
Force N															

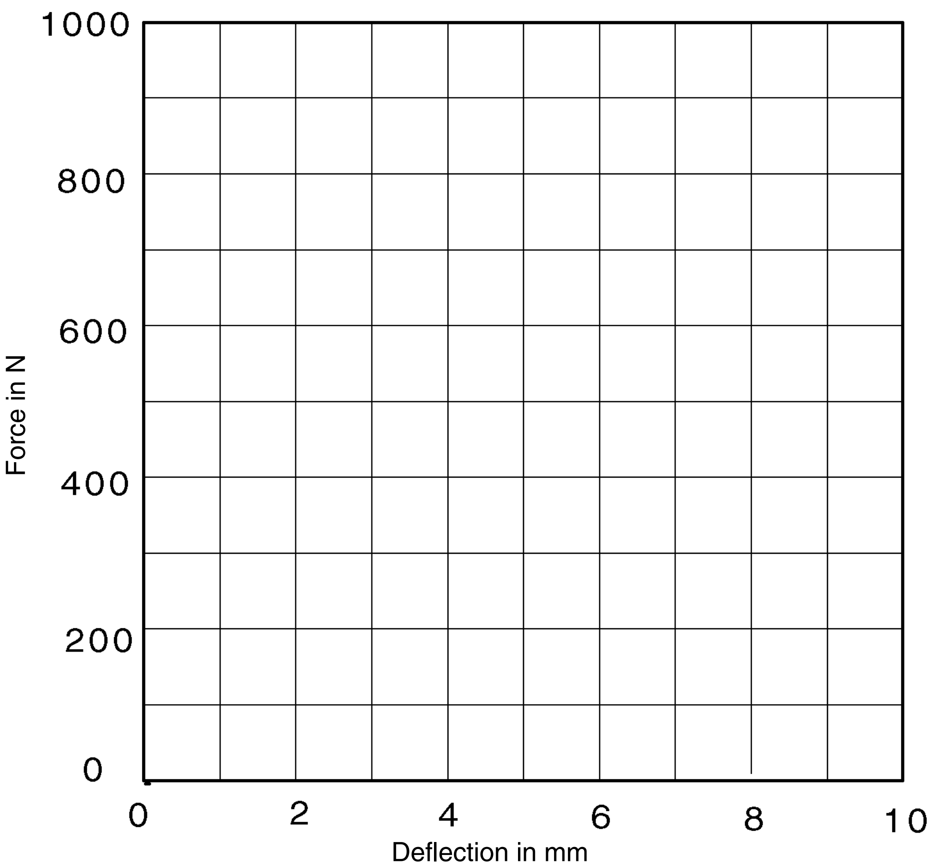


Eccentric Force Application Test Rods: SZ1, SZ2, SZ3, aluminium Length: 500 mm Geometric moment of inertia: 450 mm4 Modulus of elasticity: 70000 N/mm														
Deflection mm														
ForceSZ1,e=0, N														
Force SZ2,e=1,N														
Force SZ3,e=3,N														



Cross Load Test
Rods: SZ1, aluminium
Length: 500 mm
Geometric moment of inertia: 450 mm4
Modulus of elasticity: 70000 N/mm

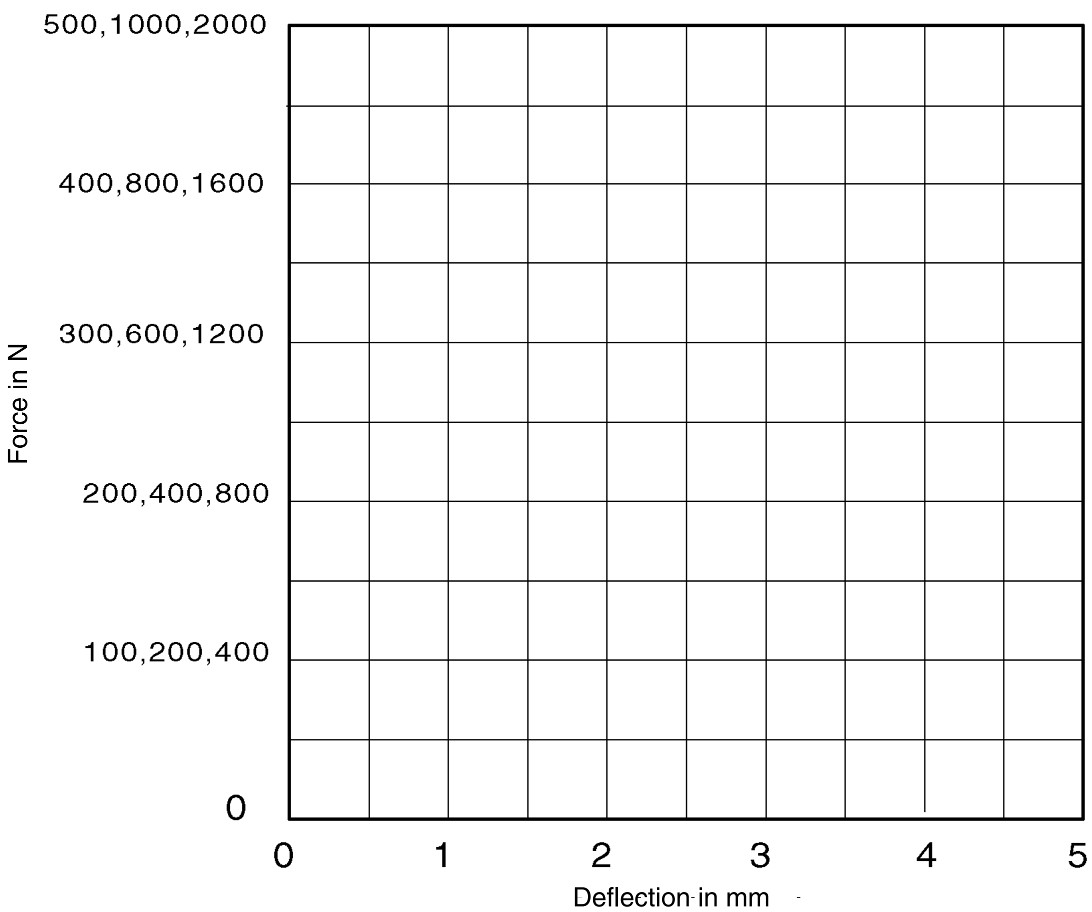
Deflection mm														
Force (Q=0) N														
Deflection mm														
Force(Q=10N) N														
Deflection mm														
Force(Q=20N) N														



Test

Rod:
Length:
Geometric moment of inertia:
Modulus of elasticity:

Deflection mm														
Force N														



5.2 Glossary of Formulas

Area

- Rectangle width b, height h

$$A = b \cdot h$$

- Box cross section, outer dimensions B,H, inner dimensions b,h

$$A = B \cdot H - b \cdot h$$

- Circle diameter D

$$A = \frac{D^2}{4} \pi$$

- Tube, outer diameter D, inner diameter d

$$A = \frac{D^2 - d^2}{4} \pi$$

Geometric moment of inertia

- Rectangle, width b, height h

$$I_y = \frac{b\,h^3}{12}$$

- Symmetrical box cross section, outer dimensions B, H, inner dimensions b,h

$$I_y = \frac{B\,H^3 - b\,h^3}{12}$$

- Circle, diameter D

$$I_y = \frac{D^4}{64} \pi$$

- Tube, outer diameter D, inner diameter d

$$I_y = \frac{D^4 - d^4}{64} \pi$$

- Inertial radius, geometric moment of inertia I_y, area A

$$i = \sqrt{I_y / A}$$

Slenderness (buckling), buckling length l_k, inertial radius i

$$\lambda = \frac{l_k}{i}$$

Compressive strain, compressive force F, area A

$$\sigma_P = \frac{F}{A}$$

Normal bending stress, bending moment M, geometric moment of inertia I_y, max. edge fiber distance z_{max}

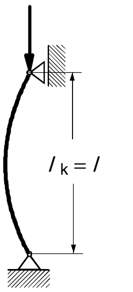
$$\sigma_M = \frac{M}{I_y} z_{max}$$

Critical slenderness, modulus of elasticity E, admissible compressive strain σ_p

$$\lambda_{krit} = \sqrt{\pi^2 E / \sigma_p}$$

Critical buckling force (Euler formula), modulus of elasticity E, geometric moment of inertia I_y, buckling length l_k

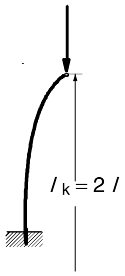
$$F_{krit} = \frac{\pi^2 E I_y}{l_k^2}$$



Mounting conditions, Euler cases

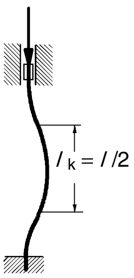
- Euler case 1, articulated/articulated

  $/_k = /$



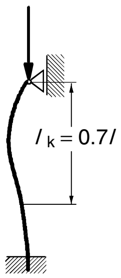
- Euler case 2, clamped/free

  $/_k = 2\ /$



- Euler case 3, clamped/clamped

  $/_k = / /2$



- Euler case 4, clamped/articulated

  $/_k = 0.7/$

5.3 Rod Specimen Table

Standard set WP 120				
No..:	Material	Diameter mm	Length mm	Mounting
S1	Tool steel 1.2842	20 x 4	350	knife-edge/knife-edge
S2	Tool steel 1.2842	20 x 4	500	knife-edge/knife-edge
S3	Tool steel 1.2842	20 x 4	600	knife-edge/knife-edge
S4	Tool steel 1.2842	20 x 4	650	knife-edge/knife-edge
S5	Tool steel 1.2842	20 x 4	700	knife-edge/knife-edge
S6	Tool steel 1.2842	20 x 4	650	clamped/knife-edge
S7	Tool steel 1.2842	20 x 4	650	clamped/clamped
S8	Alu. AlMgSi0.5 F22	25 x 6	600	knife-edge/knife-edge
S9	Brass CuZn40Pb2	25 x 6	600	knife-edge/knife-edge
S10	Copper E-Cu	25 x 6	600	knife-edge/knife-edge
S11	Fieberline	25 x 10	600	knife-edge/knife-edge

Accessories set WP 120.01				
No.:	Material	Diameter mm	Length mm	Mounting
SZ1	Alu. AlMgSi0.5 F22	25 x 6	500	knife-edge/knife-edge e=0mm
SZ2	Alu. AlMgSi0.5 F22	25 x 6	500	knife-edge/knife-edge e=1mm
SZ3	Alu. AlMgSi0.5 F22	25 x 6	500	knife-edge/knife-edge e=3mm
SZ4	Alu. AlMgSi0.5 F22	40 x 6	500	knife-edge/knife-edge
SZ5	Fieberline	25 x 10	700	knife-edge/knife-edge
SZ6	PVC	ø16 x 2	400	knife-edge/knife-edge
SZ7	PVC	ø20 x 1.5	400	knife-edge/knife-edge
SZ8	Alu. AlMgSi0.5 F22	20 x 10 x 2	700	knife-edge/knife-edge
SZ9	Alu. AlMgSi0.5 F22	ø15 x 2	700	knife-edge/knife-edge
SZ10	Alu. AlMgSi0.5 F22	ø14	700	knife-edge/knife-edge

5.4 Cross Section Value Table

Standard set WP 120				
No:	Cross section mm	Area A mm^2	Minimum geometric moment of inertia Iy mm^4	Length l mm
S1	20 x 4	80	106.6	350
S2	20 x 4	80	106.6	500
S3	20 x 4	80	106.6	600
S4	20 x 4	80	106.6	650
S5	20 x 4	80	106.6	700
S6	20 x 4	80	106.6	650
S7	20 x 4	80	106.6	650
S8	25 x 6	150	450.0	600
S9	25 x 6	150	450.0	600
S10	25 x 6	150	450.0	600
S11	25 x 10	250	2083.3	600

Accessories set WP 120.01				
Nr:	Cross section mm	Area A mm^2	Minimum geometric moment of inertia Iy mm^4	Length l mm
SZ1	25 x 6	150	450.0	500
SZ2	25 x 6	150	450.0	500
SZ3	25 x 6	150	450.0	500
SZ4	40 x 6	240	720.0	500
SZ5	25 x 10	250	2083.3	700
SZ6	ø16 x 2	88	2199.1	400
SZ7	ø20 x 1.5	87	3754.2	400
SZ8	20 x 10 x 2	104	1378.7	700
SZ9	ø15 x 2	82	1766.4	700
SZ10	ø14	154	1885.7	700

5.4 Material Values Table

Material	modulus of elasticity E 10^3 N/mm^2	Admissible strain on the proportionality limit N/mm^2
Tool steel 1.2842	210	300
Alu. AlMgSi0.5 F22	69 - 70	110
Brass CuZn40Pb2	104	150
Copper E-Cu	125	120
Fiberline	10 - 40	k.A.
PVC	1 - 3	k.A